8.46: FM, Instantaneous Frequency, Frequency Chirp

8.46. Consider the complex exponential function of time,

$$s(t) = e^{j\theta(t)}, \qquad \text{(P8.46–1)}$$

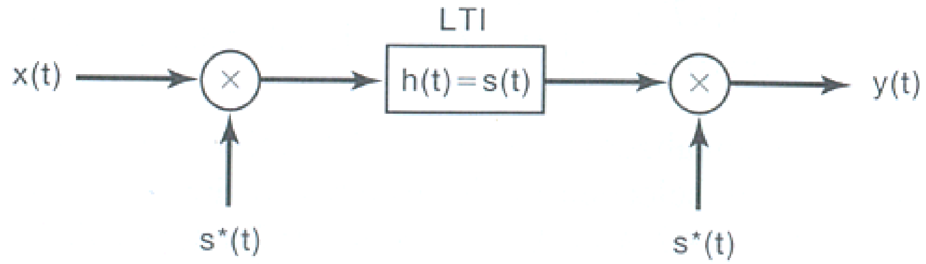
where $\theta(t) = \omega_0 t^2/2$.

Since the instantaneous frequency $\omega_i = d\theta/dt$ is also a function of time, the signal $s(t)$ may be regarded as an FM signal. In particular, since the signal sweeps linearly through the frequency spectrum with time, it is often called a frequency "chirp" or "chirp signal."

(a) Determine the instantaneous frequency.

(b) Determine and sketch the magnitude and phase of the Fourier transform of the "chirp signal." To evaluate the Fourier transform integral, you may find it helpful to complete the square in the exponent in the integrand and to use the relation

$$\int_{-\infty}^{+\infty} e^{jz^2} dz = \sqrt{\frac{\pi}{2}}(1 + j).$$



(c) Consider the system in Figure P8.46, in which $s(t)$ is the "chirp signal" in eq. (P8.46–1). Show that $y(t) = X(j\omega_0 t)$, where $X(j\omega)$ is the Fourier transform of $x(t)$.

(*Note:* The system in Figure P8.46 is referred to as the *"chirp" transform algorithm* and is often used in practice to obtain the Fourier transform of a signal.)

8.17: DT Modulation – Insertion of Zero-Valued Samples

8.17. Consider an arbitrary finite-duration signal $x[n]$ with Fourier transform $X(e^{j\omega})$. We generate a signal $g[n]$ through insertion of zero-valued samples:

$$g[n] = x_{(4)}[n] = \begin{cases} x[n/4], & n = 0, \pm 4, \pm 8, \pm 12, \ldots \\ 0, & \text{otherwise} \end{cases}.$$

The signal $g[n]$ is passed through an ideal lowpass filter with cutoff frequency $\pi/4$ and passband gain of unity to produce a signal $q[n]$. Finally, we obtain

$$y[n] = q[n] \cos\left(\frac{3\pi}{4} n\right).$$

For what values of ω is $Y(e^{j\omega})$ guaranteed to be zero?