- The z-Transform
- The Region of Convergence for z-Transforms
- The Inverse z-Transform
- Geometric Evaluation of the Fourier Transform
- Properties of the z-Transform
- Some Common z-Transform Pairs
- Analysis & Characterization of LTI Systems Using the z-Transforms
- System Function Algebra and Block Diagram Representations
- The Unilateral z-Transform

# Problem 10.13: Property, Difference, Summation

**10.13.** Consider the rectangular signal

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases}.$$

Let

$$g[n] = x[n] - x[n-1].$$

**(a)** Find the signal $g[n]$ and directly evaluate its $z$-transform.

**(b)** Noting that

$$x[n] = \sum_{k=-\infty}^{n} g[k],$$

use Table 10.1 to determine the $z$-transform of $x[n]$.

# Problem 10.14: Property, Convolution, IVT

**10.14.** Consider the triangular signal

$$g[n] = \begin{cases} n-1, & 2 \leq n \leq 7 \\ 13-n, & 8 \leq n \leq 12 \\ 0, & \text{otherwise} \end{cases}.$$

**(a)** Determine the value of $n_0$ such that

$$g[n] = x[n] * x[n-n_0],$$

where $x[n]$ is the rectangular signal considered in Problem 10.13.

**(b)** Use the convolution and shift properties in conjunction with $X(z)$ found in Problem 10.13 to determine $G(z)$. Verify that your answer satisfies the initial-value theorem.

# Problem 10.16: Stable, Causal, Poles

**10.16.** Consider the following system functions for stable LTI systems. Without utilizing the inverse $z$-transform, determine in each case whether or not the corresponding system is causal.

**(a)** $$\frac{1 - \frac{4}{3}z^{-1} + \frac{1}{2}z^{-2}}{z^{-1}(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

**(b)** $$\frac{z - \frac{1}{2}}{z^2 + \frac{1}{2}z - \frac{3}{16}}$$

**(c)** $$\frac{z + 1}{z + \frac{4}{3} - \frac{1}{2}z^{-2} - \frac{2}{3}z^{-3}}$$

# Problem 10.34: DE->System Function, Unit Response, Stable



**10.34.** A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

**(a)** Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.

**(b)** Find the unit sample response of the system.

**(c)** You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation.

# Problem 10.18: Block diagram, Difference Equation, Stable

**10.18.** Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related through the block diagram representation shown in Figure P10.18.

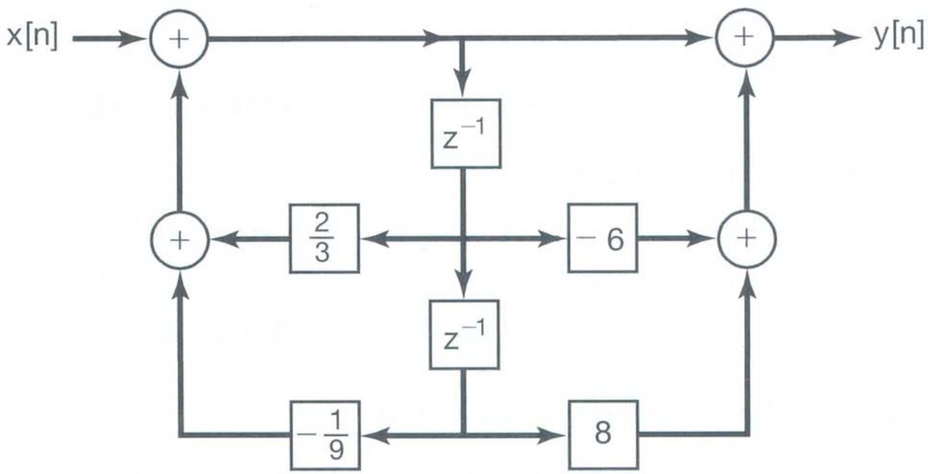


**Figure P10.18**

**(a)** Determine a difference equation relating $y[n]$ and $x[n]$.

**(b)** Is this system stable?

# Problem 10.19: UzT, ROC

**10.19.** Determine the unilateral $z$-transform of each of the following signals, and specify the corresponding regions of convergence:

**(a)** $x_1[n] = (\frac{1}{4})^n u[n+5]$

**(b)** $x_2[n] = \delta[n+3] + \delta[n] + 2^n u[-n]$

**(c)** $x_3[n] = (\frac{1}{2})^{|n|}$

# Problem 10.20: Difference equation, response, output



**10.20.** Consider a system whose input $x[n]$ and output $y[n]$ are related by

$$y[n-1] + 2y[n] = x[n].$$

**(a)** Determine the zero-input response of this system if $y[-1] = 2$.

**(b)** Determine the zero-state response of the system to the input $x[n] = (1/4)^n u[n]$.

**(c)** Determine the output of the system for $n \geq 0$ when $x[n] = (1/4)^n u[n]$ and $y[-1] = 2$.

# Problem 10.41: BzT and UzT

**10.41.** Consider the following two signals:

$$x_1[n] = \left(\frac{1}{2}\right)^{n+1} u[n+1],$$

$$x_2[n] = \left(\frac{1}{4}\right)^{n} u[n].$$

Let $\mathcal{X}_1(z)$ and $X_1(z)$ respectively be the unilateral and bilateral $z$-transforms of $x_1[n]$, and let $\mathcal{X}_2(z)$ and $X_2(z)$ respectively be the unilateral and bilateral $z$-transforms of $x_2[n]$.

**(a)** Take the inverse bilateral $z$-transform of $X_1(z)X_2(z)$ to determine $g[n] = x_1[n] * x_2[n]$.

**(b)** Take the inverse unilateral $z$-transform of $\mathcal{X}_1(z)\mathcal{X}_2(z)$ to obtain a signal $q[n]$ for $n \geq 0$. Observe that $q[n]$ and $g[n]$ are not identical for $n \geq 0$.