從信號與系統到控制

單元：連續基本信號-2

脈衝函數 與 步階函數

授課老師：連　豐　力

單元學習目標與大綱

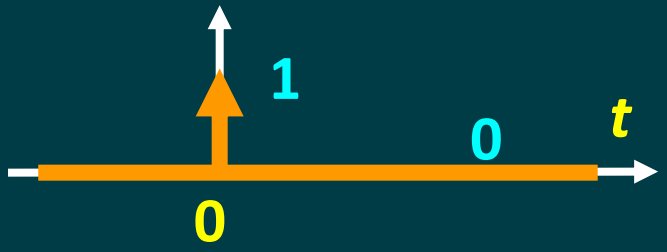
- 四種 連續時間 的 基本信號：

- 三角函數
- 指數函數
- 脈衝函數
- 步階函數

單位脈衝函數 (Unit Impulse)

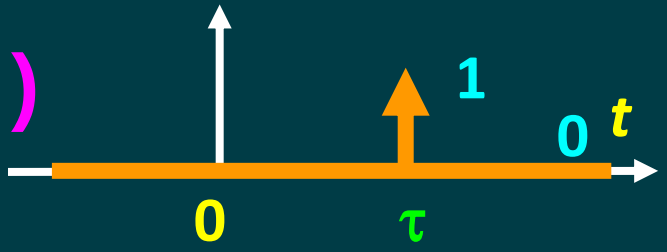
- 單位脈衝函數

$\delta(t)$



- 在時間軸平移的單位脈衝函數

$\delta(t-\tau)$

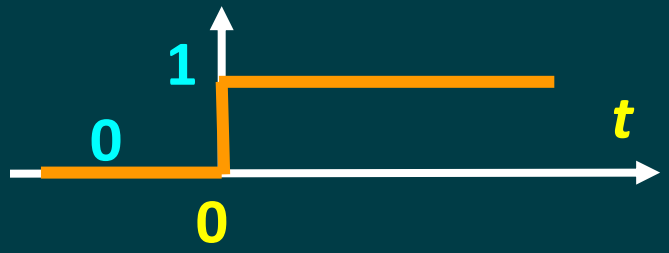


單位步階函數 (Unit Step)

- 單位步階函數

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

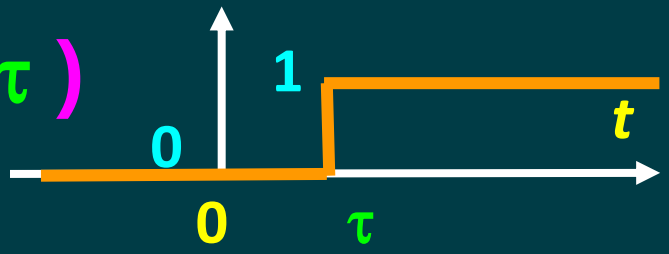
u(t)



- 在時間軸平移的單位步階函數

$$u(t-\tau) = \begin{cases} 0, & t < \tau \\ 1, & t > \tau \end{cases}$$

u(t - τ)



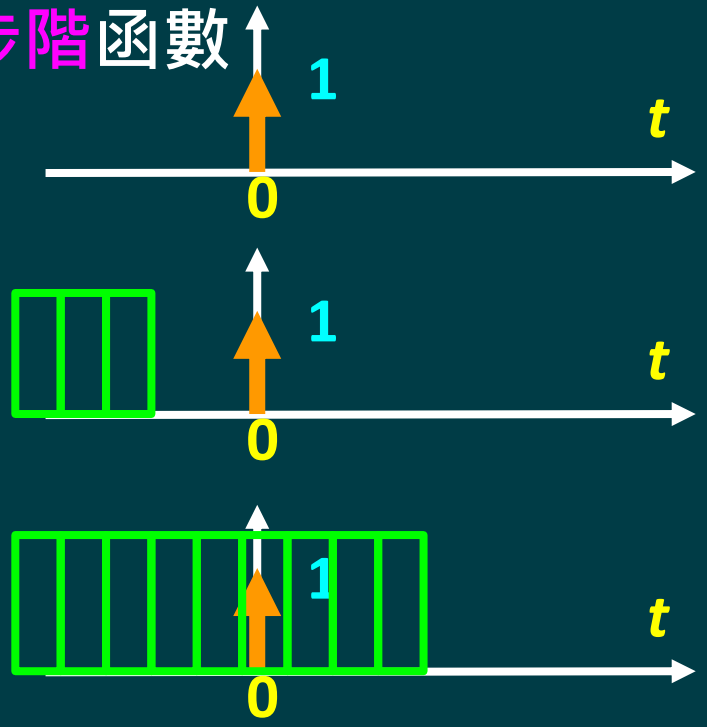
脈衝函數與步階函數的關係

- 單位脈衝函數的積分　是單位步階函數

$$\int_{-\infty}^{t} \delta(s)\, ds = u(t)$$

$t < 0 \quad = \quad 0$

$t > 0 \quad = \quad 1$

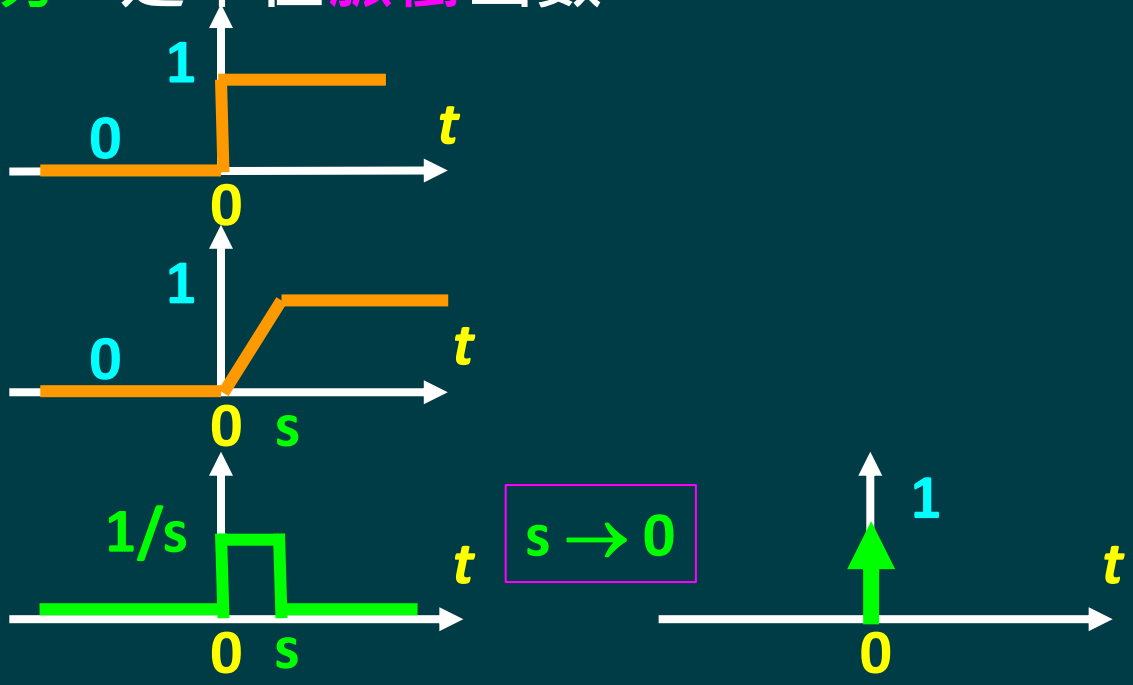


脈衝函數與步階函數的關係

- 單位步階函數的微分　是單位脈衝函數

$$\frac{d}{dt}\,u(t) = \delta(t)$$

$$\frac{d}{dt}\,u_s(t)$$



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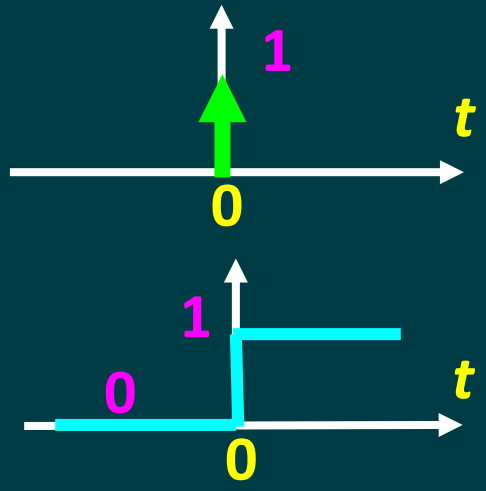
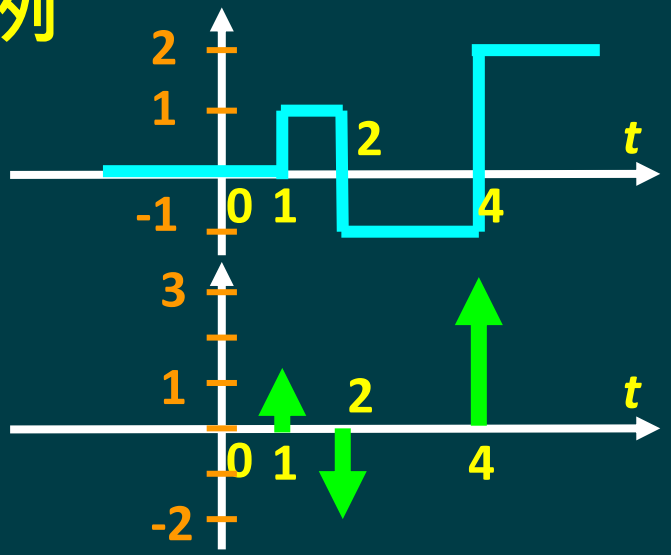


- 單位步階函數的微分 是單位脈衝函數

範例

$x(t)$

$\frac{d}{dt}\,x(t)$



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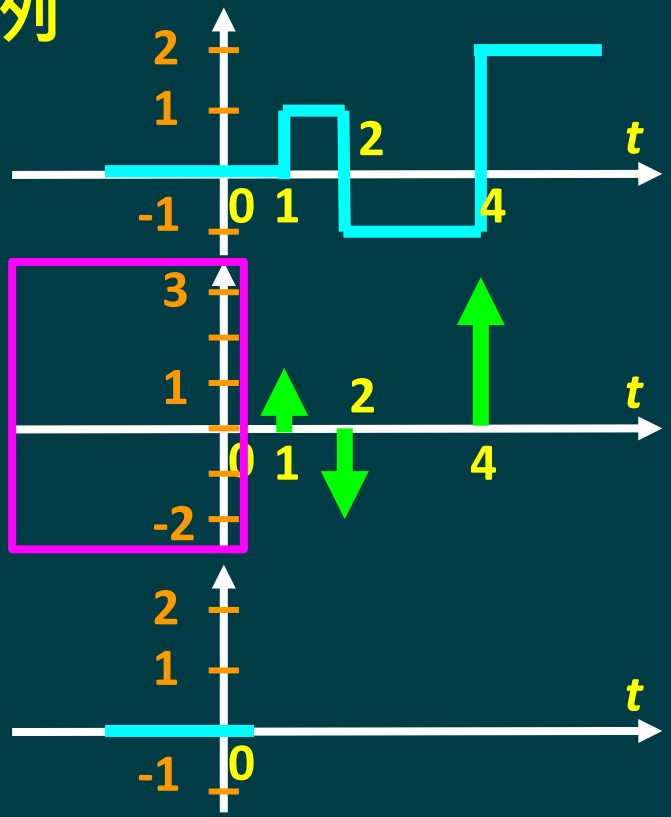
$\int_{-\infty}^{t} ds$



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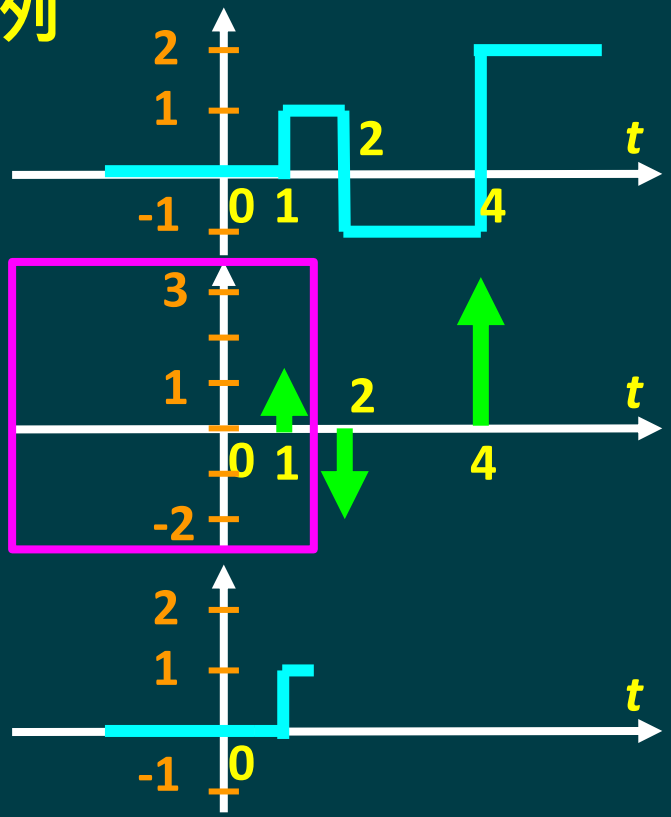

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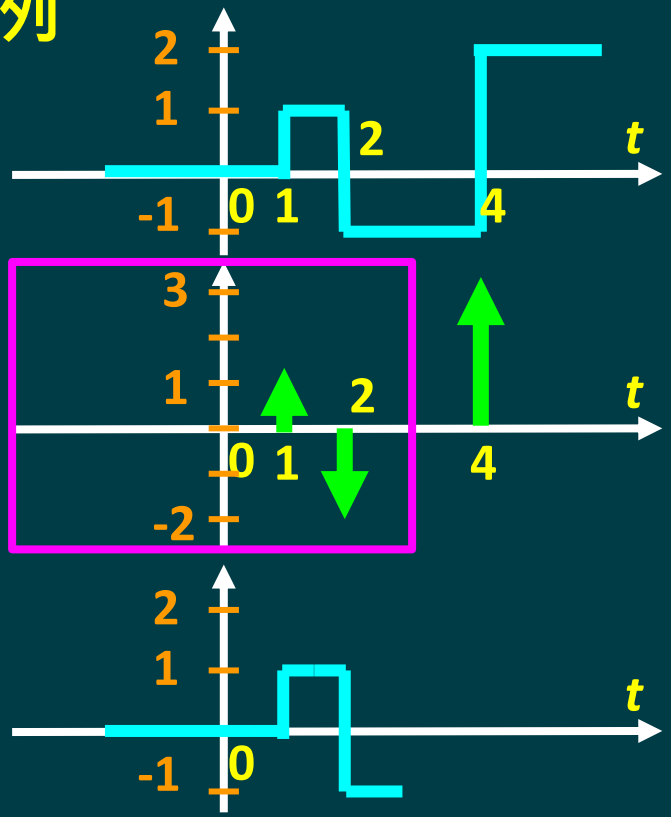

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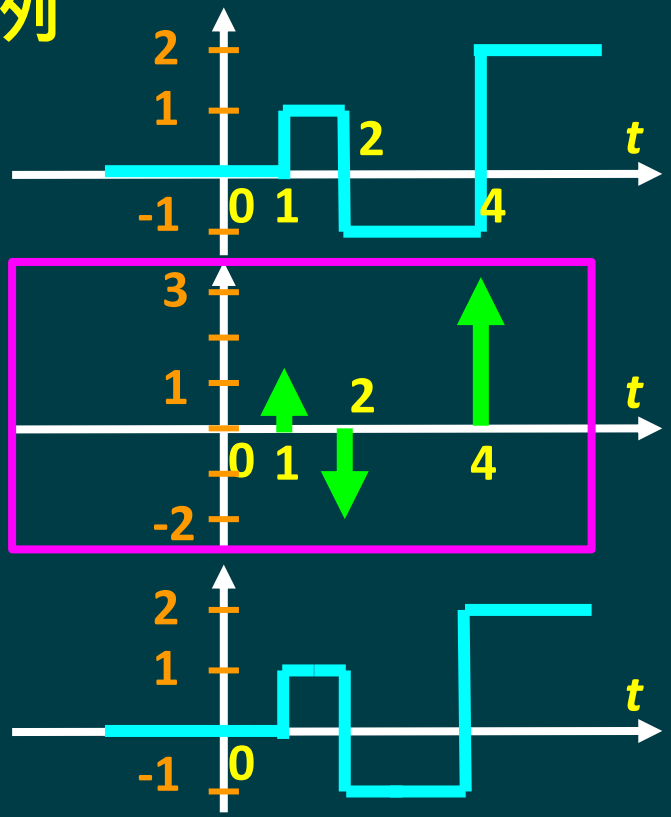

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脈衝函數 與 步階函數

- 單位脈衝函數 ，它的積分 是 單位步階函數

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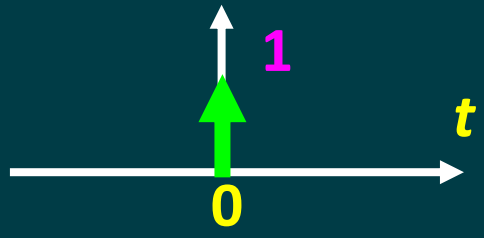

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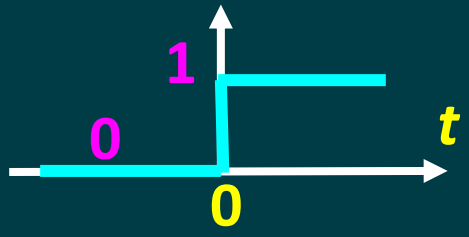
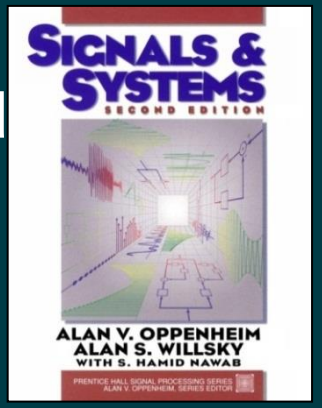


- 單位步階函數 ，它的微分 是 單位脈衝函數

參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/