

從信號與系統到控制

單元：離散摺積-2

離散脈衝響應與摺積計算

授課老師：連 豐 力

單元學習目標與大綱

- 線性 非時變 系統 – 基本定義
- 離散 脈衝響應 – 輸入輸出操作
- 離散 摺積計算 – 公式推導

線性非時變系統 (LTI system)

- 線性 (Linear)

$$x[n] \longrightarrow \text{Linear} \longrightarrow y[n]$$

$$3x[n] \longrightarrow \text{Linear} \longrightarrow 3y[n]$$

$$x_1[n] + x_2[n] \longrightarrow \text{Linear} \longrightarrow y_1[n] + y_2[n]$$

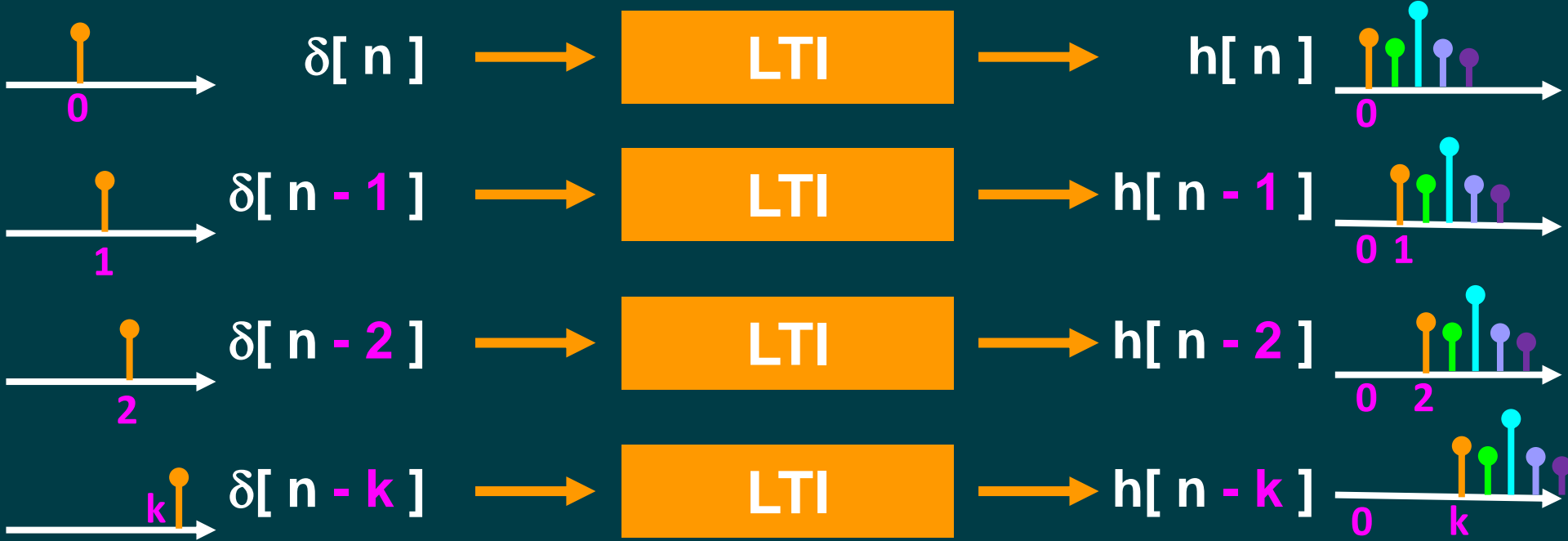
$$4x_1[n] - 7x_2[n] \longrightarrow \text{Linear} \longrightarrow 4y_1[n] - 7y_2[n]$$

線性非時變系統 (LTI system)

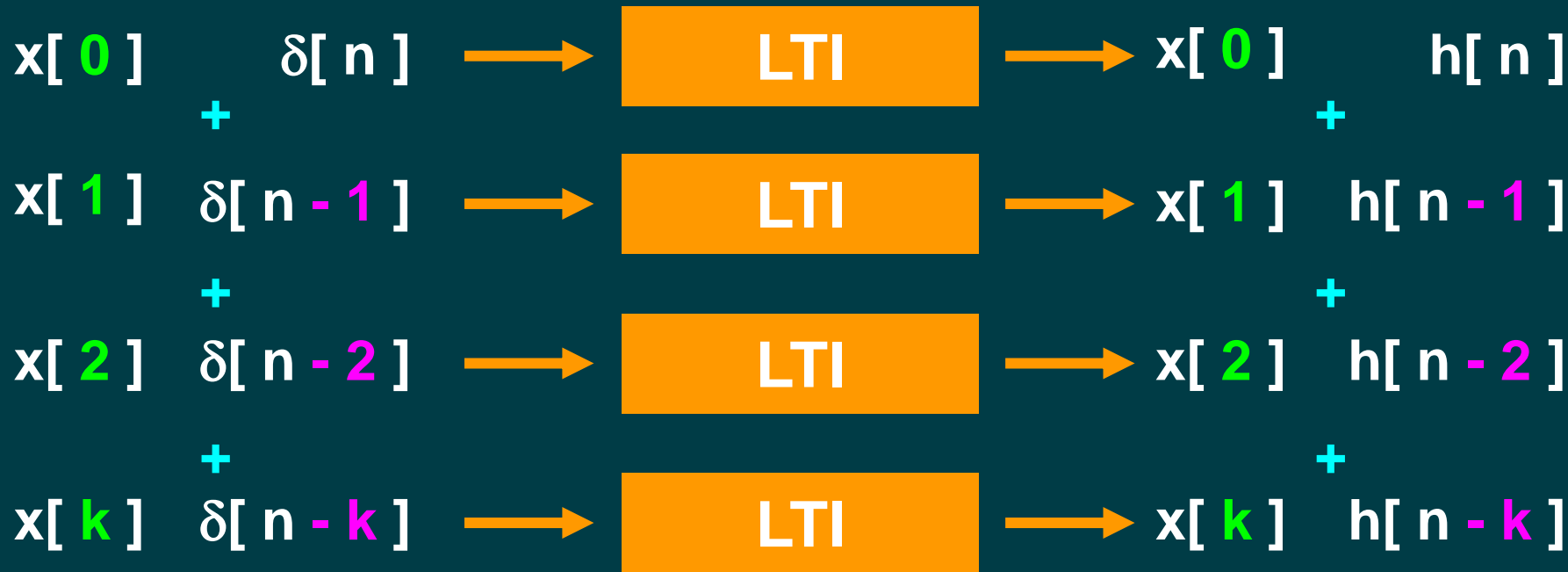
- 非時變 (Time-Invariant)



離散脈衝響應 (Impulse Response)



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$$\begin{aligned} & \dots + x[0] \delta[n] \\ & + x[1] \delta[n-1] \\ & + x[2] \delta[n-2] \\ & + \dots \\ & + x[k] \delta[n-k] + \dots \end{aligned}$$



$$\begin{aligned} & = \sum_{k=-\infty}^{+\infty} x[k] \delta[n-k] \\ & = x[n] \end{aligned}$$

$$\begin{aligned} & \dots + x[0] h[n] \\ & + x[1] h[n-1] \\ & + x[2] h[n-2] \\ & + \dots \\ & + x[k] h[n-k] \\ & + \dots \end{aligned}$$

$$\begin{aligned} & = \sum_{k=-\infty}^{+\infty} x[k] h[n-k] \\ & = y[n] \end{aligned}$$

離散摺積計算 (Convolution Sum)

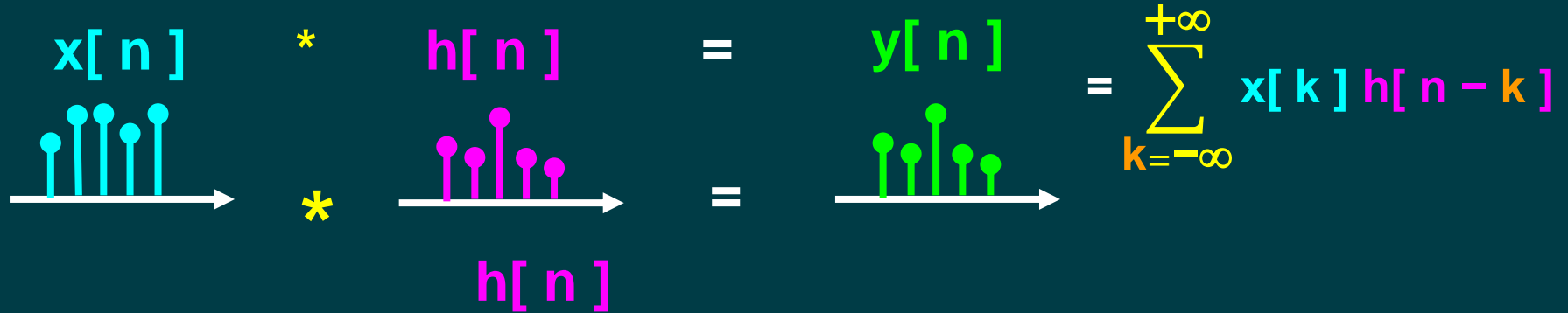


$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] h[n-k]$$

$$= x[n] * h[n]$$

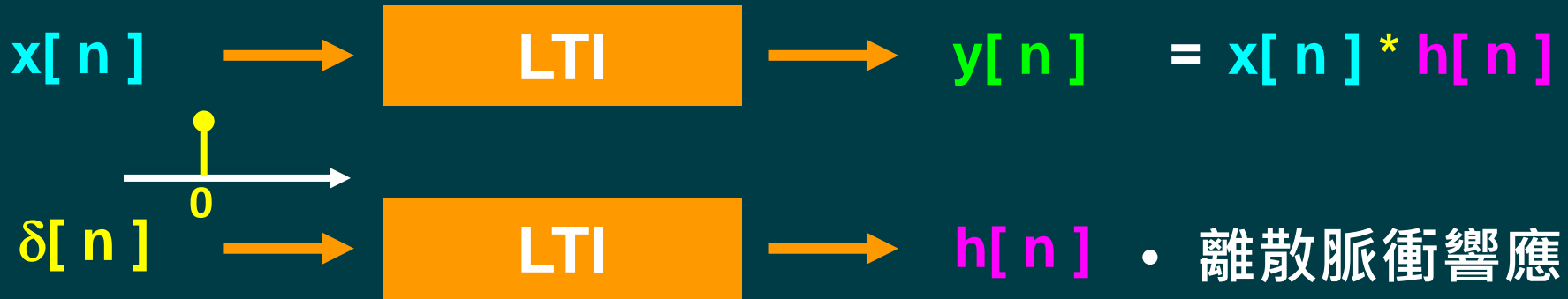
- 離散摺積計算 (DT Convolution Sum)

離散摺積計算 (Convolution Sum)



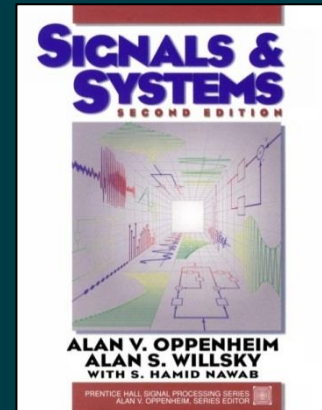
The diagram illustrates the discrete convolution sum. On the left, a cyan signal $x[n]$ is shown as a sequence of four vertical bars of varying heights on a horizontal axis. This is followed by a yellow asterisk $*$ and a magenta signal $h[n]$, also shown as a sequence of four vertical bars. A second yellow asterisk $*$ is placed below the magenta signal. This is followed by an equals sign $=$ and a green signal $y[n]$, shown as a sequence of five vertical bars. To the right of the green signal is another equals sign $=$ followed by a mathematical summation formula: $\sum_{k=-\infty}^{+\infty} x[k] h[n-k]$. The summation limits are in orange, and the terms $x[k]$ and $h[n-k]$ are in cyan and magenta respectively.

$$x[n] * h[n] = y[n] = \sum_{k=-\infty}^{+\infty} x[k] h[n-k]$$



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid, **Signals & Systems**, Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>