Exams

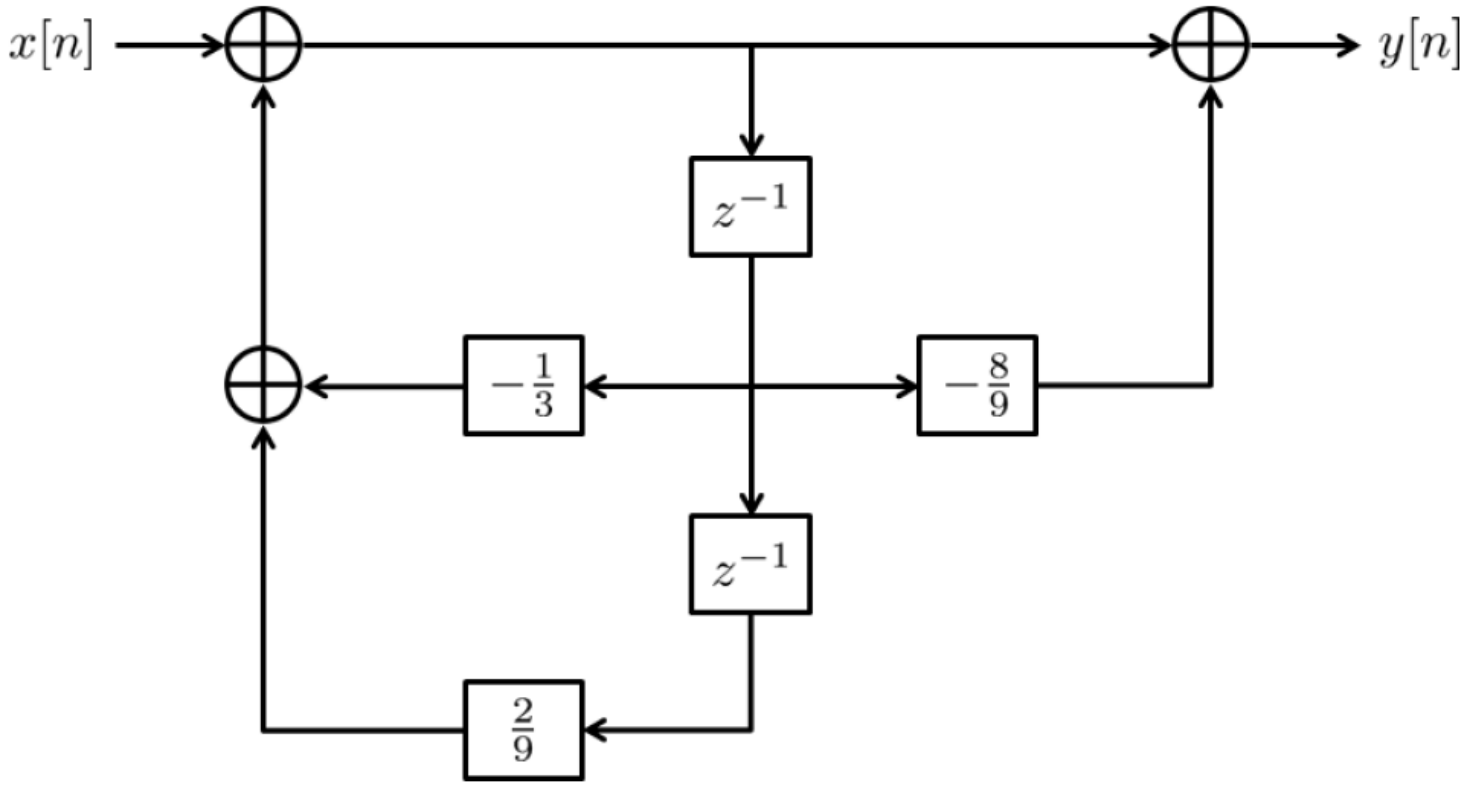
12. (8%) Given an LTI system with

$$h[n] = \begin{cases} a^n, & 0 \le n \le N-1,\ 0 < a < 1 \\ 0, & \text{else.} \end{cases}$$

(a) (5%) Find $H(z)$, and draw its pole-zero plot and ROC.

(b) (3%) What's the system function $H^{-1}(z)$ for the inverse system of it? Draw its pole-zero plot.

13. (6%) Given a causal LTI system represented by the following block diagram:



(a) (3%) Write down the difference equation relating $x[n]$ and $y[n]$.

(b) (3%) Write down $H(z)$ and verify whether it is stable.

4. (13%) Consider a discrete-time right sided LTI system with transfer function

$$H(z)=\frac{1-a^{*}z}{z-a},\ |a|<1.$$

(a) (3%) Sketch a pole-zero plot for this system in the z-plane.

(b) (3%) Use the graphical method to show that the magnitude response of the system is unity for all frequencies.

(c) (3%) Use the graphical method to sketch the phase response of the system for $a=1/2$.

(d) (4%) Use the result from (b) to prove that any system with a transfer function of the form

$$H(z)=\prod_{k=1}^{K}\frac{1-a_k^{*}z}{z-a_k},\ |a_k|<1,$$

corresponds to a stable and causal all-pass system.

9. (15%) Let an LTI system have the output equal to $y[n] = \beta\,(1/2)^n u[n] + 10(1/3)^n u[n]$ when the input $x[n] = (1/6)^n u[n]$, where β is a real number. Moreover, the output becomes $y[n] = 7(-1)^n/4$ when the input $x[n]$ is set to $(-1)^n$.

   (a) (3%) Find the transfer function H(z) of the LTI system. You must justify your answer.

   (b) (3%) Find the ROC of H(z). You must justify your answer.

   (c) (3%) Is the LTI system stable ? Why ?

   (d) (3%) Is the LTI system causal ? Why ?

   (e) (3%) Find the linear constant coefficient difference equation corresponding to the LTI system. You must justify your answer.

2. (10%) Using the appropriate properties of the z-transform, determine the sequence $x[n]$ for which the z-transform is the following respectively:

(a) $X(z) = \ln(1-2z),\ |z| < \frac{1}{2}$

(b) $X(z) = \ln(1-\frac{1}{2}z^{-1}),\ |z| > \frac{1}{2}$

Hint: $\frac{d}{dx}\ln(x) = \frac{1}{x}$.

Final Exam in 2011 – 6, Problem 10.34 (p. 805), HW9

6. [18] Consider that a causal linear time-invariant (LTI) system has its input $x[n]$ and output $y[n]$ given by the following difference equation:

$$y[n] - y[n-1] - y[n-2] = x[n-1] \qquad \text{(I)}$$

(a) Find the system function $H(z)$. Justify your answer. [3]
(b) Sketch the poles and zeros of $H(z)$. [3]
(c) Determine the region of convergence of $H(z)$. [3]
(d) Find the unit impulse response of the LTI system. Justify your answer. [3]
(e) Is the system stable? Using Part (c) to justify your answer. [3]
(f) Find a stable but noncausal LTI system whose unit impulse response satisfies the same difference equation given by (I). Justify your answer. [3]

10.34. A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

(a) Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.
(b) Find the unit sample response of the system.
(c) You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation.