

# Exams

6. (10%) Suppose  $x(t)$  is a continuous-time signal,  $p(t)$  is a narrow pulse with duration  $\tau < T$ , and  $y(t)$  is a pulse train modulated by the samples of  $x(t)$ ,

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)p(t - nT).$$

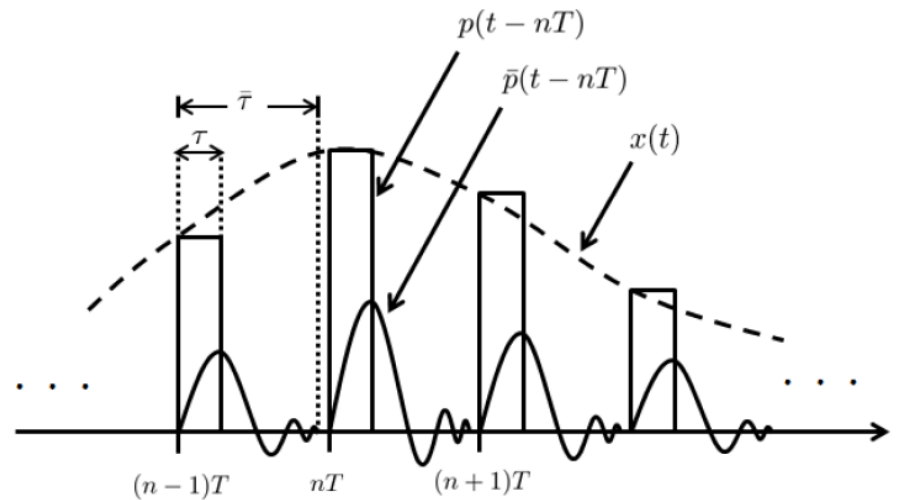
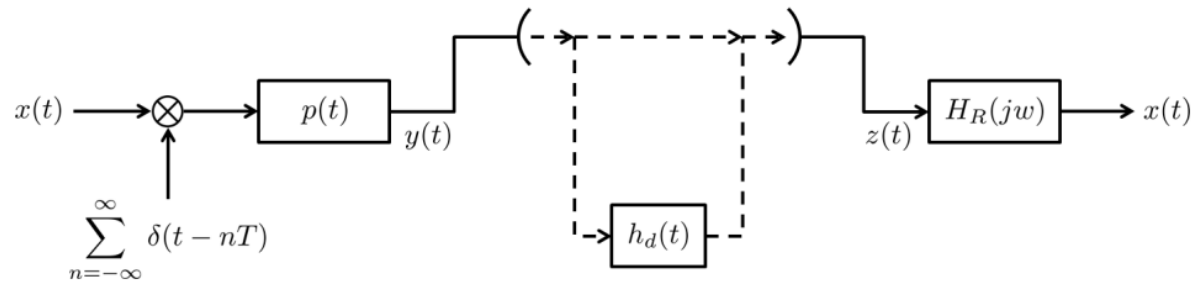
$y(t)$  is transmitted through some channel and received at the receiver as  $z(t)$ . The transmission through the channel can be modeled as a convolution by some distortion filter  $h_d(t)$ ,

$$z(t) = y(t) * h_d(t) = \sum_{n=-\infty}^{\infty} x(nT)\bar{p}(t - nT),$$

where

$$\bar{p}(t) = p(t) * h_d(t).$$

The duration of  $\bar{p}(t)$  is  $\bar{\tau}$ , it is possible that  $\bar{\tau} < T$  or  $\bar{\tau} > T$ .



An engineer thinks he can apply a filter  $H_R(j\omega)$  at the receiver end, and reconstruct the original signal  $x(t)$  by passing  $z(t)$  through  $H_R(j\omega)$ .

Do you think he is correct or not? If yes, is there any condition for it? Write it down and verify your answer. What this  $H_R(j\omega)$  should be? If not, explain why not and verify it.

7. (4%)

- (a) (2%) Explain why **phase modulation** with a signal  $x(t)$  corresponds to **frequency modulation** with a signal  $\frac{d}{dt}x(t)$ .
- (b) (2%) A signal  $x(t) = au(t)$  is **frequency modulated** where  $a$  is a constant and  $u(t)$  is a unit step function, and the output signal is  $y(t)$ . Plot the general shape of  $y(t)$ .

8. (2%) An information-bearing signal  $x(t)$  is amplitude modulated,

$$y(t) = x(t) \cos(\omega_c t + \theta_c).$$

and  $y(t)$  is synchronously demodulated with a local carrier with a phase  $\phi_c$ , i.e.,

$$w(t) = y(t) \cos(\omega_c t + \phi_c),$$

and  $w(t)$  is lowpass filtered as in the synchronous demodulation. Explain what happens when (i)  $\phi_c = \theta_c$  and (ii)  $\phi_c \neq \theta_c$  in both time and frequency domains.

5. (5%) Intersymbol interference can be avoided in a pulse-amplitude modulation system by constraining the pulse shape to have zero-crossings at integer multiples of the symbol spacing  $T_1$ . Consider the pulse  $p_1(t)$  that is real and even and has a Fourier transform  $P_1(j\omega)$  with odd symmetry around  $\pi/T_1$  so that

$$P_1(-j\omega + j\frac{\pi}{T_1}) = -P_1(j\omega + j\frac{\pi}{T_1}), \quad 0 \leq \omega \leq \frac{\pi}{T_1}.$$

Given that  $p_1(kT_1)=0$ ,  $k=\pm 1, \pm 2, \pm 3, \dots$ , show that a pulse  $p(t)$  with Fourier transform

$$P(j\omega) = \begin{cases} 1 + P_1(j\omega), & |\omega| \leq \frac{\pi}{T_1} \\ P_1(j\omega), & \frac{\pi}{T_1} < |\omega| \leq \frac{2\pi}{T_1} \\ 0, & \text{otherwise} \end{cases}$$

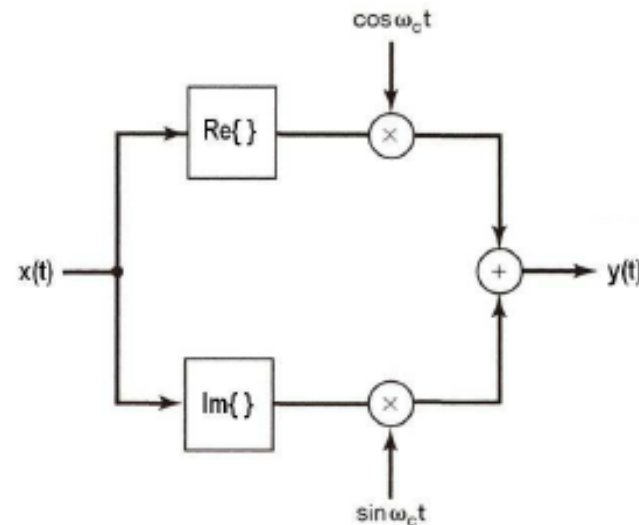
also has the property that  $p(kT_1)=0$ ,  $k=\pm 1, \pm 2, \pm 3, \dots$

6. (6%) Consider a modulating signal  $x(t)$  with highest frequency  $\omega_M$ . We want to design an AM communication system to transmit the signal  $x(t)$  by using the carrier signal  $c(t)$  with carrier frequency  $\omega_c$ .
- (a) (2%) Suppose the modulating signal  $x(t)$  is a wideband signal such that  $\omega_M \geq \omega_c$ . Design a modulator and a demodulator for this AM system, assumed that the phases of the modulating carrier and demodulating carrier are both zero.
- (b) (2%) Now, suppose the modulating signal  $x(t)$  is a narrow band signal such that  $\omega_M < \omega_c$ . Since we know that the single-sideband modulation (SSB) can save the bandwidth for transmission, describe two approaches at the transmitter to generate the SSB modulated signal from the original double-sideband modulation (DSB) modulated signal.
- (c) (2%) We know that, in practice, the phases of the modulating carrier and demodulating carrier may not be zero, and hence, there are commonly two methods: synchronous demodulation and asynchronous demodulation for demodulation. Describe the pros and cons of the two methods.

10. (10%) Let a communication system use a modulation technique to transmit a message signal  $x(t)$ .
- (a) (3%) Assume that  $x(t) = \sin(1000 \pi t) / \pi t$  and the modulated signal  $y(t) = (x(t) + A) \cos(10000 \pi t)$ . Find the largest permissible value of the modulation index  $m$  that would allow envelope detector to be used to recover  $x(t)$  from  $y(t)$ . You must justify your answer.
- (b) (3%) Assume that the transmitter creates a modulated signal  $y(t) = \cos\left(\omega_c t + m \int_{-\infty}^t x(\alpha) d\alpha\right)$ . Find the modulated signal  $y(t)$  when  $m \ll \pi/2$ . You must justify your answer.
- (c) (4%) Find the relationship between the bandwidth of  $y(t)$ , the bandwidth of  $x(t)$  and the  $\omega_c$  in Part (b). You must justify your answer.

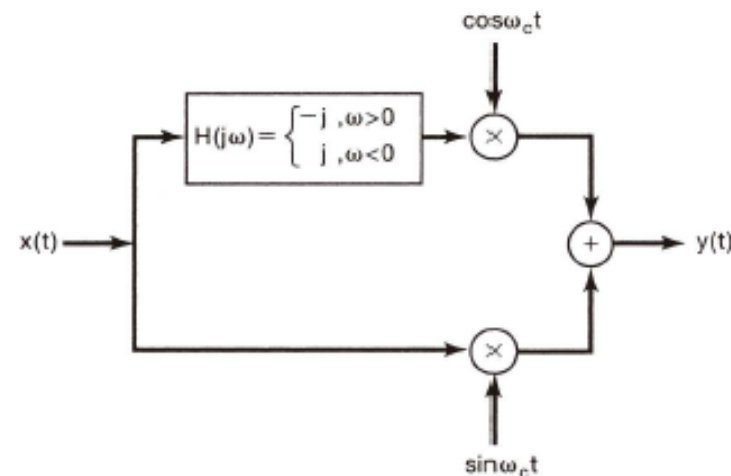


7. (15%) Consider the following modulator, where the input signal  $x(t) = A\text{sinc}(\frac{t}{T}) + jA\text{sinc}(\frac{t}{T})$ ,  $T = 10^{-5}$  seconds,  $\omega_c = 10^6 \pi$  rad/sec, and  $\text{Re}\{\}$  and  $\text{Im}\{\}$  denote the operations generating the



real part and imaginary part of the input, respectively. Find an expression for the Fourier transform  $Y(j\omega)$  of the output signal  $y(t)$ . Plot the real and imaginary parts of  $Y(j\omega)$ .

9. [8] Consider a system for generating a modulated signal  $y(t)$  from the message signal  $x(t)$  as follows:



Assume that the message signal  $x(t)$  has its spectrum  $X(j\omega) = 0$  for  $|\omega| > \omega_M$  and  $\omega_c > \omega_M$ .

- (a) Is  $y(t)$  a real modulated signal if  $x(t)$  is real? Justify your answer. [4]  
(b) How do you recover the message signal  $x(t)$  from the modulated signal  $y(t)$ ? Justify your answer. [4]