從信號與系統到控制

單元：離散摺積-5

離散摺積計算-指數函數與步階函數

授課老師：連　豐　力

單元學習目標與大綱

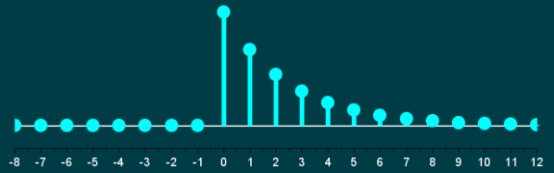
- 離散摺積計算範例
- 指數函數與步階函數
- 系統輸入輸出的關係

離散摺積計算-指數函數與步階函數

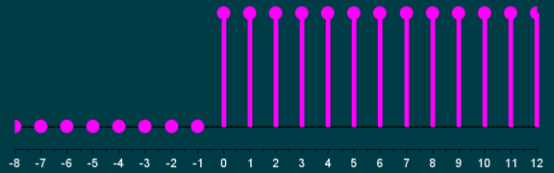
$x[n] * h[n] =$

$$= \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$x[n] = a^n u[n]$



$h[n] = u[n]$



離散摺積計算

- $n < 0$

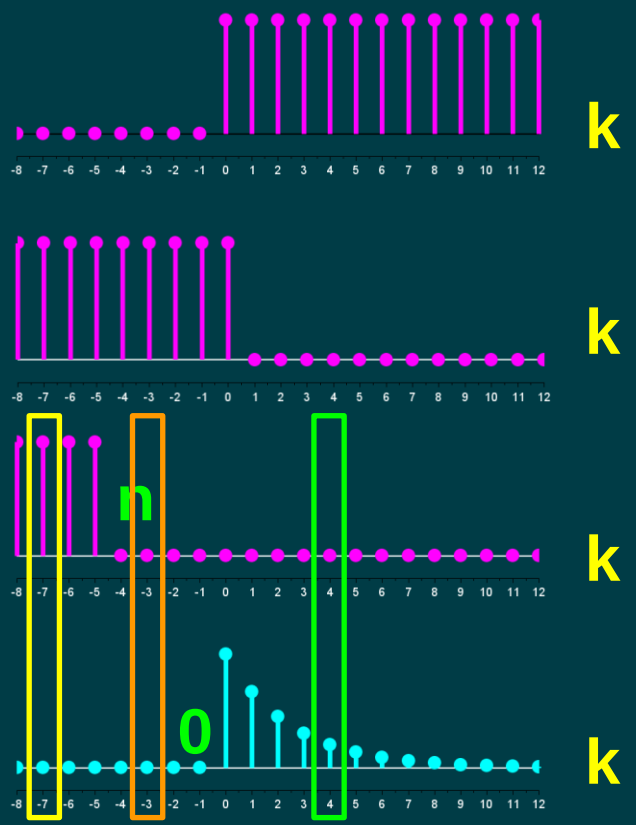
$x[k]\,h[n-k]$

$= 0$

$y[n]$

$= \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$

$= 0$



離散摺積計算

- $n > 0$

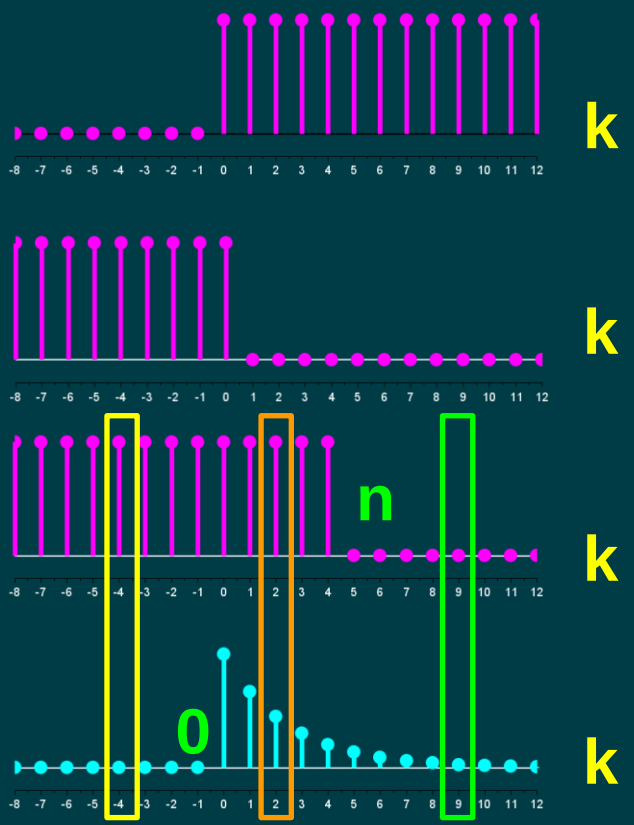
$x[k]\,h[n-k]$

$= a^k \qquad 0 < k < n$

$y[n]$

$= \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$

$= \sum_{k=0}^{n} a^k$

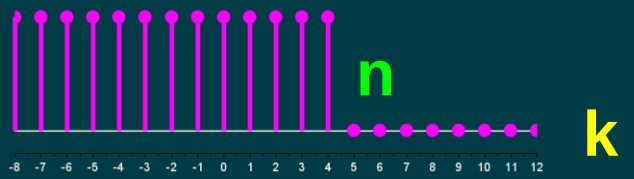


離散摺積計算

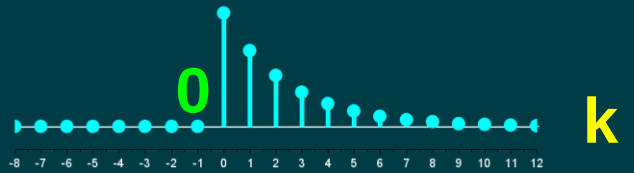
- n > 0

$$y[n] = \sum_{k=0}^{n} a^k = \frac{a^0 (1 - a^{n+1})}{1 - a}$$

h[n - k]



x[k]



離散摺積計算

$$y[n] = \frac{1 - a^{n+1}}{1 - a}$$

$a = 0.4$

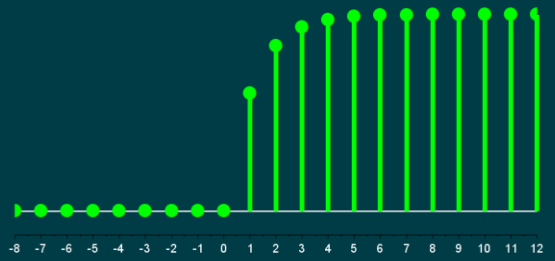
$$y[0] = \frac{1 - 0.4^{0+1}}{1 - 0.4} = 1$$

$$y[1] = \frac{1 - 0.4^{1+1}}{1 - 0.4} = 1.4$$

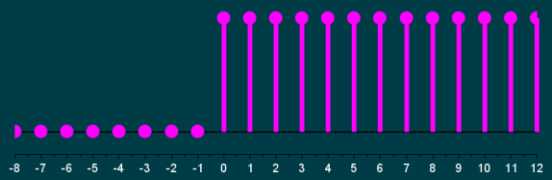
$$y[2] = \frac{1 - 0.4^{2+1}}{1 - 0.4} = 1.56$$

$$y[\infty] = \frac{1 - 0.4^{\infty+1}}{1 - 0.4}$$

$= 1.666666....$



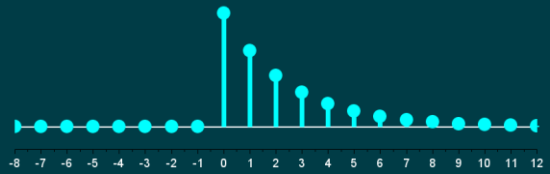
摺積計算與系統輸入輸出的關係

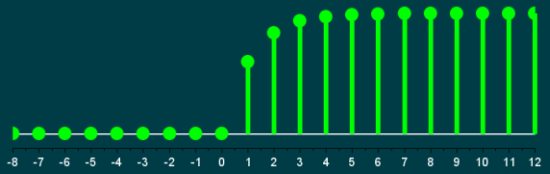


h[n]

= x[n] * h[n]

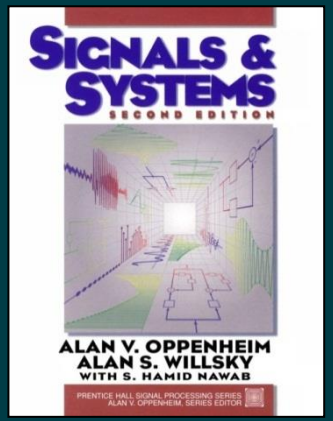
x[n] → LTI → y[n]





參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid,
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997



- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/