- The Laplace Transform
- The Region of Convergence (ROC) for Laplace Transforms
- The Inverse Laplace Transform
- Geometric Evaluation of the Fourier Transform
- Properties of the Laplace Transform
- Some Laplace Transform Pairs
- Analysis & Characterization of LTI Systems Using the Laplace Transform
- System Function Algebra and Block Diagram Representations
- The Unilateral Laplace Transform

Problem 9.9: Inverse LT

9.9. Given that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \qquad \mathcal{R}e\{s\} > \mathcal{R}e\{-a\},$$

determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \qquad \mathcal{R}e\{s\} > -3.$$

Problem 9.26: Time-Shifting, Time-Scaling

9.26. Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \qquad \text{and} \qquad x_2(t) = e^{-3t}u(t).$$

Given that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathcal{R}e\{s\} > a,$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

Problem 9.29: Convolution

9.29. Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Determine the Laplace transforms of $x(t)$ and $h(t)$.

(b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.

(c) From the Laplace transform of $y(t)$ as obtained in part (b), determine $y(t)$.

(d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

Problem 9.22: LT and ROC

9.22. Determine the function of time, $x(t)$, for each of the following Laplace transforms and their associated regions of convergence:

(a) $\frac{1}{s^2+9}$, $\quad \mathcal{R}e\{s\} > 0$

(b) $\frac{s}{s^2+9}$, $\quad \mathcal{R}e\{s\} < 0$

(c) $\frac{s+1}{(s+1)^2+9}$, $\quad \mathcal{R}e\{s\} < -1$

(d) $\frac{s+2}{s^2+7s+12}$, $\quad -4 < \mathcal{R}e\{s\} < -3$

(e) $\frac{s+1}{s^2+5s+6}$, $\quad -3 < \mathcal{R}e\{s\} < -2$

(f) $\frac{(s+1)^2}{s^2-s+1}$, $\quad \mathcal{R}e\{s\} > \frac{1}{2}$

(g) $\frac{s^2-s+1}{(s+1)^2}$, $\quad \mathcal{R}e\{s\} > -1$

Problem 9.28: ROC, Pole-Zero, Stable-Causal

9.28. Consider an LTI system for which the system function $H(s)$ has the pole-zero pattern shown in Figure P9.28.

(a) Indicate all possible ROCs that can be associated with this pole-zero pattern.
(b) For each ROC identified in part (a), specify whether the associated system is stable and/or causal.