

# 從信號與系統到控制

單元：離散基本信號-1  
三角函數 與 指數函數

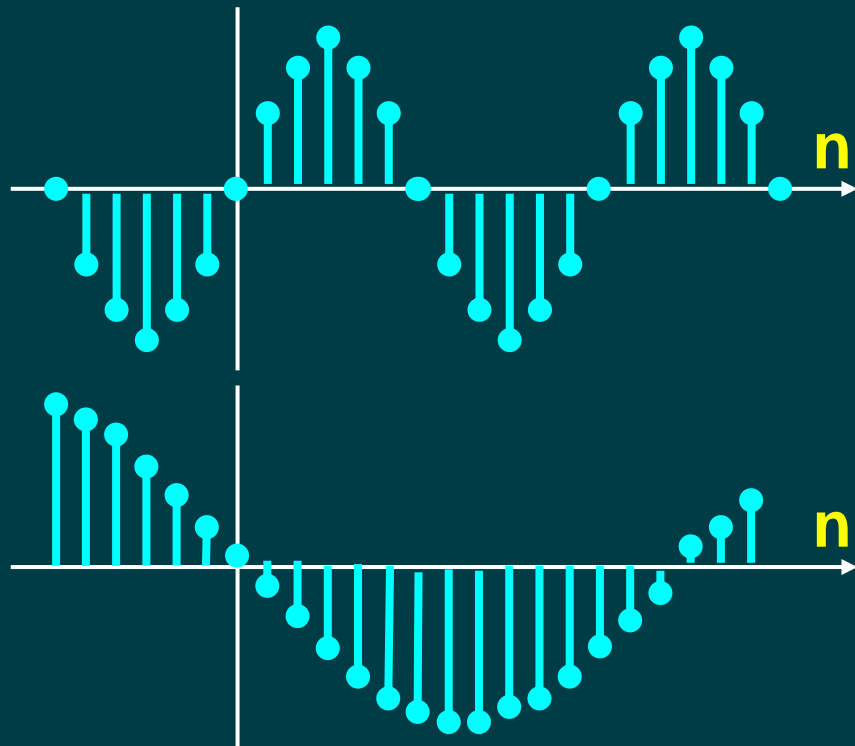
授課老師：連 豐 力

# 單元學習目標與大綱

- 四種 離散時間 的基本信號：
- 三角函數
- 指數函數
- 脈衝函數
- 步階函數

# 三角函數 - 簡單式

$$x[n] = \cos(\pi n / 6)$$



$$x[n] = \cos(n / 6)$$

# 三角函數 - 通式

$$x[n] = A \cos(\omega_0 n + \phi)$$

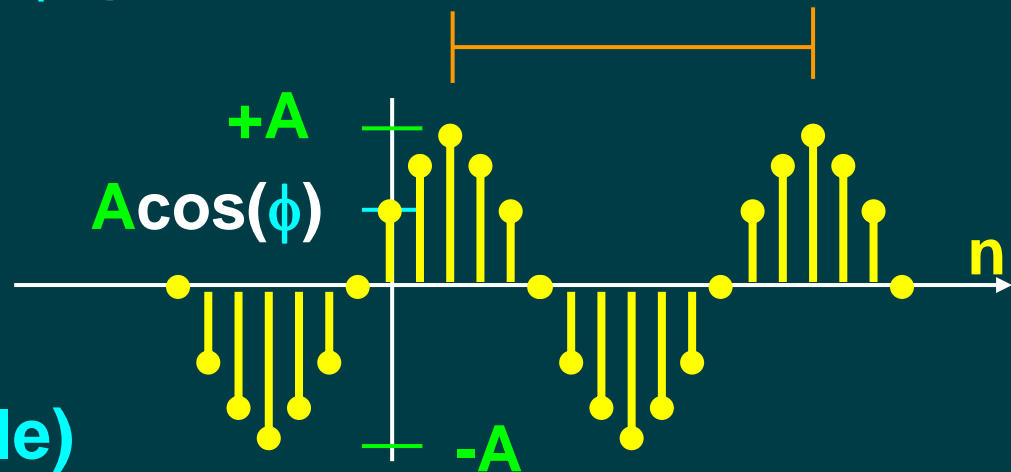
$$N_0 = 2\pi / \omega_0$$

$\omega_0$  頻率 (rad/sec)

$$= 2\pi / N_0$$

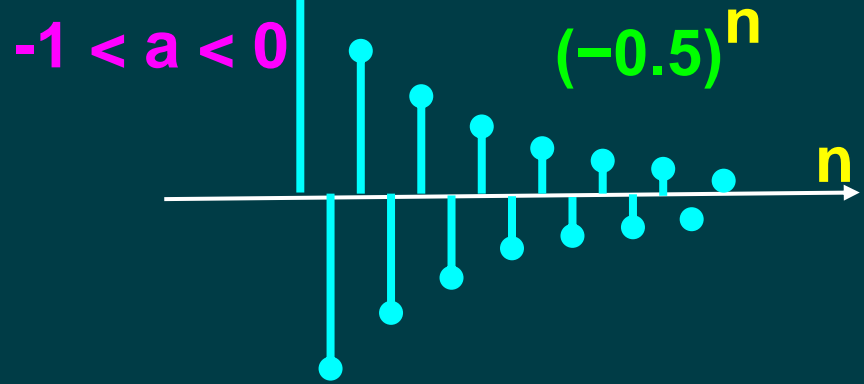
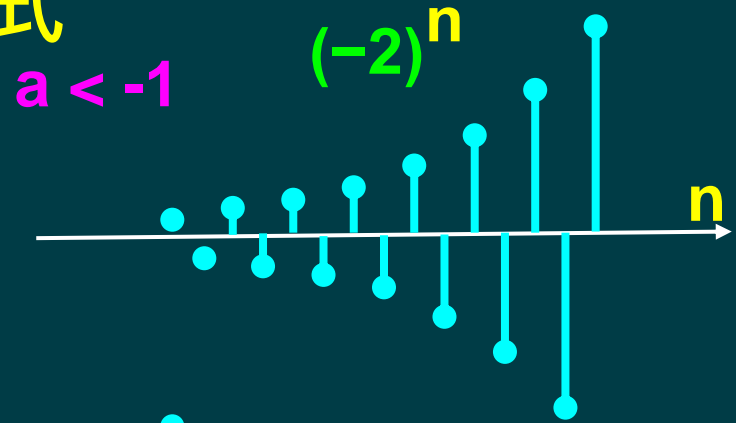
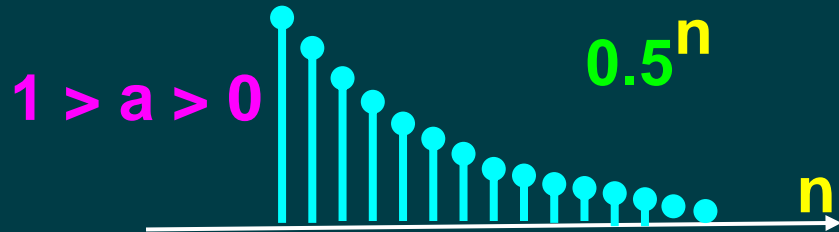
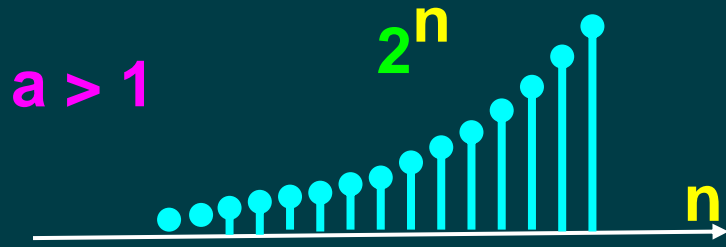
$\phi$  相角 (Phase Angle)

$A$  強度 (Magnitude)



# 指數函數 - 簡單式

$$x[n] = a^n$$



# 指數函數 - 通式 - 純虛數

$$x[n] = a^n = (e^{j\omega_0})^n = e^{j\omega_0 n}$$

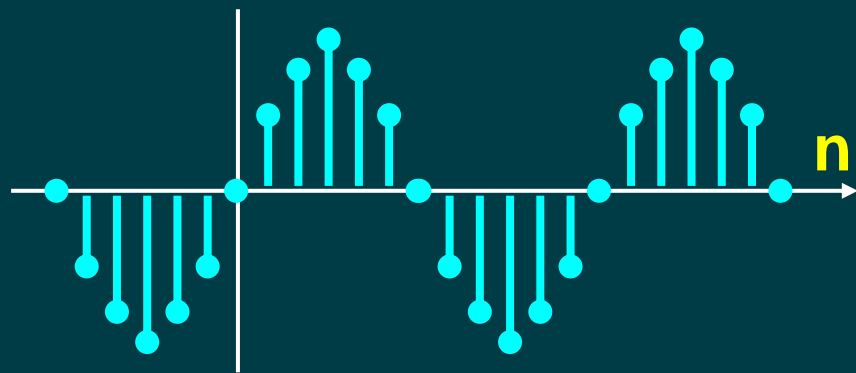
$$e^{js} = \cos(s) + j \sin(s)$$

$$a = e^{j\omega_0} = \cos(\omega_0 n) + j \sin(\omega_0 n)$$

$$\text{Real}\{x[n]\} = \cos(\omega_0 n)$$

$$\text{Imag}\{x[n]\} = \sin(\omega_0 n)$$

⇒  $x[n]$  是三角函數型的信號



# 指數函數 - 通式 - 複數

$$x[n] = C a^n = |C| e^{j\theta} (|a| e^{j\omega})^n$$

$$C = |C| e^{j\theta} \quad \text{複數} = |C| e^{j\theta} (|a|)^n (e^{j\omega})^n$$

$$a = |a| e^{j\omega} \quad \text{複數} = |C| (|a|)^n e^{j\theta} e^{j\omega n}$$

$$= |C| |a|^n e^{j(\theta + \omega n)}$$

$$= |C| |a|^n \cos(\theta + \omega_0 n)$$

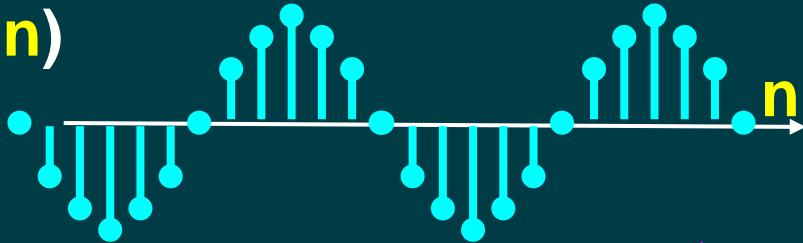
$$+ |C| |a|^n j \sin(\theta + \omega_0 n)$$

$$e^{js} = \cos(s) + j \sin(s)$$

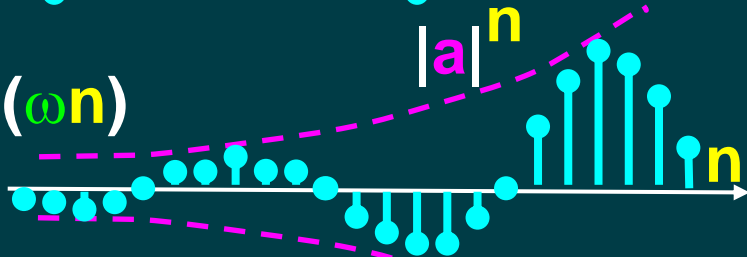
# 指數函數 - 通式 - 複數

$$x[n] = |C| |a|^n \cos(\theta + \omega_0 n) + j |C| |a|^n \sin(\theta + \omega_0 n)$$

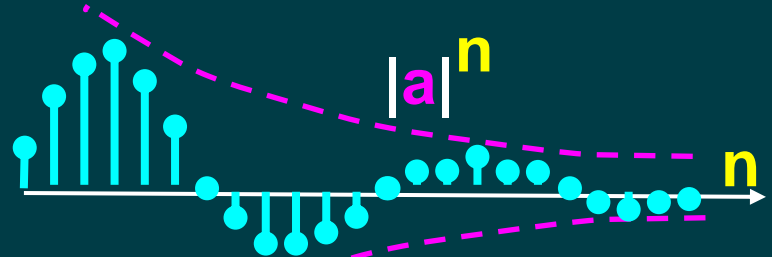
$\cos(\omega n)$



$|a|^n \cos(\omega n)$



$|a| > 1$

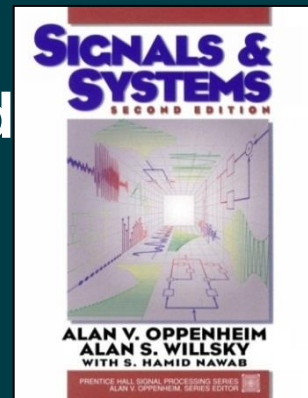


$|a| < 1$



# 參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid  
**Signals & Systems**,  
Prentice Hall, 2nd Edition, 1997



- **SciLab:**  
Open source software for numerical computation  
<http://www.scilab.org/>