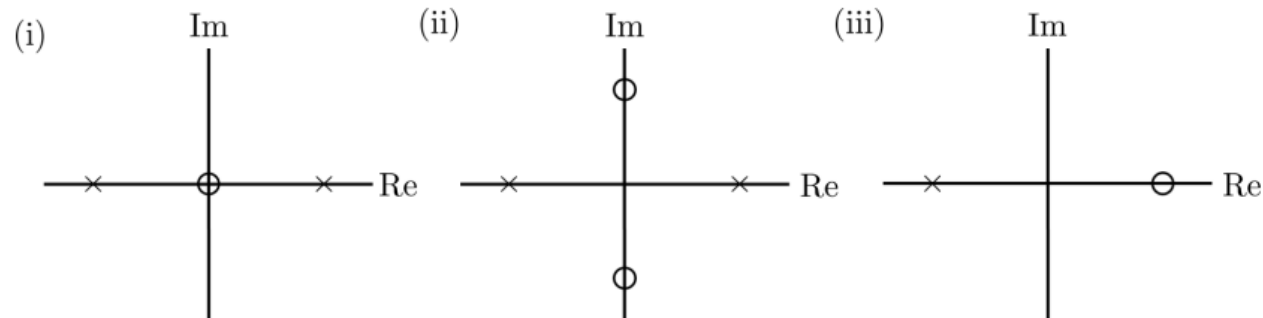


Exams

9. (13%) Let $X(s)$ be the Laplace transform of $x(t)$.

(a) (4%) Prove that if $x(t)$ is even, then $X(s) = X(-s)$. If $x(t)$ is odd, then $X(s) = -X(-s)$.

(b) (9%) Considering (a), determine which, if any, of the pole-zero plots below may correspond to an $x(t)$ which is even, and if yes, write down the required ROC corresponding to an even $x(t)$. Explain why for those cannot.



10. (8%) Given an LTI system $H(s)$, if the input of the system is

$$x(t) = e^{-3t}u(t),$$

then the output is

$$y(t) = [e^{-t} - e^{-2t}]u(t).$$

- (a) (3%) Find $H(s)$ and draw the pole-zero plot.
- (b) (3%) Find the ROC for $H(s)$ and explain your result.
- (c) (2%) Is $H(s)$ causal? Why?

3. (10%) Consider a continuous-time system with the transfer function

$$H(s) = \frac{1}{(s+1)(s^2 + s + 1)}.$$

- (a) (2%) Find the poles and zeros and sketch the magnitude response of this system. Determine whether the system is low pass or high pass, and find the cutoff frequency (the value of ω for which $|H(j\omega)| = 1/\sqrt{2}$).
- (b) (2%) Sketch a block diagram of the system in the direct form.
- (c) (3%) Perform the transformation of variables in which s is replaced by $1/s$ in $H(s)$. Repeat Part (a) for the transformed system.
- (d) (3%) Find the transformation that converts $H(s)$ to a high-pass system with cutoff frequency $\omega = 100$.

7. (15%) Let the continuous-time signal $x(t) = \exp\{-4t\}u(t)$ be input to an LTI system and the output $y(t)$ be given by $y(t) = (\exp\{-2t\} - \exp\{-3t\})u(t)$.
- (a) (3%) Find the transfer function $H(s)$ of the LTI system. You must justify your answer.
 - (b) (3%) Find the region of convergence (ROC) of $H(s)$. You must justify your answer.
 - (c) (3%) Is the LTI system causal ? Why ?
 - (d) (3%) Is the LTI system stable ? Why ?
 - (e) (3%) Find the linear constant coefficient differential equation corresponding to the LTI system. You must justify your answer.

8. (10%) Let the linear constant coefficient differential equation of an LTI system be given by

$$\frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 5y(t) = x(t).$$

Assume that the input $x(t) = 3u(t)$ and the initial conditions are given as follows:

$$y'(0^-) = 4, y(0^-) = -6.$$

- (a) (3%) Find the unilateral Laplace transform of $y(t)$. You must justify your answer.
- (b) (3%) Find the output $y(t)$ if the LTI system is causal. You must justify your answer.
- (c) (4%) Is the LTI system stable? Why?

3. (10%) The system with impulse response $h(t)$ is causal and stable and has a rational system function $H(s)$. Identify the conditions on the system function $H(s)$ so that each of the following systems with impulse response $g(t)$ is stable and causal:

(a) $g(t) = \frac{d}{dt}h(t)$

(b) $g(t) = \int_{-\infty}^t h(\tau) d\tau$

4. (10%) Determine the overall system function $H(s)$ for the following system:



$$\frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 5y(t) = x(t).$$

Assume that the input $x(t) = 3u(t)$ and the initial conditions are given as follows:

$$y'(0^-) = 4, y(0^-) = -6.$$

- (3%) Find the unilateral Laplace transform of $y(t)$. You must justify your answer.
- (3%) Find the output $y(t)$ if the LTI system is causal. You must justify your answer.
- (4%) Is the LTI system stable? Why?

3. [18] Suppose the system function of a system is

$$H(s) = \frac{10(1-s)}{(1+s)(10+s)}$$

- (a) Draw the block diagram of the system in direct, cascade, and parallel forms. [6]
- (b) Sketch the Bode plot for $H(j\omega)$. [6]
- (c) Use pole-zero plot to determine the magnitude and phase of $H(j\omega)$ graphically. [6]