- The Laplace Transform
- The Region of Convergence (ROC) for Laplace Transforms
- The Inverse Laplace Transform
- Geometric Evaluation of the Fourier Transform
- Properties of the Laplace Transform
- Some Laplace Transform Pairs
- Analysis & Characterization of LTI Systems Using the Laplace Transform
- System Function Algebra and Block Diagram Representations
- The Unilateral Laplace Transform

Problem 9.16: LT for DE and poles, stable

9.16. A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t).$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t),$$

how many poles does $G(s)$ have?

(b) For what real values of the parameter α is S guaranteed to be stable?

Problem 9.17: Block Diagram and DE

9.17. A causal LTI system S has the block diagram representation shown in Figure P9.17. Determine a differential equation relating the input $x(t)$ to the output $y(t)$ of this system.

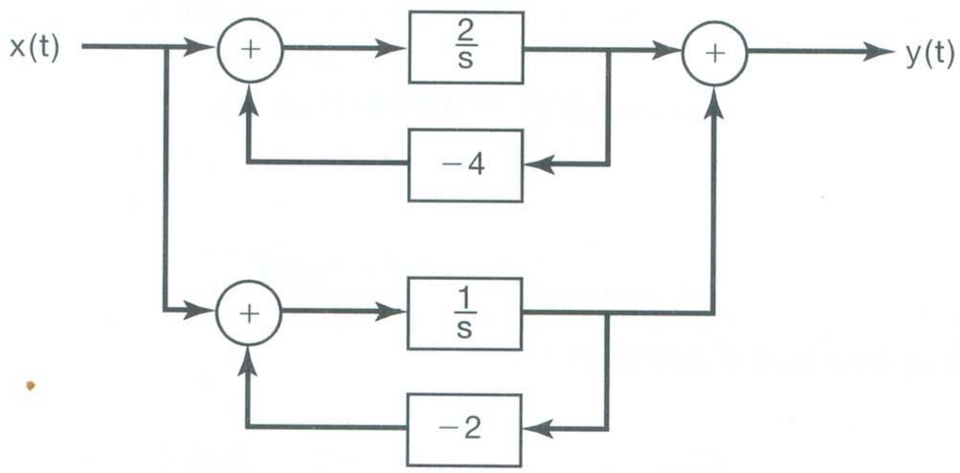


Figure P9.17

Problem 9.36: Block Diagram and DE

9.36. In this problem, we consider the construction of various types of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}.$$

To derive the direct-form block diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2 + 3s + 2}.$$

With the output of S_1 denoted by $y_1(t)$, the direct-form block diagram representation of S_1 is shown in Figure P9.36. The signals $e(t)$ and $f(t)$ indicated in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

(b) How is $dy_1(t)/dt$ related to $f(t)$?

(c) How is $d^2y_1(t)/dt^2$ related to $e(t)$?

(d) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, and $y_1(t)$.

(e) Use the result from the previous part to extend the direct-form block diagram representation of S_1 and create a block diagram representation of S.

Problem 9.36: Block Diagram and DE

(f) Observing that

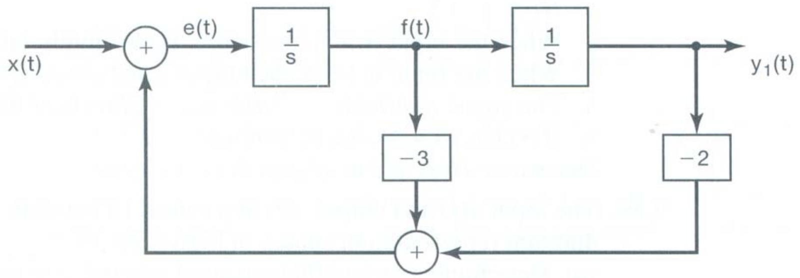
$$H(s) = \left(\frac{2(s-1)}{s+2}\right)\left(\frac{s+3}{s+1}\right),$$

draw a block diagram representation for S as a cascade combination of two subsystems.

(g) Observing that

$$H(s) = 2 + \frac{6}{s+2} - \frac{8}{s+1},$$

draw a block-diagram representation for S as a parallel combination of three subsystems.



Problem 9.19: Unilateral LT

9.19. Determine the unilateral Laplace transform of each of the following signals, and specify the corresponding regions of convergence:

(a) $x(t) = e^{-2t}u(t+1)$

(b) $x(t) = \delta(t+1) + \delta(t) + e^{-2(t+3)}u(t+1)$

(c) $x(t) = e^{-2t}u(t) + e^{-4t}u(t)$

Problem 9.45: X(s) H(s) Y(s) –> h(t) and ROC

9.45. Consider the LTI system shown in Figure P9.45(a) for which we are given the following information:

$$X(s) = \frac{s+2}{s-2},$$

$$x(t) = 0, \qquad t > 0,$$

and

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t). \qquad \text{[See Figure P9.45(b).]}$$

(a) Determine $H(s)$ and its region of convergence.
(b) Determine $h(t)$.

(c) Using the system function $H(s)$ found in part (a), determine the output $y(t)$ if the input is

$$x(t) = e^{3t}, \qquad -\infty < t < +\infty.$$

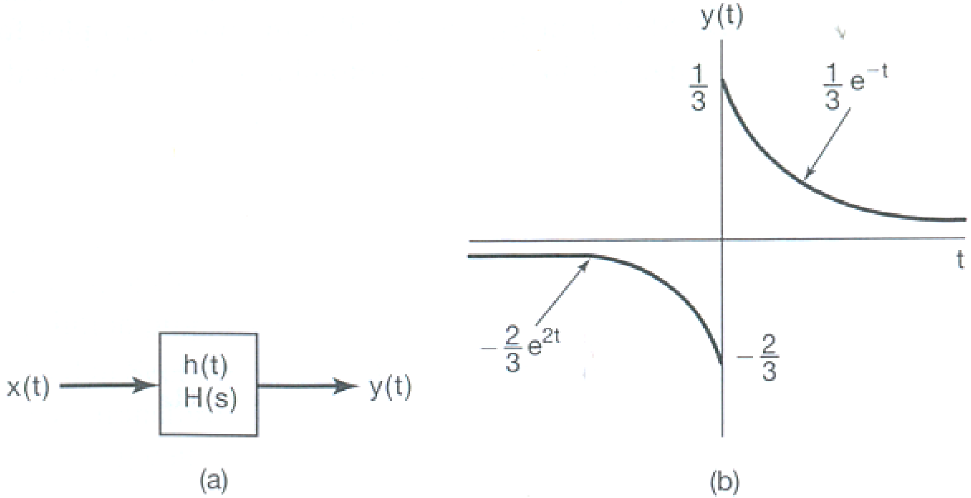


Figure P9.45