Spring 2014

信號與系統
Signals and Systems

Chapter SS-8
Communication Systems

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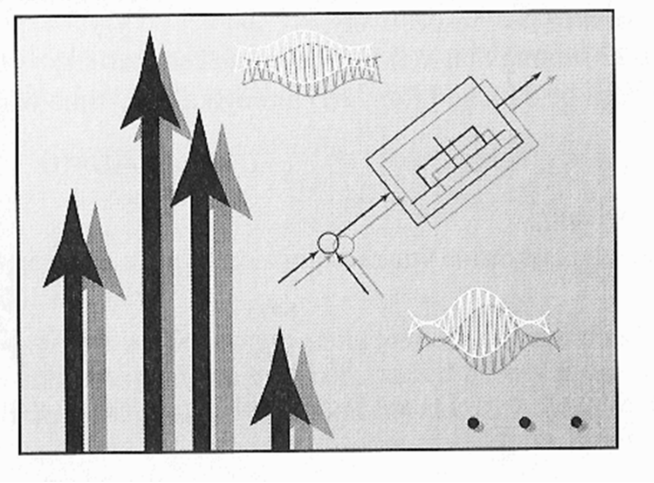
NTU-EE

Feb14 – Jun14



Figures and images used in these lecture notes are adopted from
"Signals & Systems" by Alan V. Oppenheim and Alan S. Willsky, 1997

Flowchart

Introduction (Chap 1)

LTI & Convolution (Chap 2)

Bounded/Convergent

Periodic

FS CT DT (Chap 3)

Aperiodic

FT CT (Chap 4) DT (Chap 5)

Unbounded/Non-convergent

LT CT (Chap 9)

zT DT (Chap 10)

signals

systems

Time-Frequency (Chap 6)

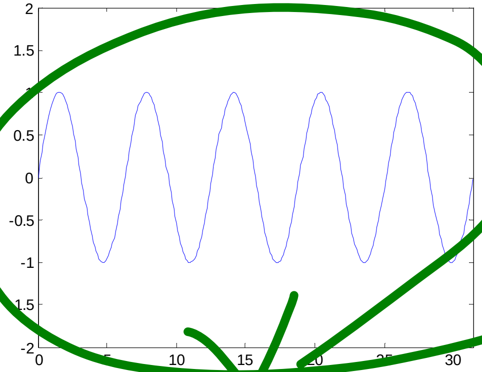
CT-DT (Chap 7)

Communication (Chap 8)

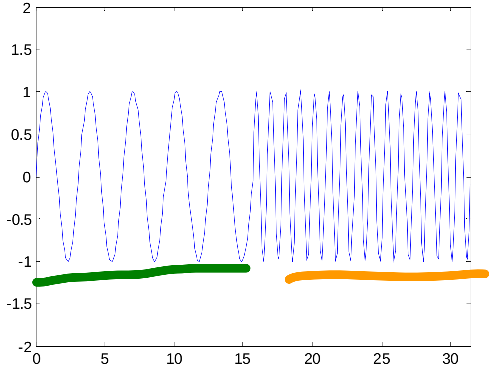
Control (Chap 11)

Digital Signal Processing (dsp-8)

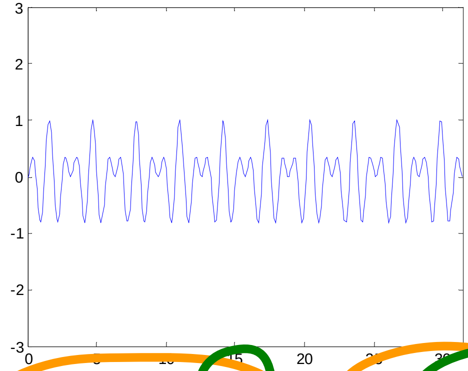
Introduction



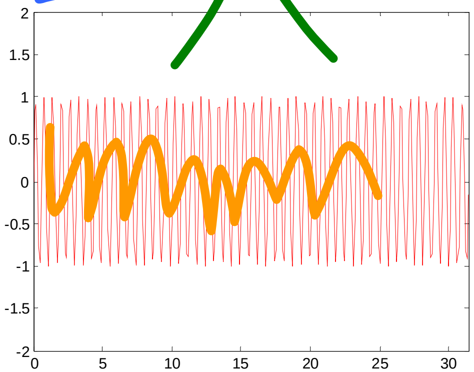
$x(t) = \sin(1\,t)$



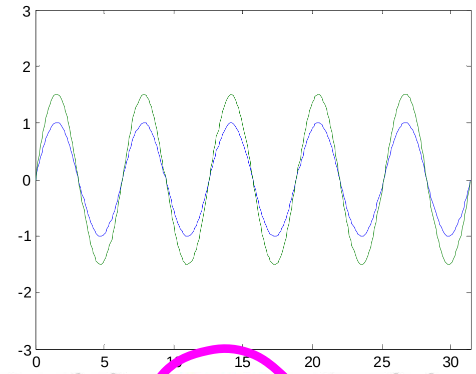
$\sin(2\,t)$ $\sin(6\,t)$



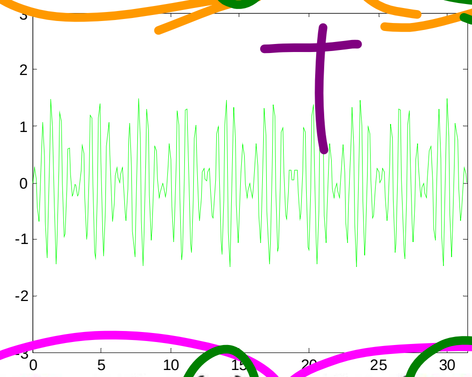
$z_1 = 1 \cdot \sin(t) \cdot \sin(5\,t)$



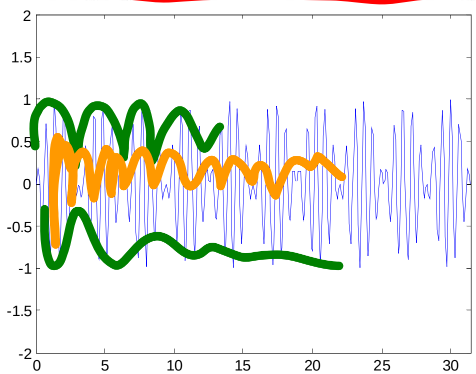
$c(t) = \sin(10\,t)$



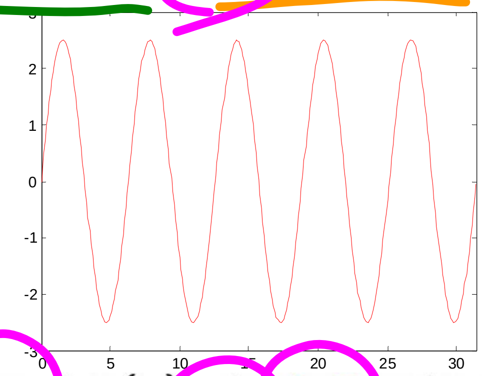
$1\,\sin(t)$ $1.5\,\sin(t)$



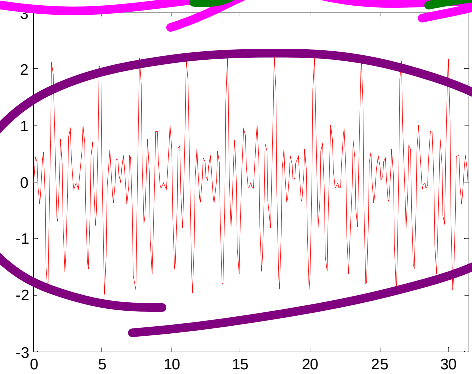
$z_2 = 1.5 \cdot \sin(t) \cdot \sin(10\,t)$



$\sin(10\,t) \cdot \sin(1\,t)$



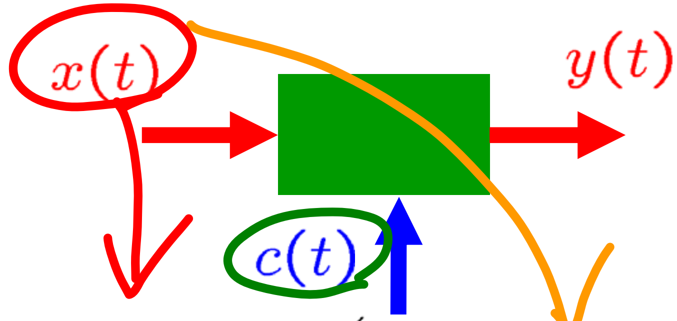
$1\,\sin(t) + 1.5\,\sin(t)$



$z_1 + z_2$

Modulation & Demodulation:

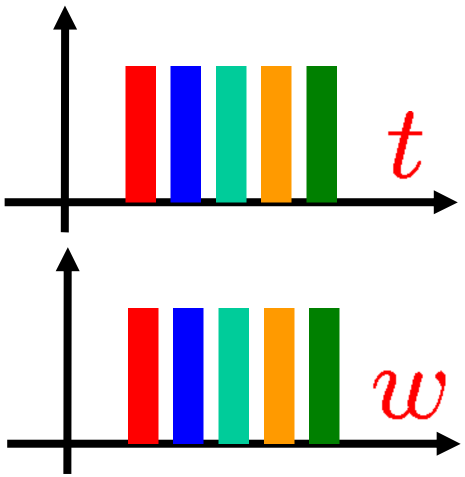
- M: Embedding an information-bearing signal into a second signal
- D: Extracting the information-bearing signal
- Methods:
  - > Amplitude Modulation (AM)
  - > Frequency Modulation (FM)



$$= A(t)\cos\big(w(t)\cdot t + \theta(t)\big)$$

Multiplexing & Demultiplexing:

- Simultaneous transmission of more than one signal with overlapping spectra over the same channel
- Methods:
  - > Time-Division Multiplexing (TDM)
  - > Frequency-Division Multiplexing (FDM)



AM

- Complex Exponential & Sinusoidal

  Amplitude Modulation & Demodulation

- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier

  » Time-Division Multiplexing

- Pulse-Amplitude Modulation

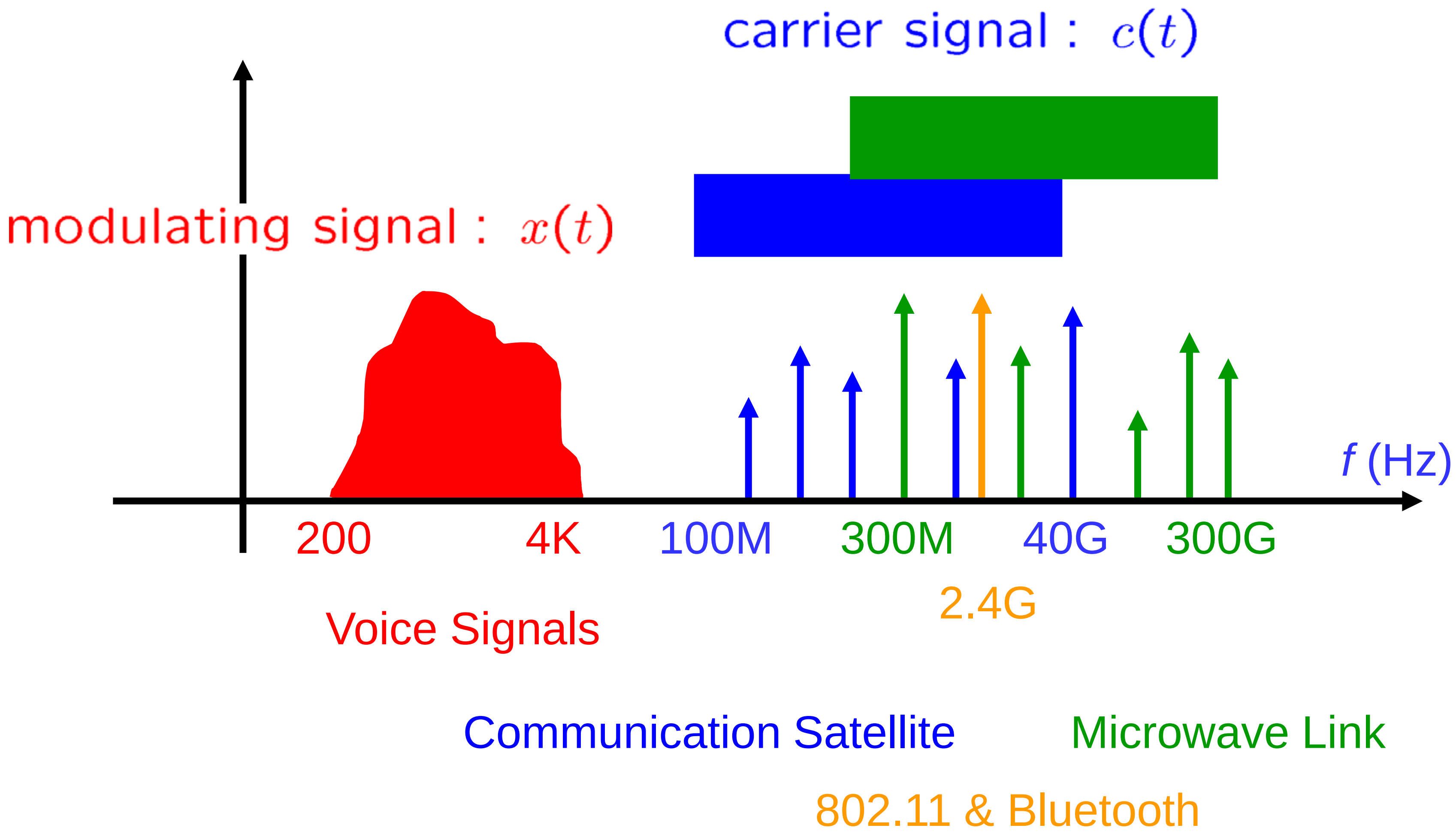
CT

FM (

- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

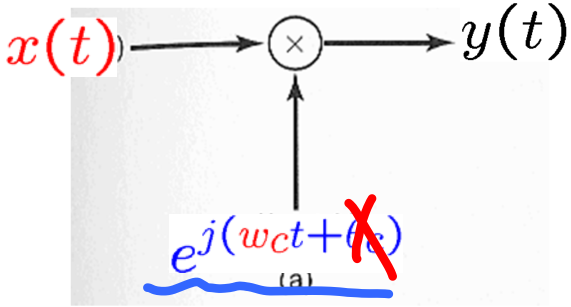
) DT

- Signal Frequency Characteristics:



modulated signal : $y(t) = x(t)\, c(t)$

- AM with a Complex Exponential Carrier:

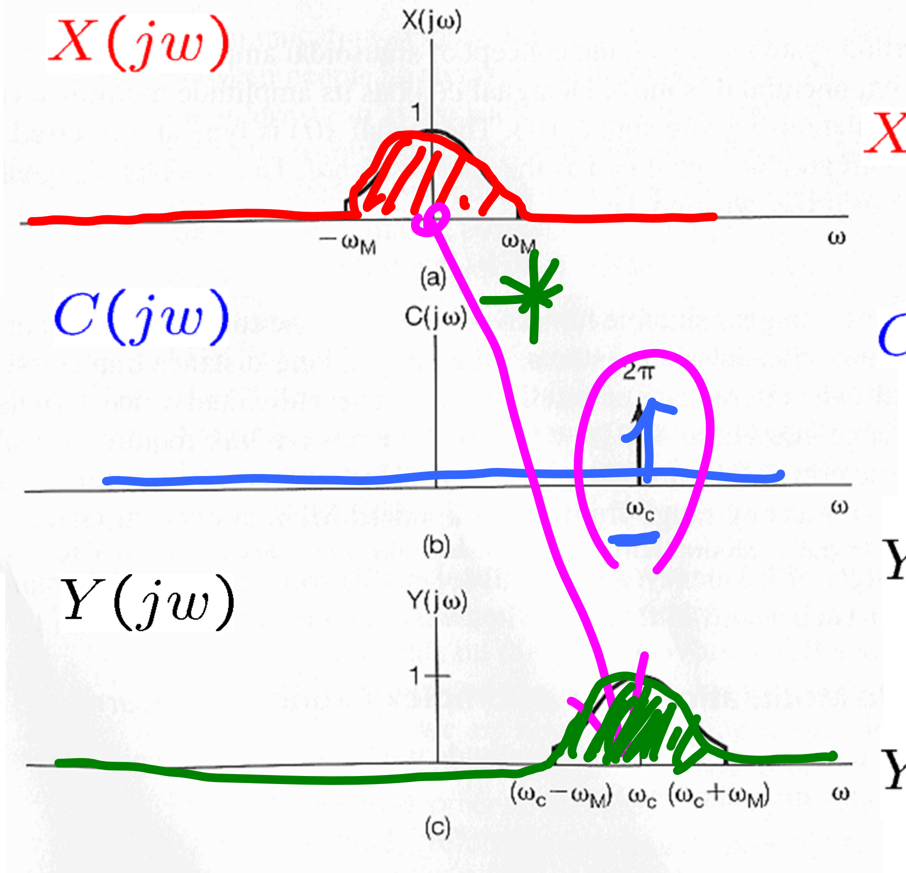


w_c : carrier frequency

$$c(t) = e^{j(w_c t+\theta_c)}$$

$$y(t) = x(t)\ c(t) \quad = x(t)\ e^{jw_c t}$$

$$\theta_c = 0$$



$$X(jw)$$

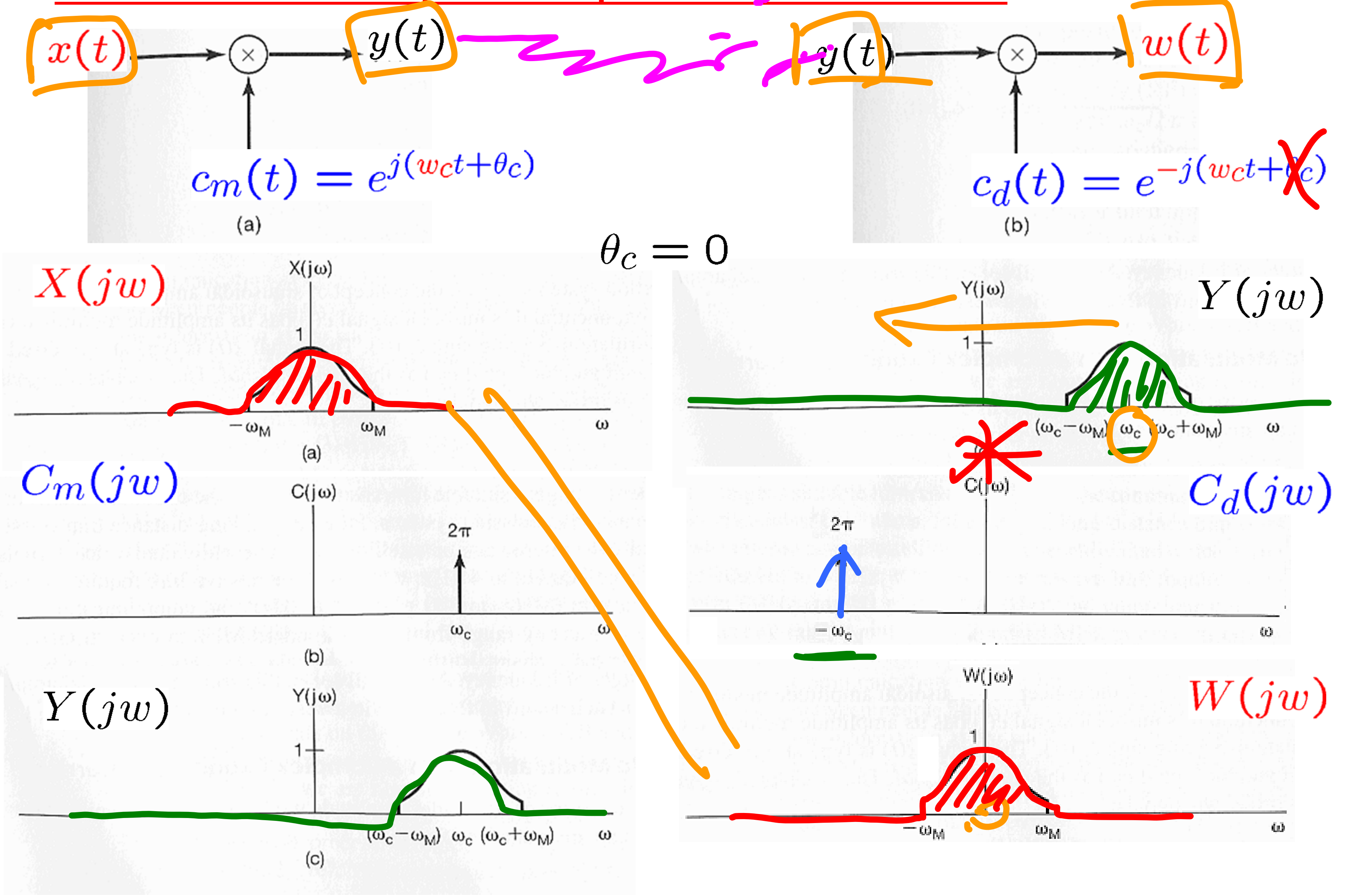
$$C(jw) = 2\pi\ \delta(w - w_c)$$

$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\ C(j(w-\theta))d\theta$$

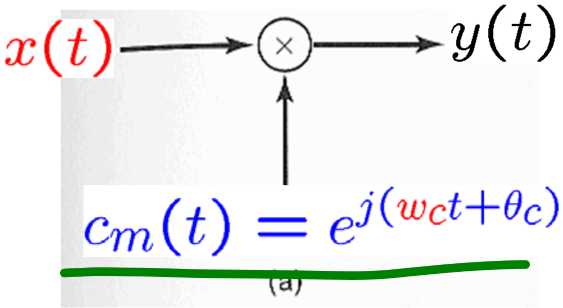
$$Y(jw) = X\left(j(w - w_c)\right)$$

$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))\, d\theta$$

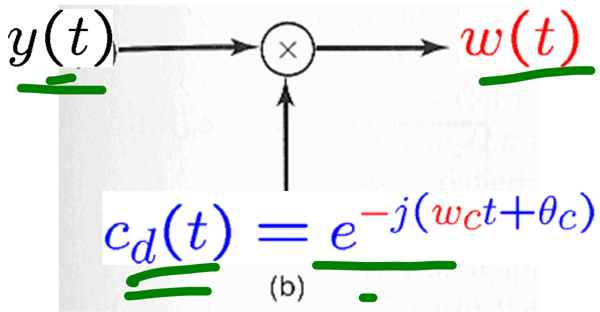
- AM with a Complex Exponential Carrier:



AM with a Complex Exponential Carrier:



$\theta_c = 0$



$$y(t) = x(t)\ c_m(t)$$
$$= x(t)\ e^{jw_c t}$$

$$w(t) = y(t)\ c_d(t)$$
$$= y(t)\ e^{-jw_c t}$$
$$= x(t)\ e^{\cancel{jw_c t}}\ e^{\cancel{-jw_c t}}$$
$$\Rightarrow\ w(t) = x(t)$$

$$Y(jw) = X\left(j(w - w_c)\right)$$

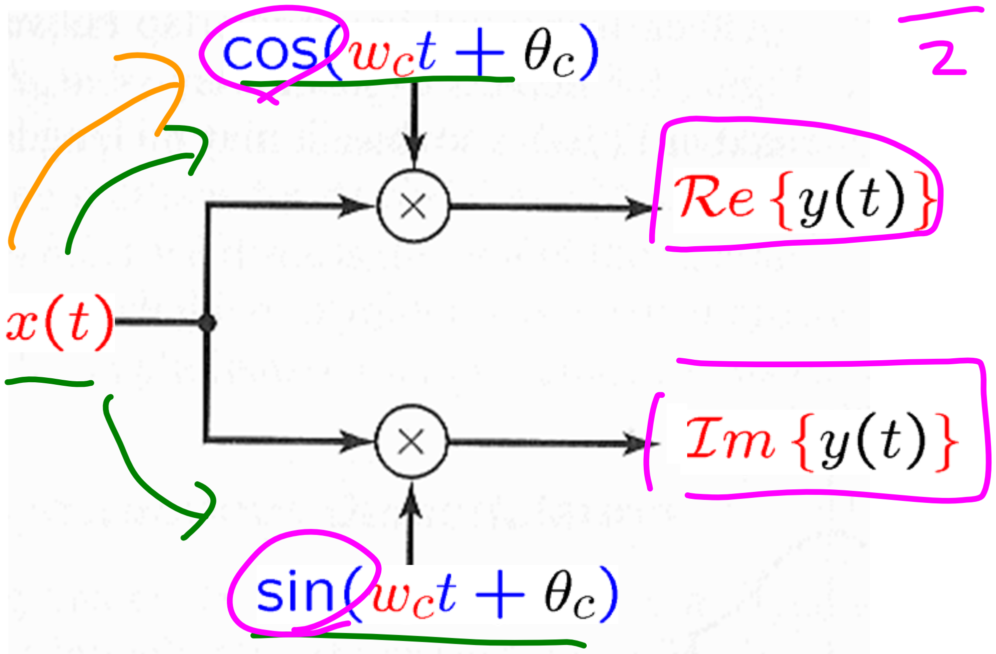
$$W(jw) = Y\left(j(w + w_c)\right)$$
$$\Rightarrow\ W(jw) = X(jw)$$

- ## AM with Sinusoidal Carriers:

$$c(t) = e^{jw_c t} = \cos(w_c t) + j \sin(w_c t)$$

$$\Rightarrow y(t) = x(t) \cos(w_c t) + j\, x(t) \sin(w_c t)$$

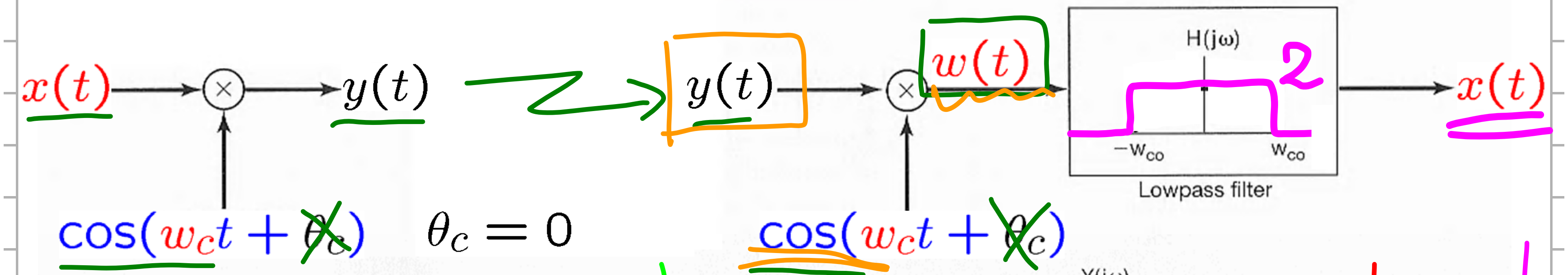
$\frac{\pi}{2}$

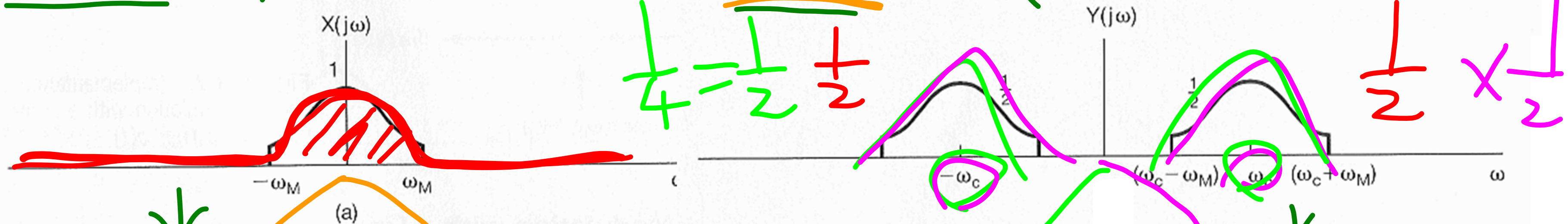


phase difference of $c_1(\cdot), c_2(\cdot)$?

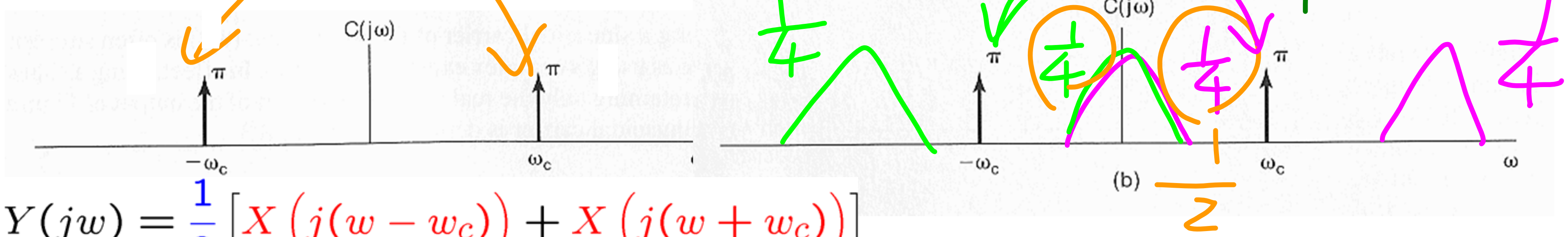
AM with a Sinusoidal Carrier:

$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))\, d\theta$$

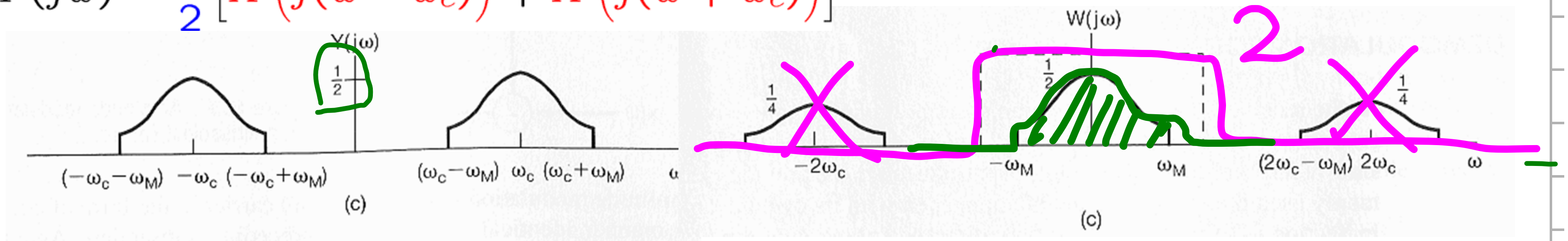




$$C(jw) = \pi[\,\delta(w - w_c) + \delta(w + w_c)\,]$$

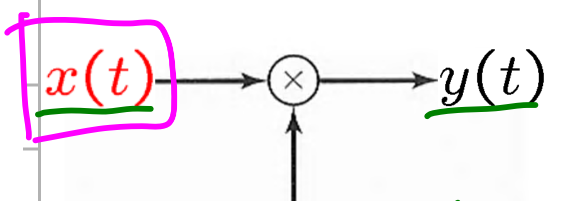


$$Y(jw) = \frac{1}{2}\left[X\left(j(w-w_c)\right) + X\left(j(w+w_c)\right)\right]$$

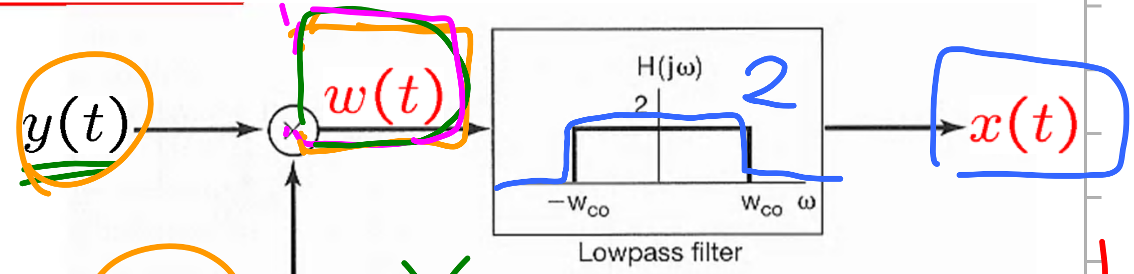


AM with a Sinusoidal Carrier:

$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))\, d\theta$$

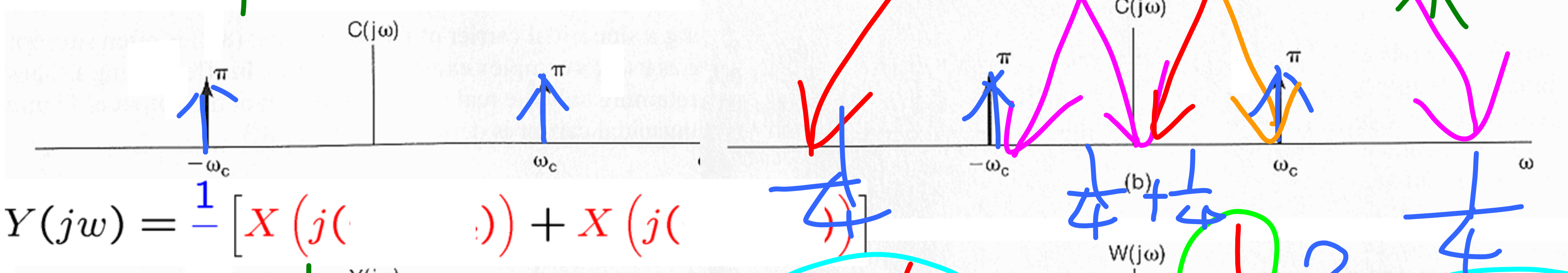


$\cos(w_c t + \cancel{\theta_c})$ $\quad \theta_c = 0$

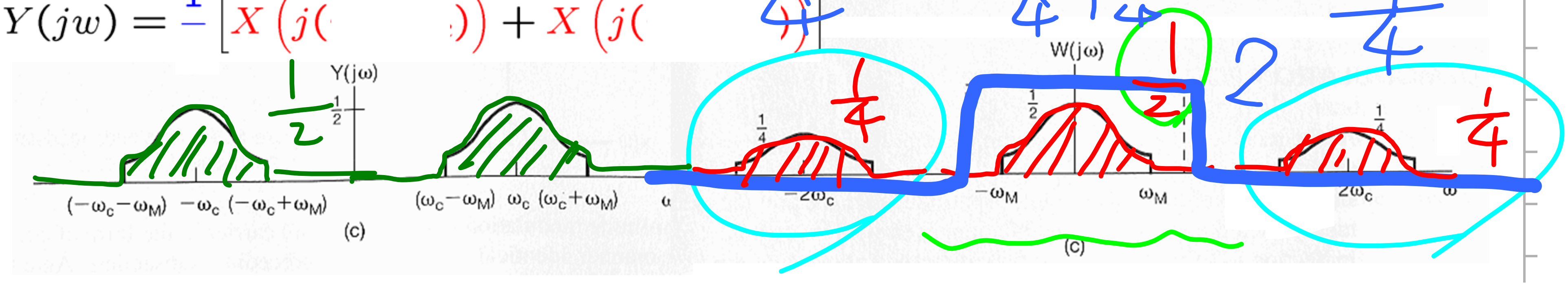


$\cos(w_c t + \cancel{\theta_c})$

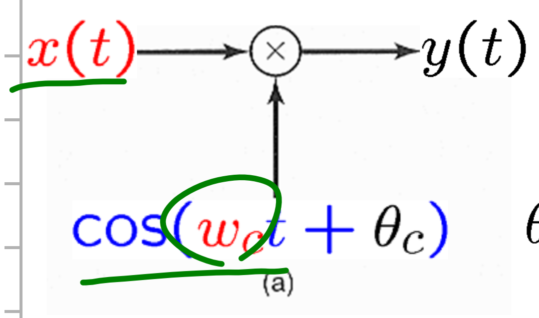
$$* \quad C(jw) = \pi[\,\delta(w - w_c) + \delta(w + w_c)\,]$$



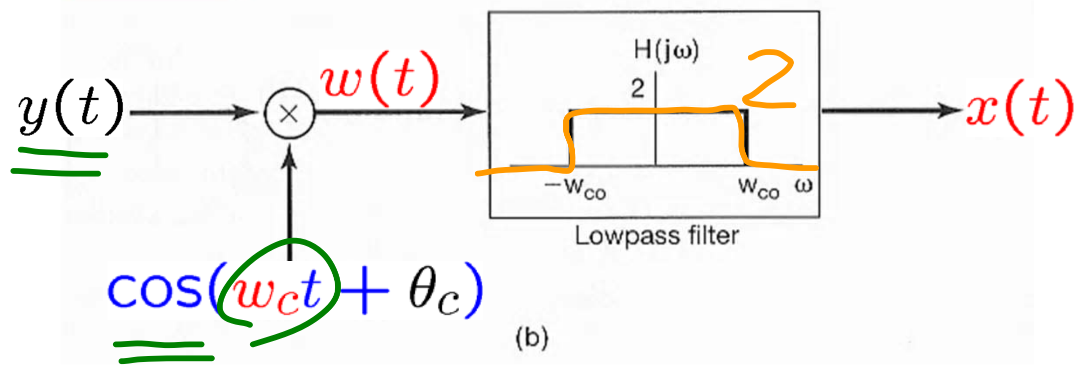
$$Y(jw) = \frac{1}{2}\left[X\left(j(\quad)\right) + X\left(j(\quad)\right)\right]$$



AM with a Sinusoidal Carrier:



$\theta_c = 0$



$$y(t) = \boxed{x(t) \cos(w_c t)}$$

$$w(t) = y(t) \cos(w_c t)$$

$$\Rightarrow w(t) = x(t) \cos^2(w_c t)$$

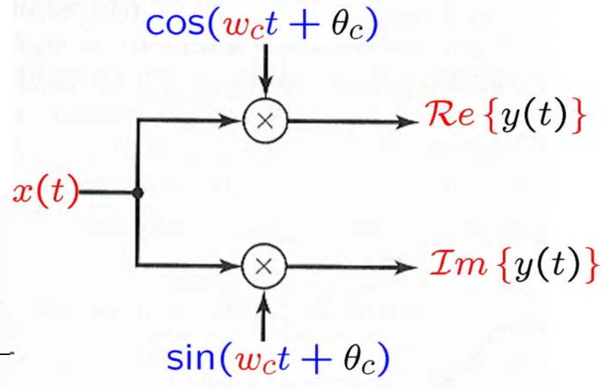
$$= x(t) \left[\frac{1}{2} + \frac{1}{2} \cos(2 w_c t) \right]$$

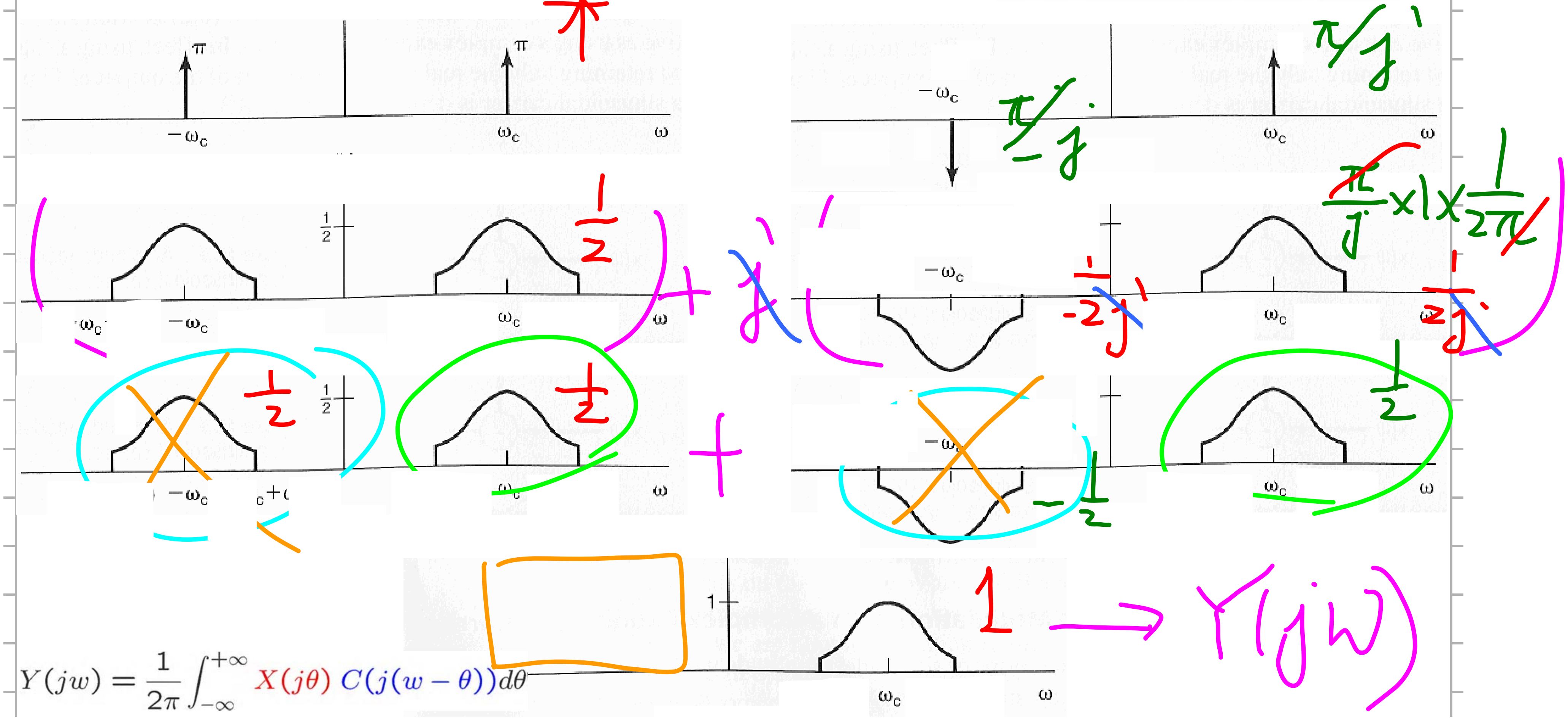
$$2\times \; = \boxed{\frac{1}{2} x(t)} + \cancel{\frac{1}{2} x(t) \cos(2 w_c t)}$$

Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation

$$c(t) = e^{jw_c t} = \cos(w_c t) + j\sin(w_c t)$$

$$y(t) = x(t)\cos(w_c t) + j\, x(t)\sin(w_c t)$$

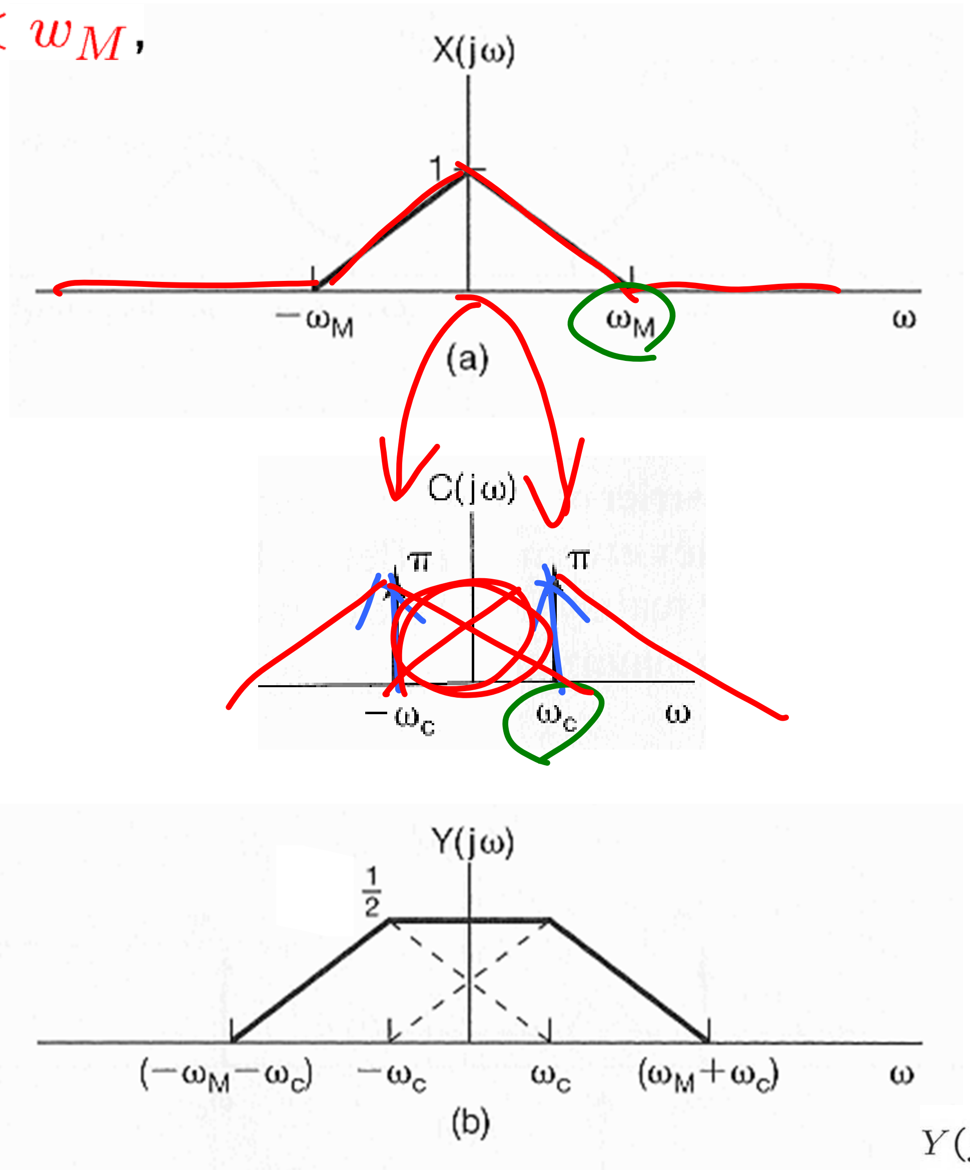


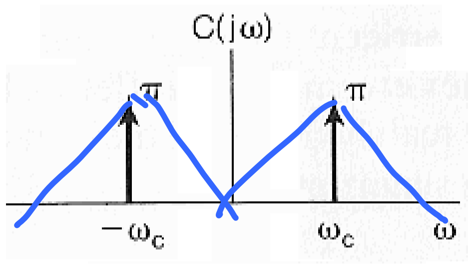


$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))\, d\theta$$

Overlapping of AM with a Sinusoidal Carrier:

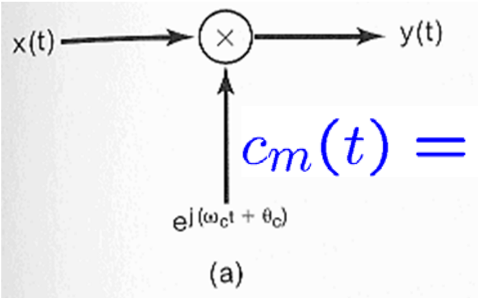
- If $w_c < w_M$,



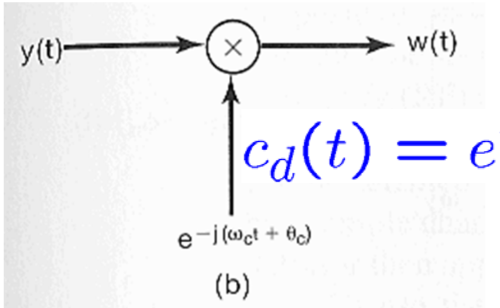


$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))d\theta$$

▪ Not Synchronized in Phase:



$c_m(t) = e^{j(w_c t + \theta_c)}$



$c_d(t) = e^{-j(w_c t + \phi_c)}$

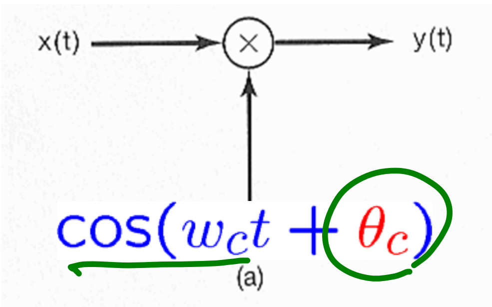
$\boxed{\theta_c \neq \phi_c}$

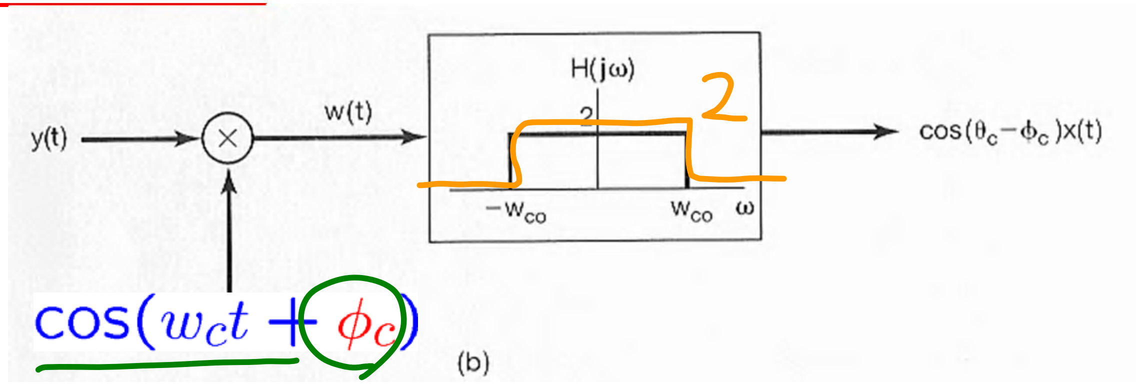
$$y(t) = x(t)\, c_m(t)$$
$$= x(t)\, e^{j(w_c t + \theta_c)}$$

$$w(t) = y(t)\, c_d(t)$$
$$= y(t)\, e^{-j(w_c t + \phi_c)}$$
$$= x(t)\, \boxed{e^{j(\theta_c - \phi_c)}}$$

$$\Rightarrow \text{ ONLY } \quad |x(t)| = |w(t)|$$

▪ Not Synchronized in Phase:





$$y(t) = x(t)\cos(w_c t + \theta_c) \qquad w(t) = y(t)\cos(w_c t + \phi_c)$$

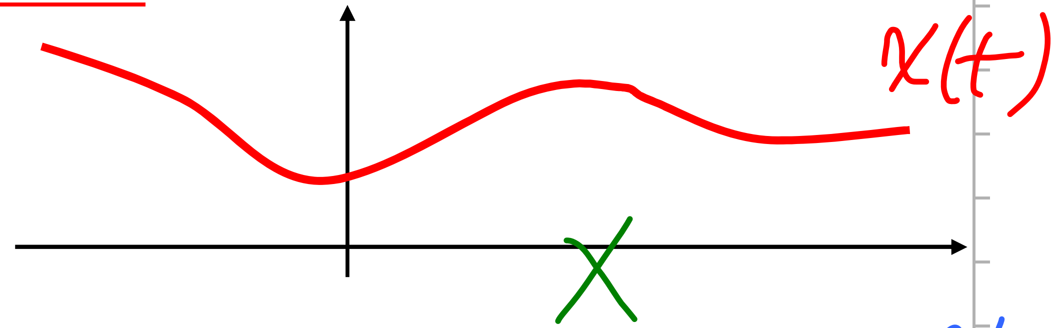
$$\Rightarrow w(t) = x(t)\cos(w_c t + \theta_c)\cos(w_c t + \phi_c)$$

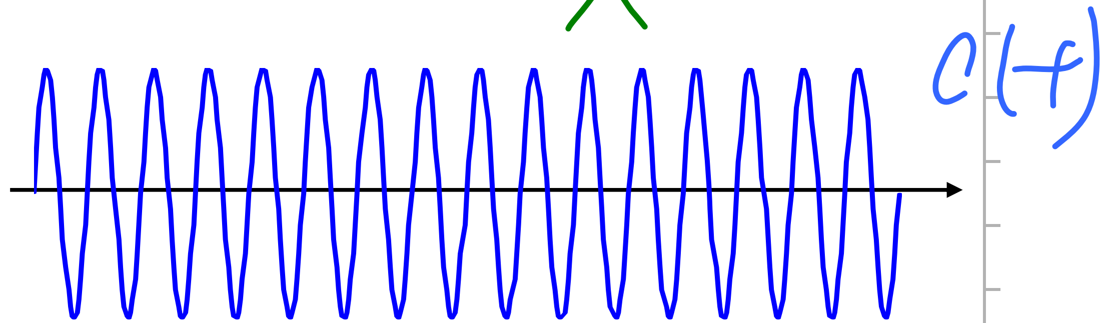
$$= x(t)\left[\frac{1}{2}\cos(\theta_c - \phi_c) + \frac{1}{2}\cos(2w_c t + \theta_c + \phi_c)\right]$$

$$= \frac{1}{2}\boxed{\cos(\theta_c - \phi_c)}\,x(t) + \cancel{\frac{1}{2}\,x(t)\,\cos(2w_c t + \theta_c + \phi_c)}$$

Asynchronous Demodulation:

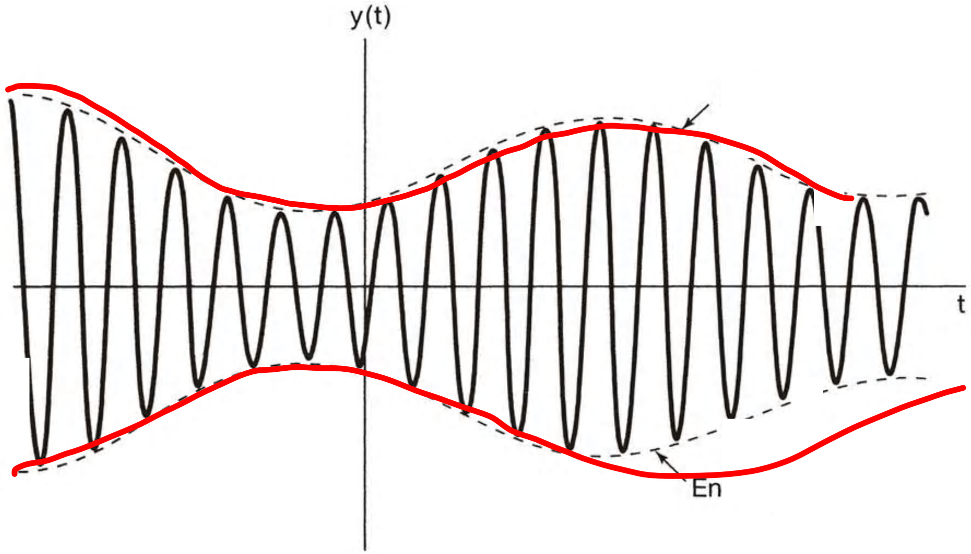
- $w_c >> w_M$
- $x(t) > 0, \quad \forall t$
  - In audio transmission over a RF channel
    - > w_M: 15 - 20 Hz
    - > $w_c/2\pi$: 500kHz – 2 MHz



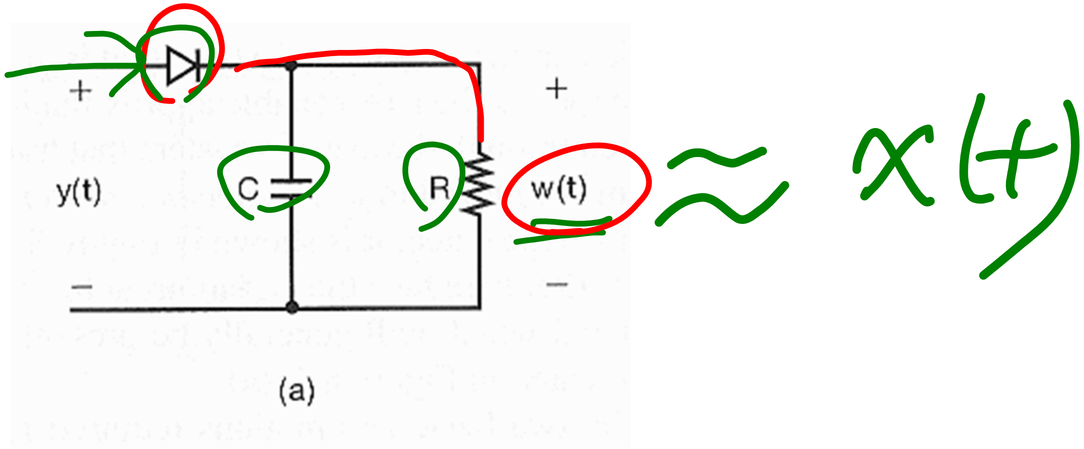


$$y(t) = x(t)\cos(w_c t + \theta_c)$$

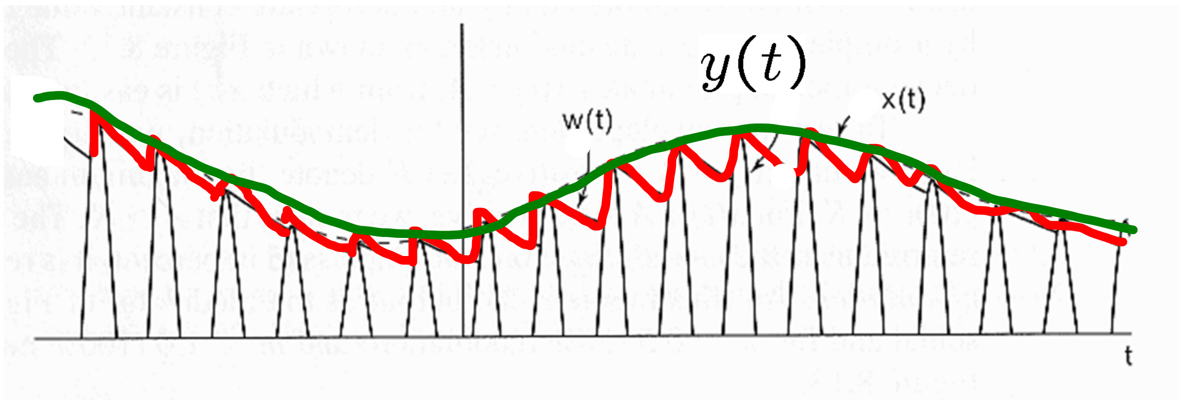
$$\approx x(t)$$



- Envelope Detector:

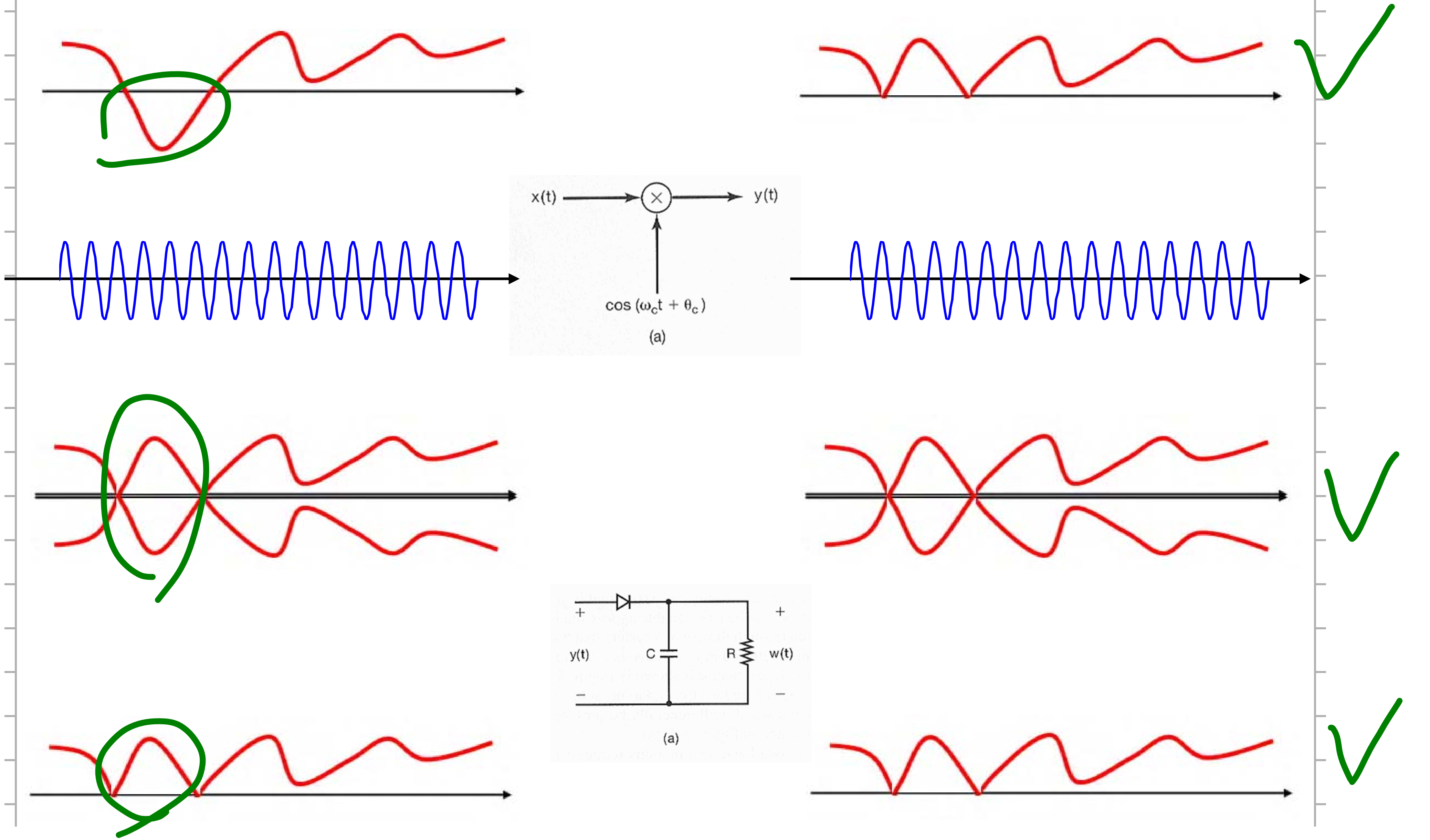


(a)



(b)

Asynchronous Demodulation:

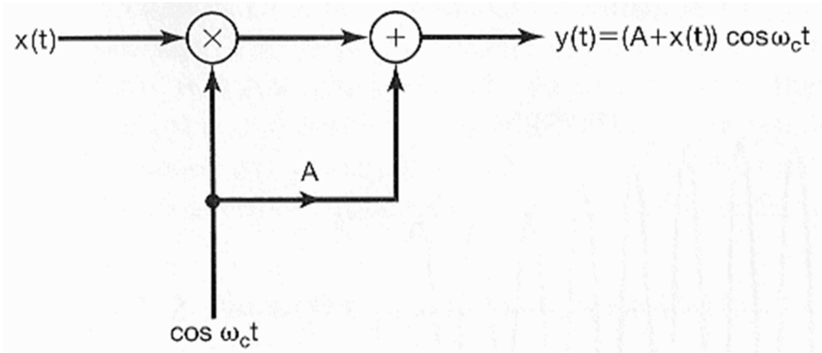


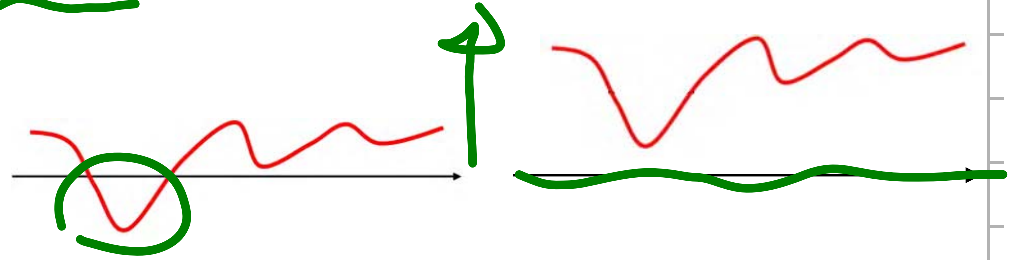
Asynchronous Demodulation:

- $w_c >> w_M$
- $x(t) > 0, \quad \forall t$

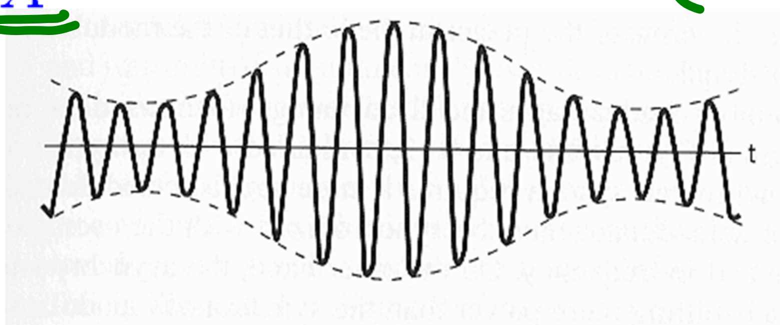
If not, $x(t) \rightarrow x(t) + A > 0$

$A \geq K, \quad |x(t)| \leq K$



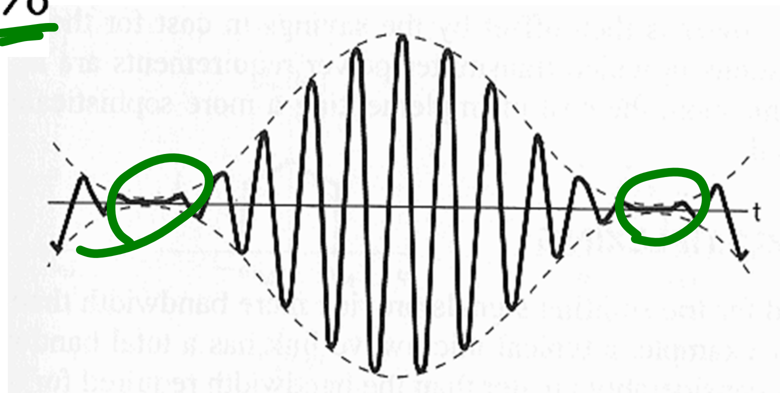


- $\frac{K}{A}$: modulation index m in %



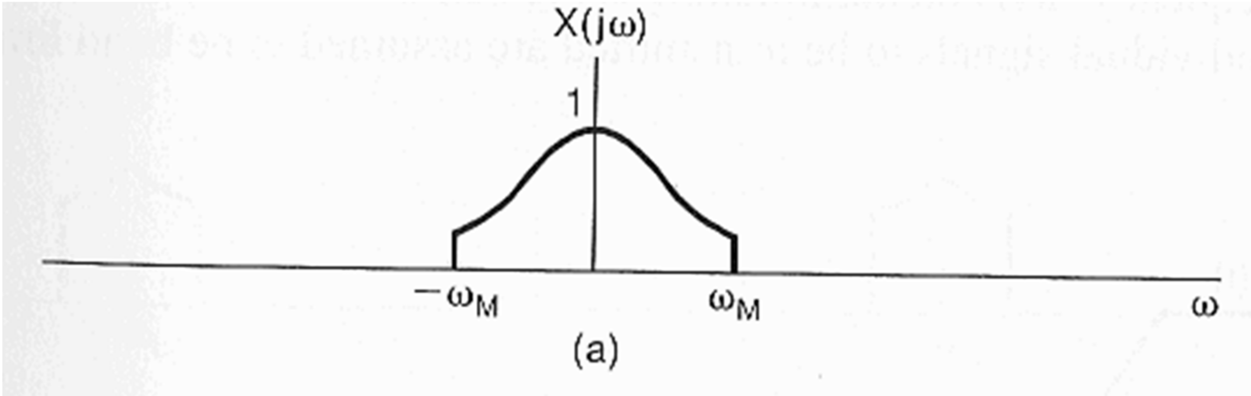
(a)

$m = 0.5$

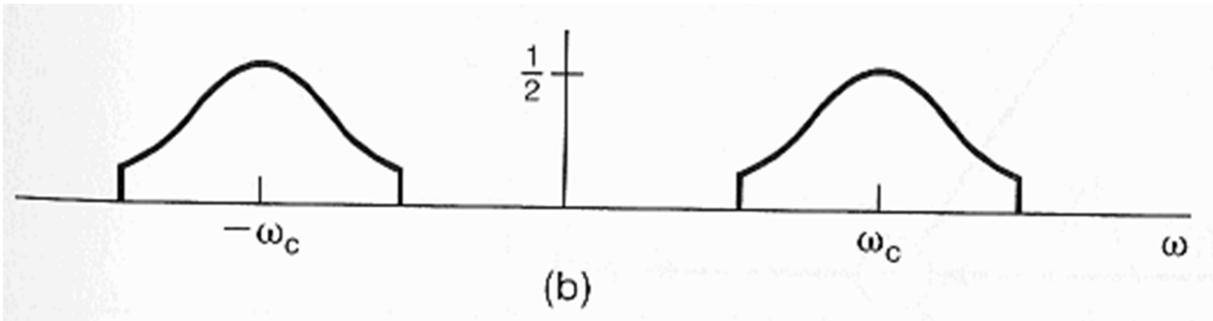


$m = 1.0$

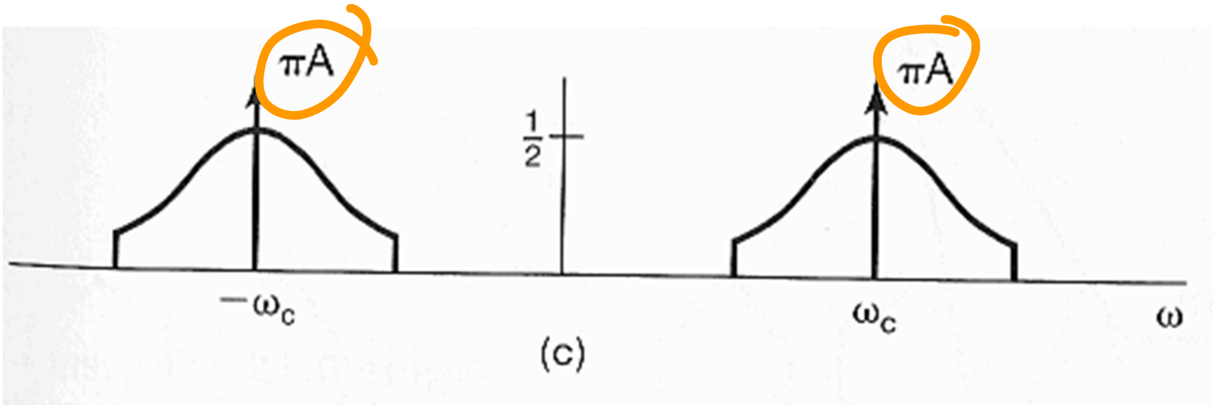
- ## Synchronous & Asynchronous Demodulation:



$x(t)$ $\cos(w_c t)$



$[x(t) + A] \cos(w_c t)$

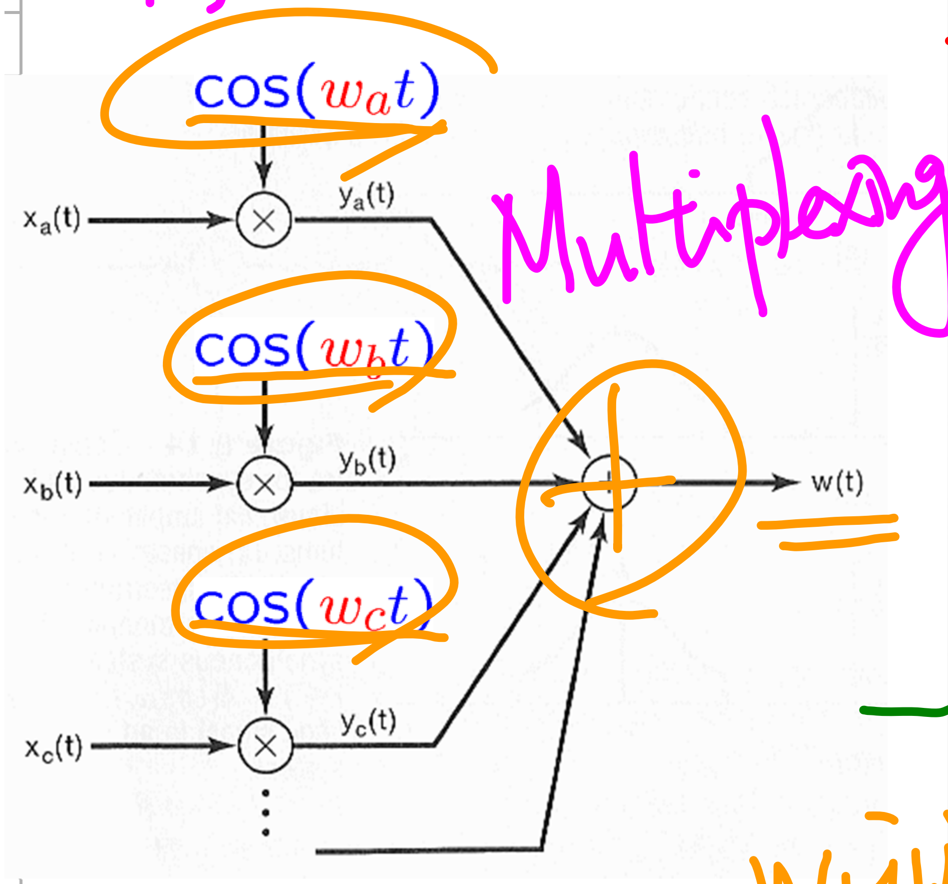


Outline

- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
  - » Time-Division Multiplexing
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

- FDM Using Sinusoidal AM:

AM



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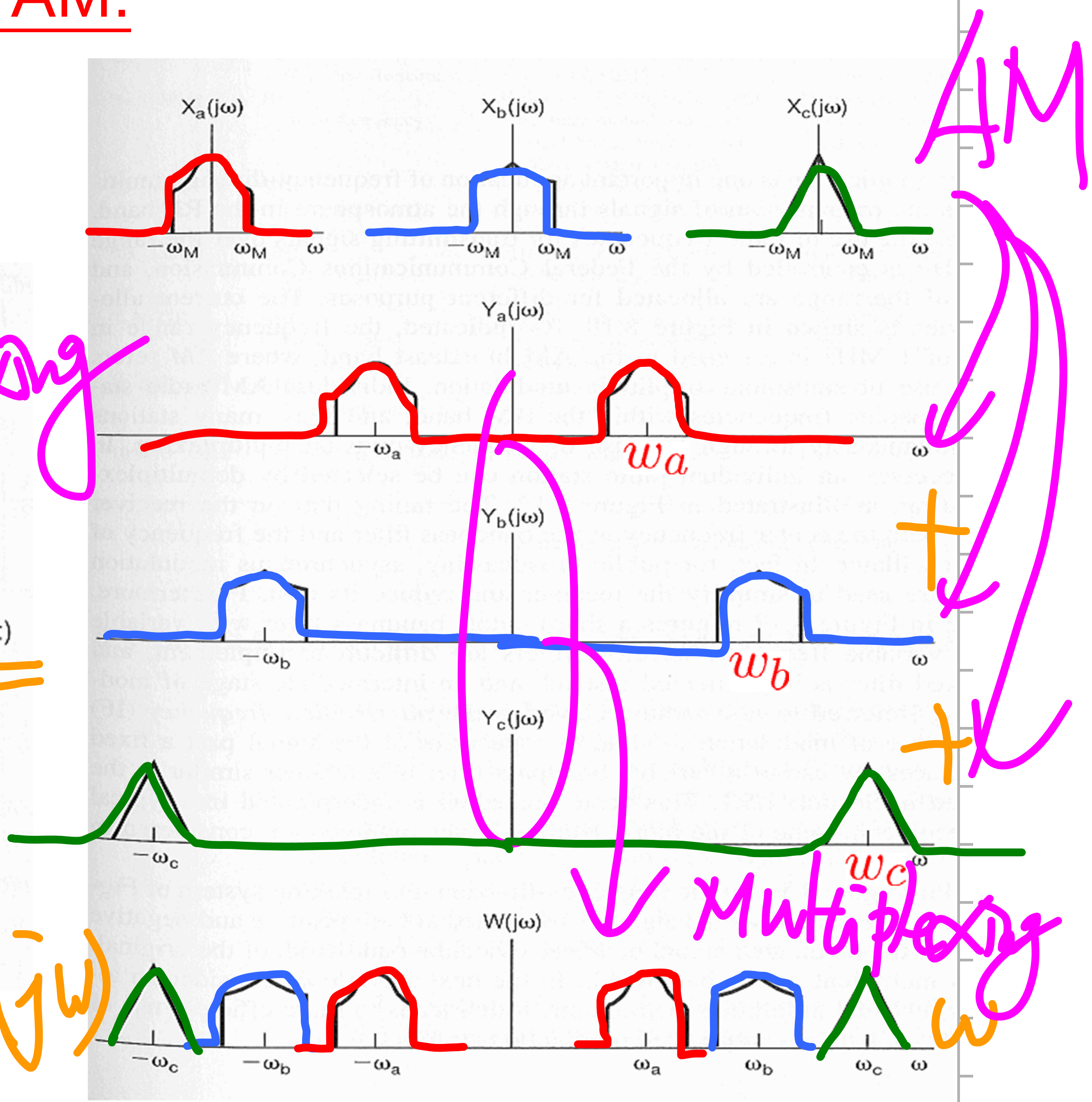
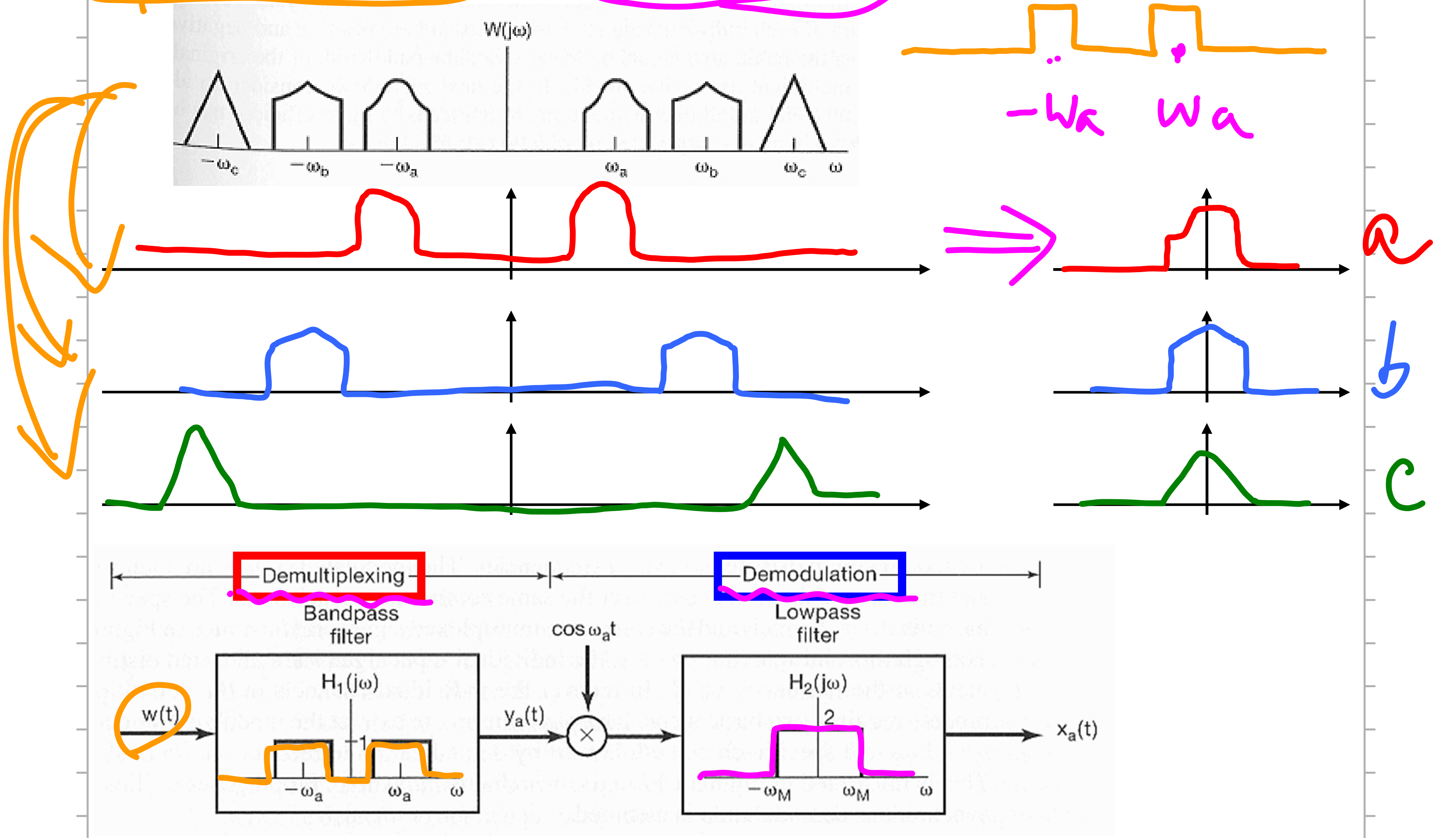


- Demultiplexing and Demodulation:



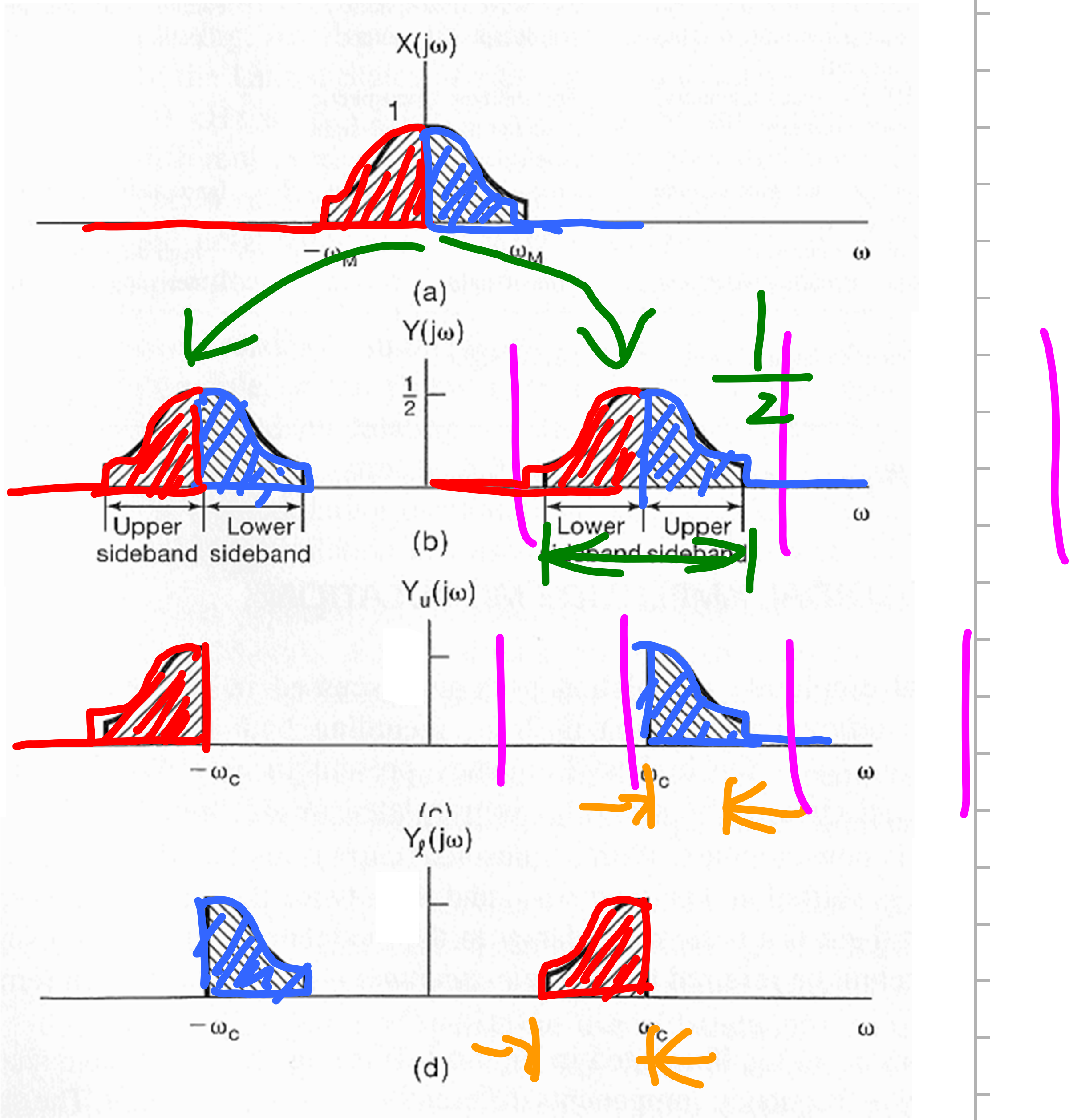
Allocation of Frequencies in the RF Spectrum

Frequency range	Designation	Typical uses	Propagation method	Channel features
30–300 Hz	ELF (extremely low frequency)	Macrowave, submarine communication	Megametric waves	Penetration of conducting earth and seawater
0.3–3 kHz	VF (voice frequency)	Data terminals, telephony	Copper wire	
3–30 kHz	VLF (very low frequency)	Navigation, telephone, telegraph, frequency and timing standards	Surface ducting (ground wave)	Low attenuation, little fading, extremely stable phase and frequency, large antennas
30–300 kHz	LF (low frequency)	Industrial (power line) communication, aeronautical and maritime long-range navigation, radio beacons	Mostly surface ducting	Slight fading, high atmospheric pulse
0.3–3 MHz	MF (medium frequency)	Mobile, AM broadcasting, amateur, public safety	Ducting and ionospheric reflection (sky wave)	Increased fading, but reliable
3–30 MHz	HF (high frequency)	Military communication, aeronautical mobile, international fixed, amateur and citizen's band, industrial	Ionospheric reflecting sky wave, 50–400 km layer altitudes	Intermittent and frequency-selective fading, multipath
30–300 MHz	VHF (very high frequency)	FM and TV broadcast, land transportation (taxis, buses, railroad)	Sky wave (ionospheric and tropospheric scatter)	Fading, scattering, and multipath
0.3–3 GHz	UHF (ultra high frequency)	UHF TV, space telemetry, radar, military	Transhorizon tropospheric scatter and line-of-sight relaying	
3–30 GHz	SHF (super high frequency)	Satellite and space communication, common carrier (CC), microwave	Line-of-sight ionosphere penetration	Ionospheric penetration, extraterrestrial noise, high directly
30–300 GHz	EHF (extremely high frequency)	Experimental, government, radio astronomy	Line of sight	Water vapor and oxygen absorption
10^3–10^7 GHz	Infrared, visible light, ultraviolet	Optical communications	Line of sight	

Outline

- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
  - » Time-Division Multiplexing
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

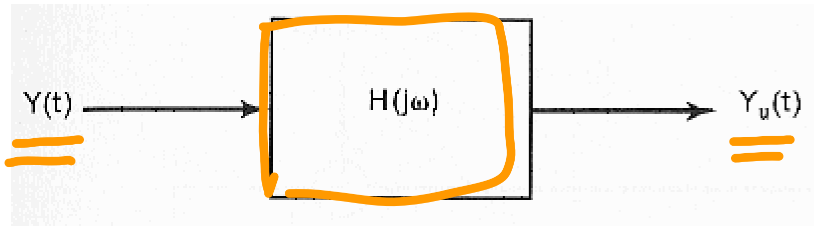
- SSB Modulation:

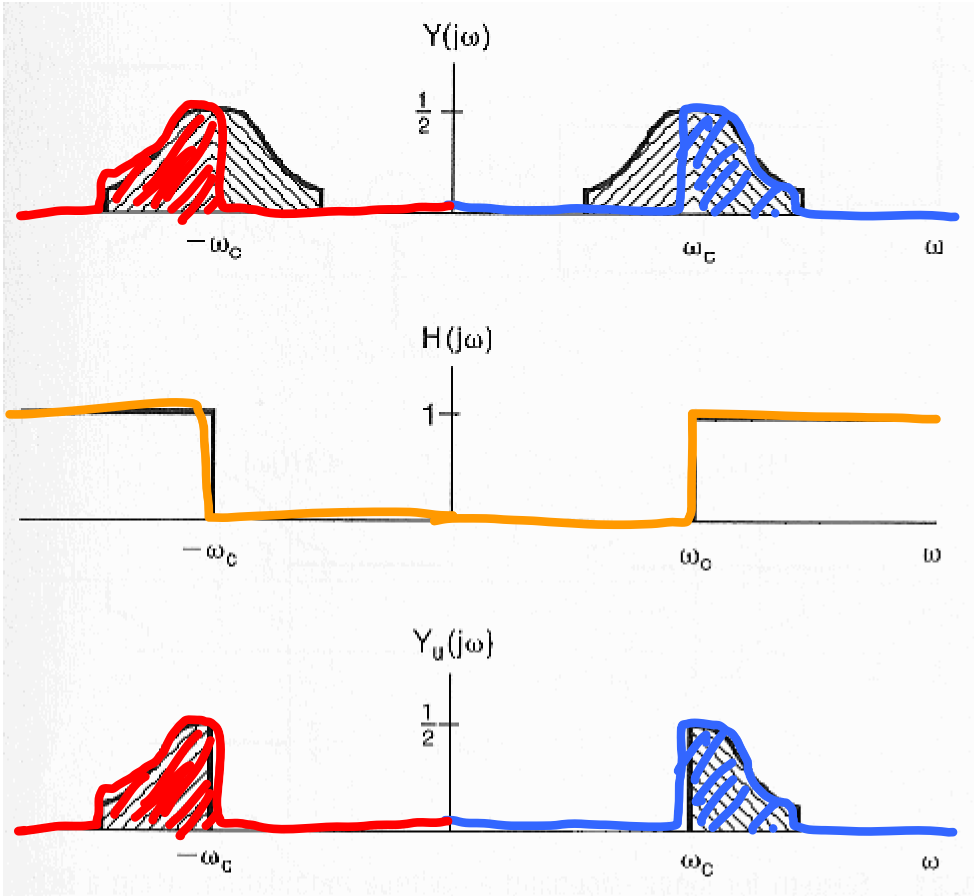


upper sidebands

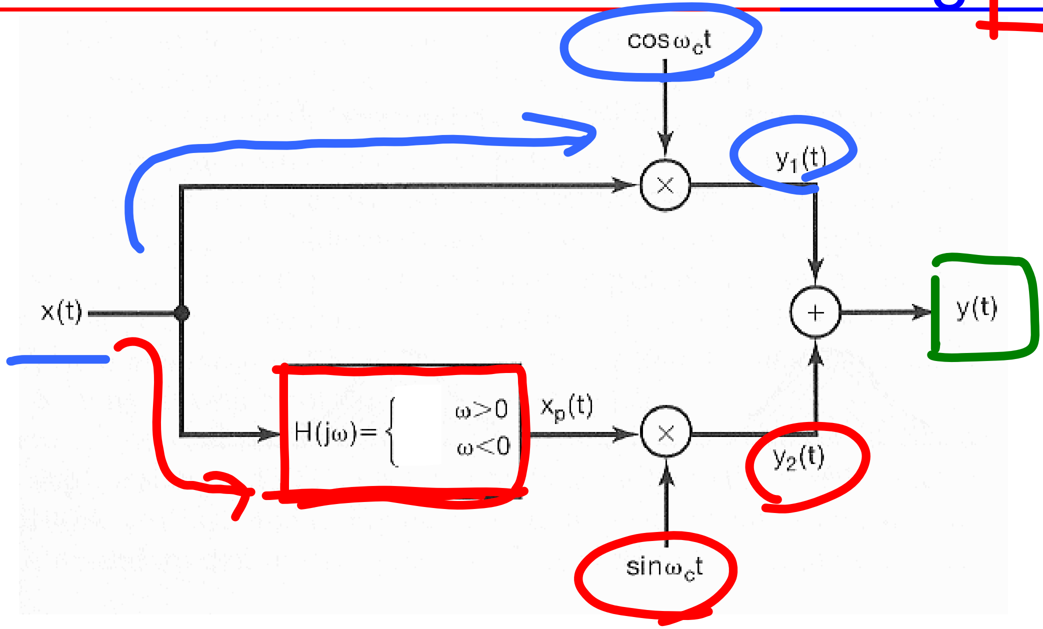
lower sidebands

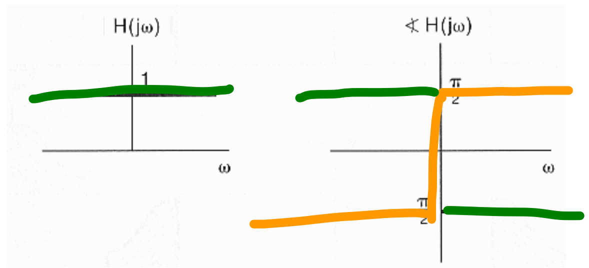
- Retain Upper Sidebands Using Ideal Highpass Filter





- Retain Lower Sidebands Using Phase-Shift Network





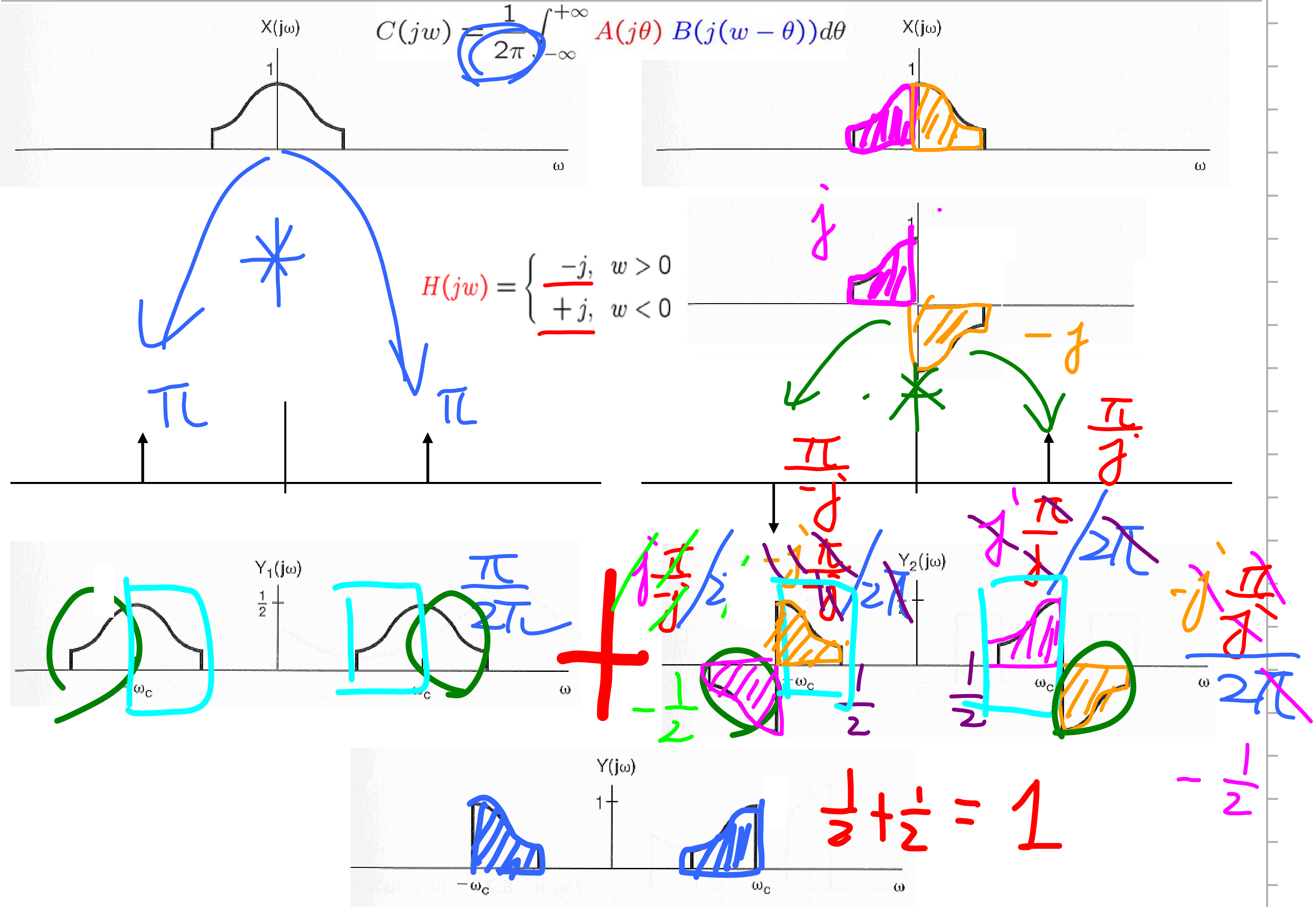
- Retain Lower Sidebands $H(jw) = \begin{cases} -j, & w > 0 \\ j, & w < 0 \end{cases}$

- Retain Upper Sidebands $H(jw) = \begin{cases} +j, & w > 0 \\ -j, & w < 0 \end{cases}$

Single-Sideband Sinusoidal Amplitude Modulation

$$C(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A(j\theta)\, B(j(w-\theta))\, d\theta$$

$$H(jw) = \begin{cases} -j, & w > 0 \\ +j, & w < 0 \end{cases}$$



$\frac{1}{2} + \frac{1}{2} = 1$

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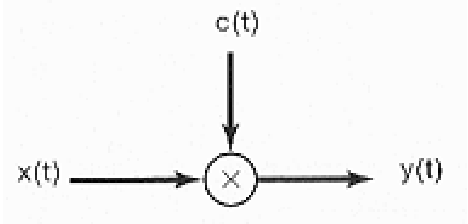
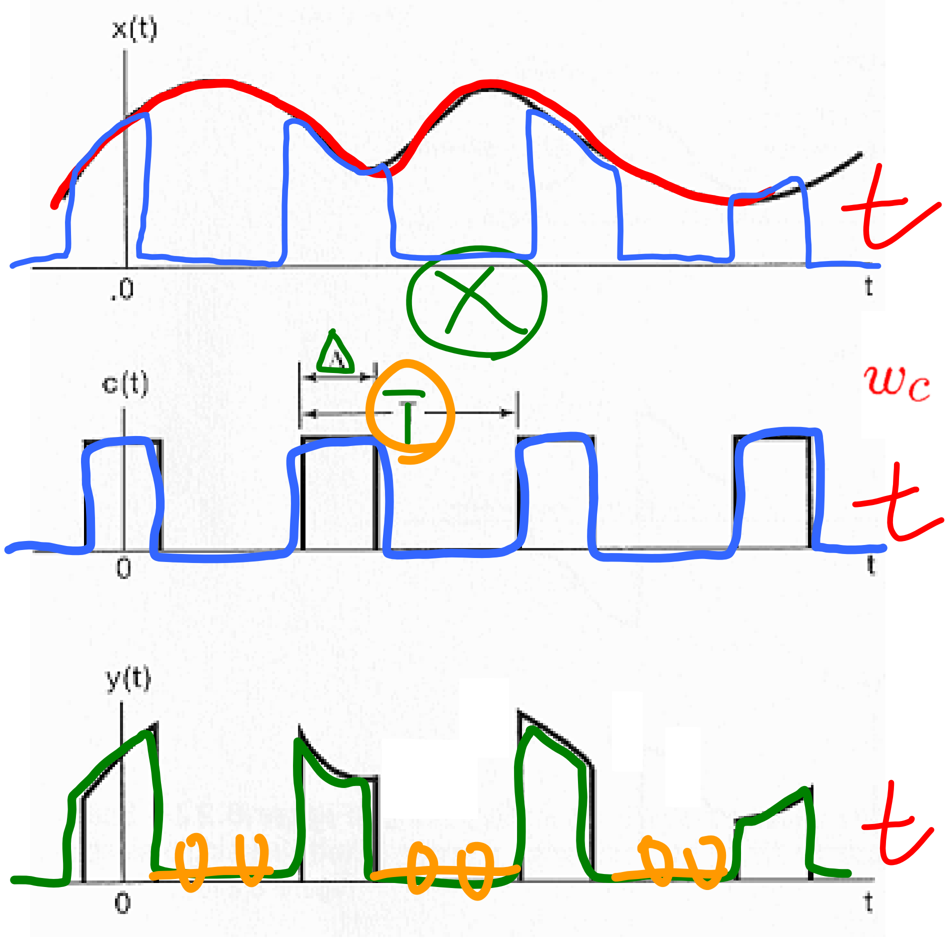
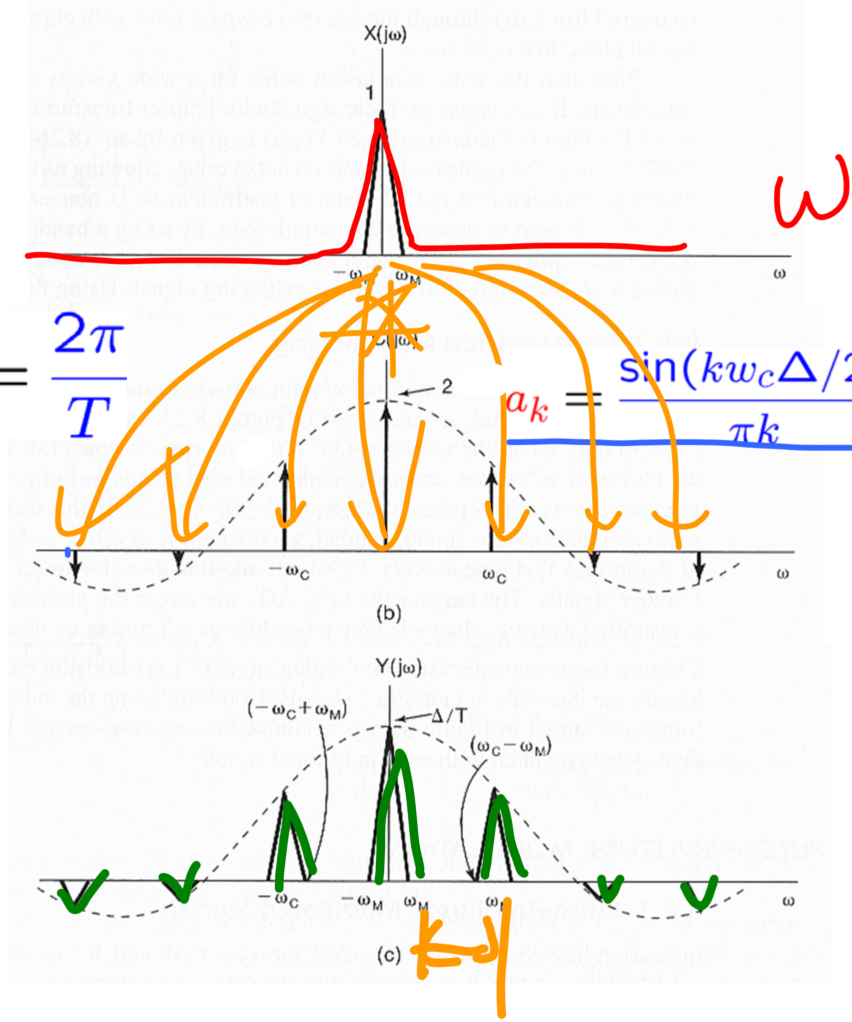
- Modulation of a Pulse-Train Carrier:



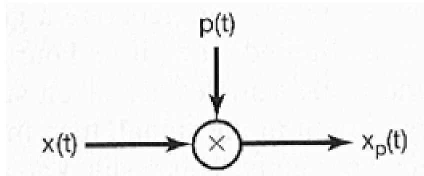


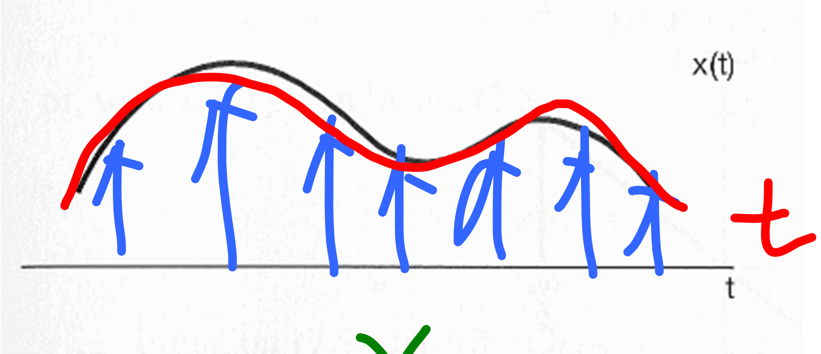
$w_c = \frac{2\pi}{T}$

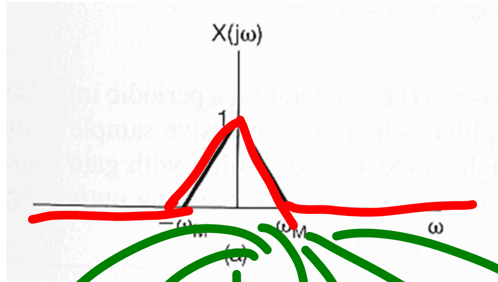
$a_k = \frac{\sin(k w_c \Delta/2)}{\pi k}$

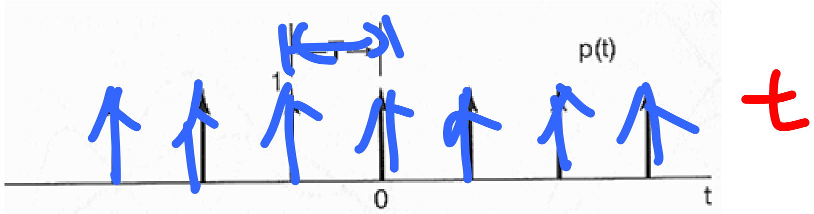


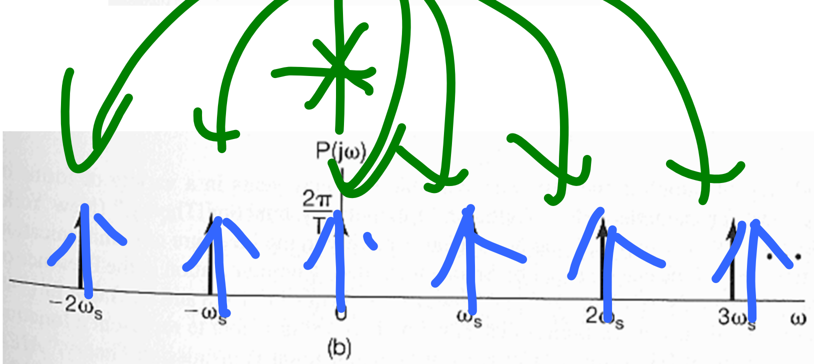
▪ Impulse-Train Sampling:



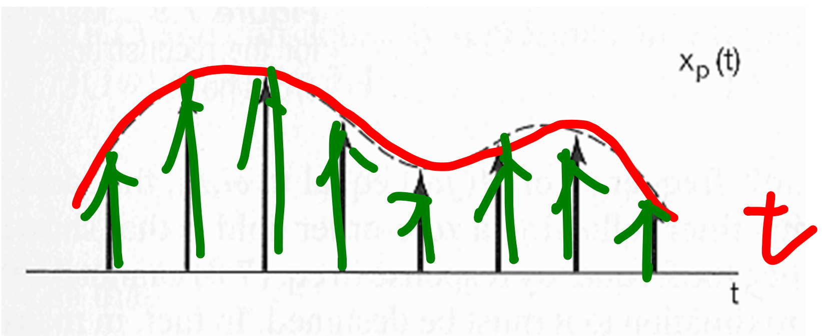


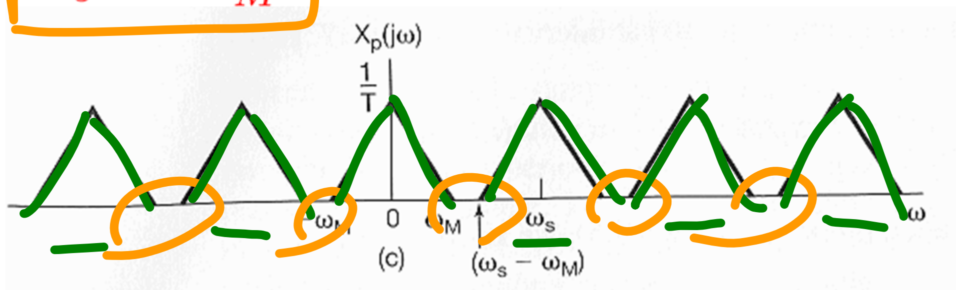




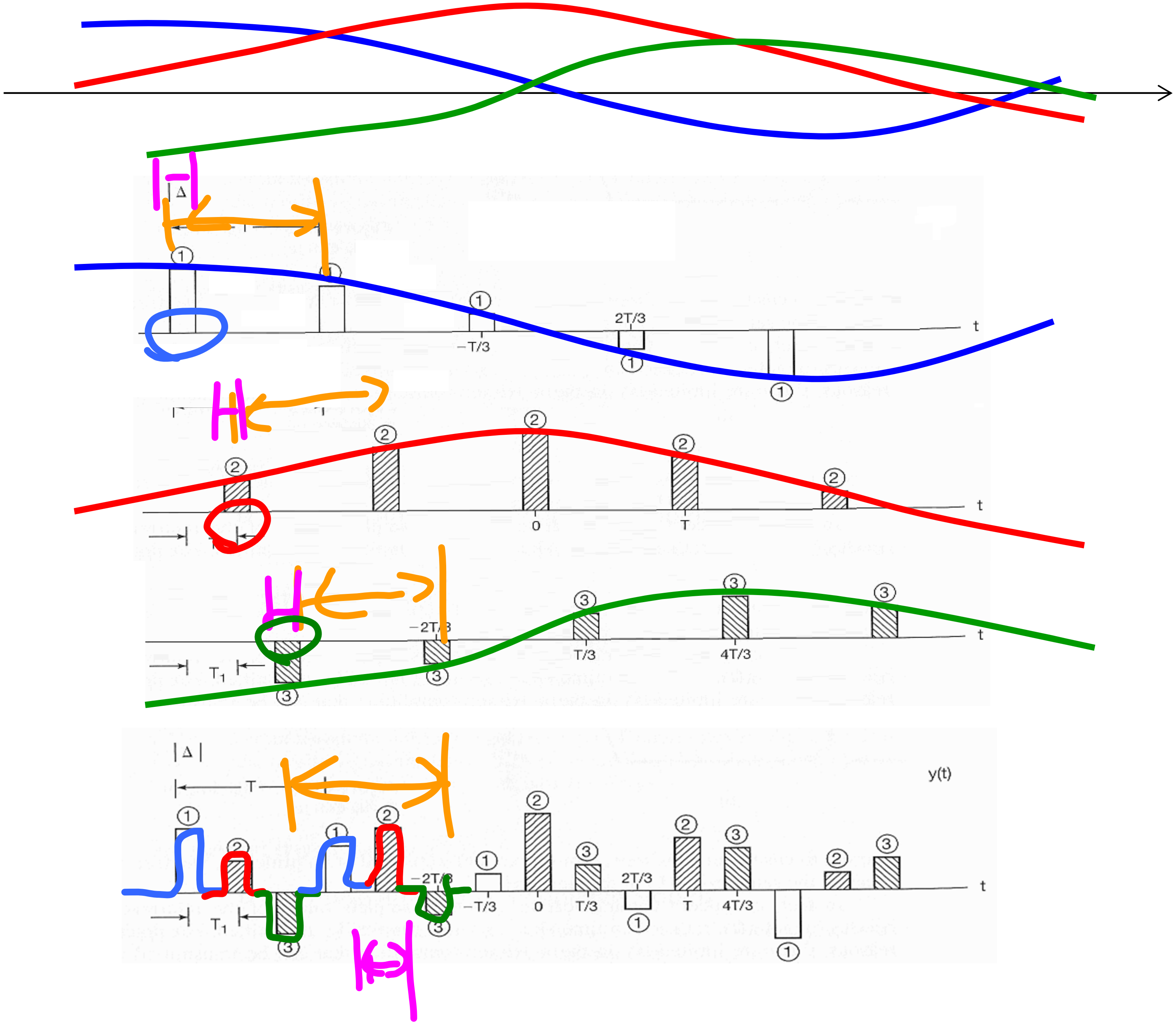


$w_s > 2w_M$

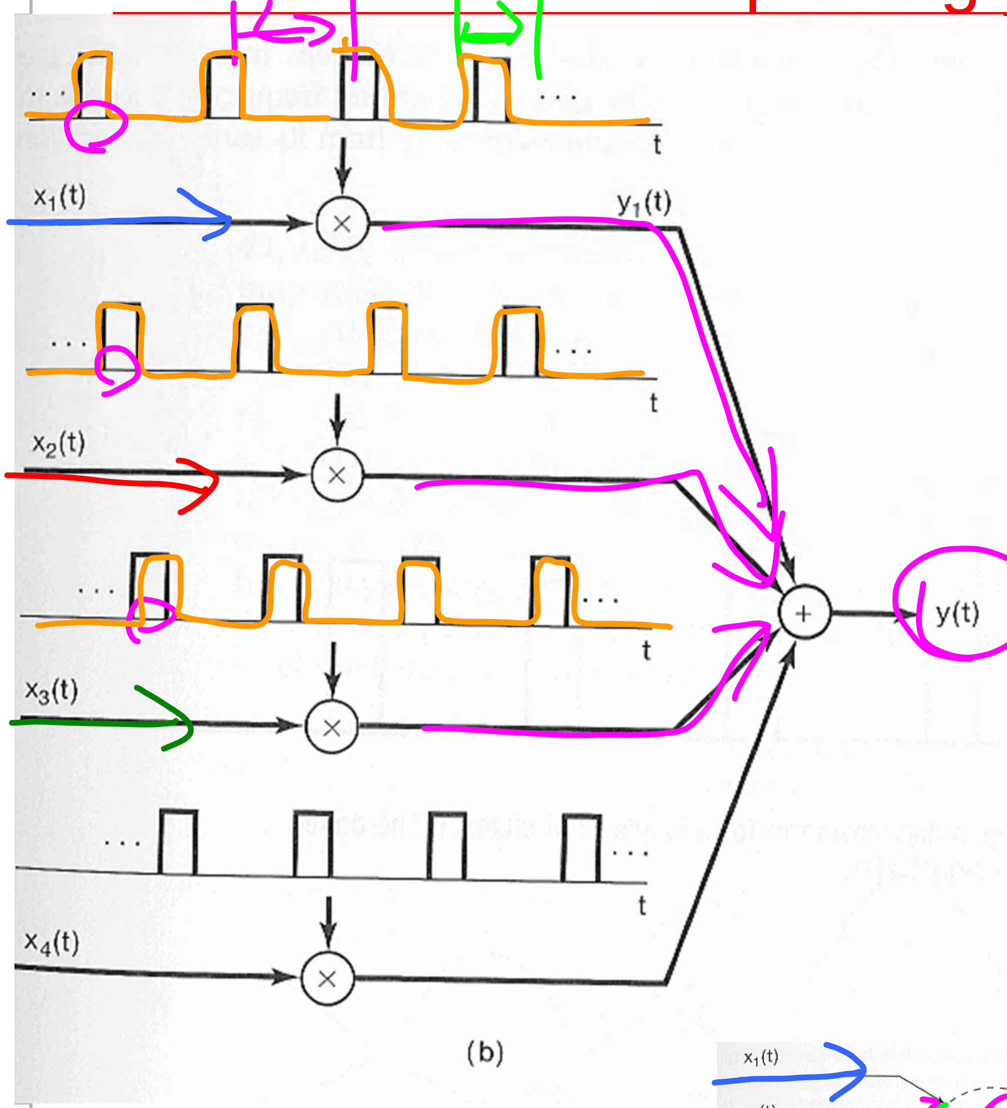


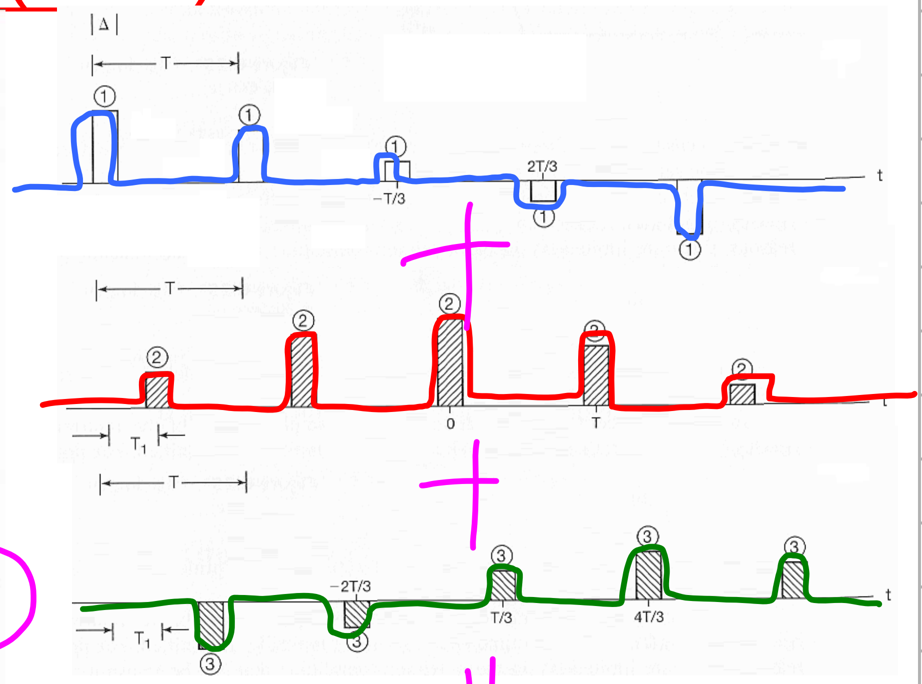


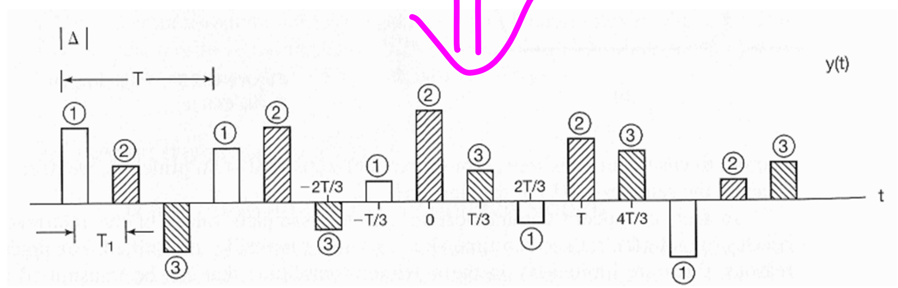
- Time-Division Multiplexing (TDM):

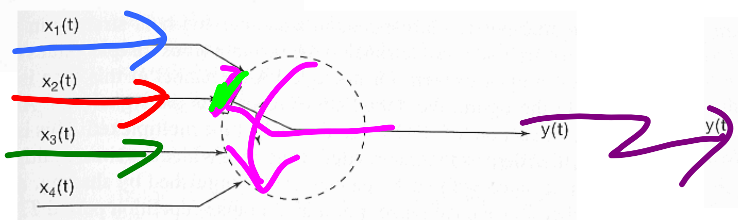


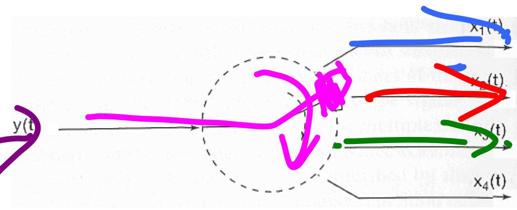
- Time-Division Multiplexing (TDM):











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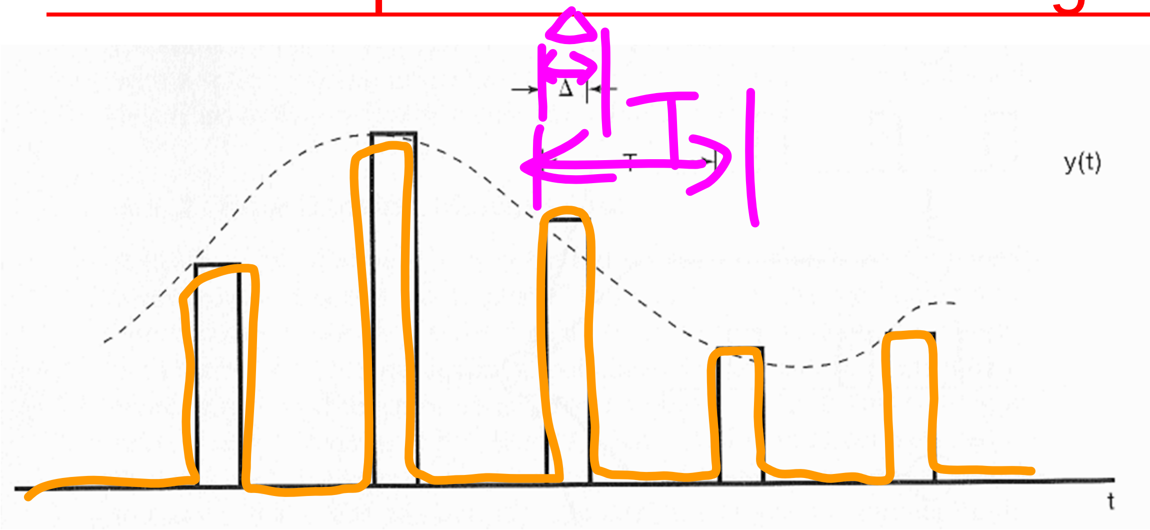
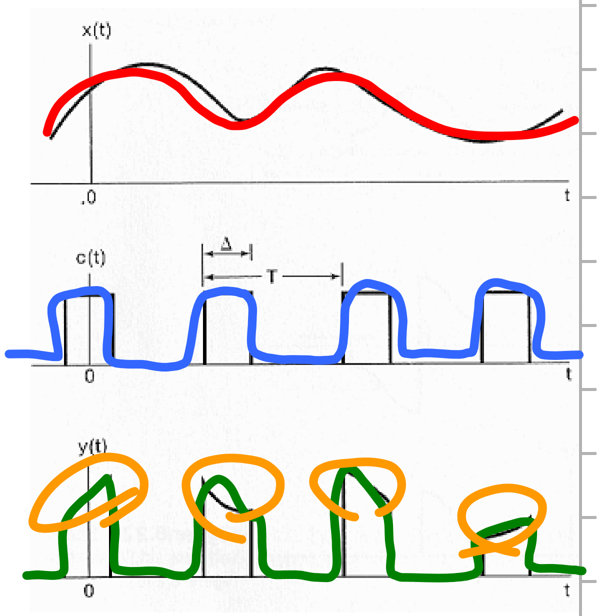
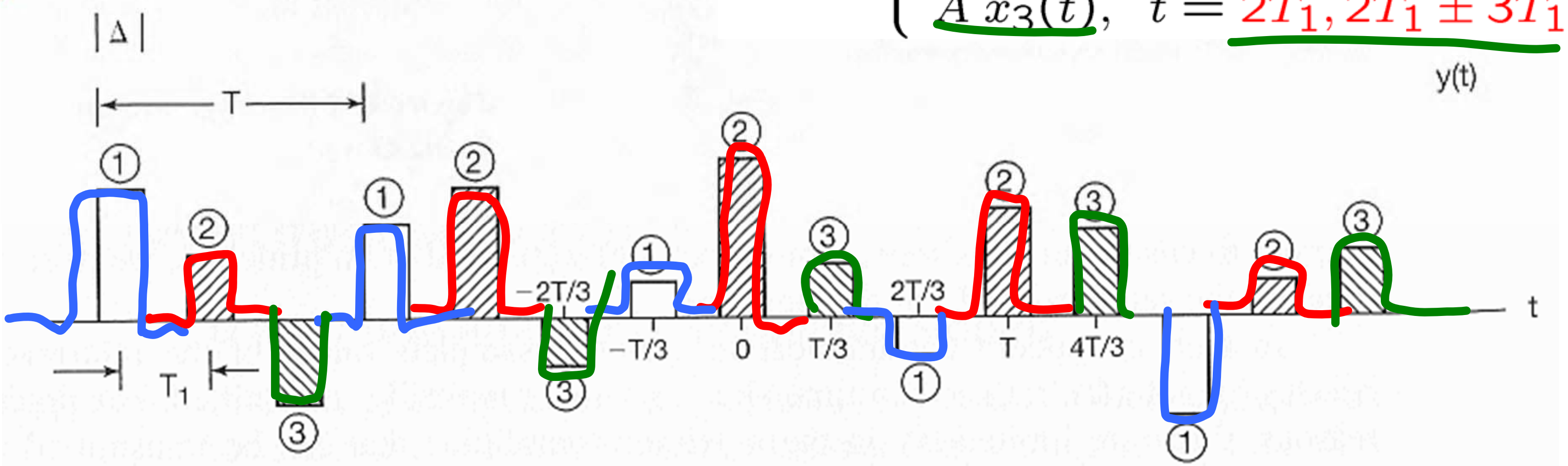
- Pulse-Amplitude Modulated Signals:



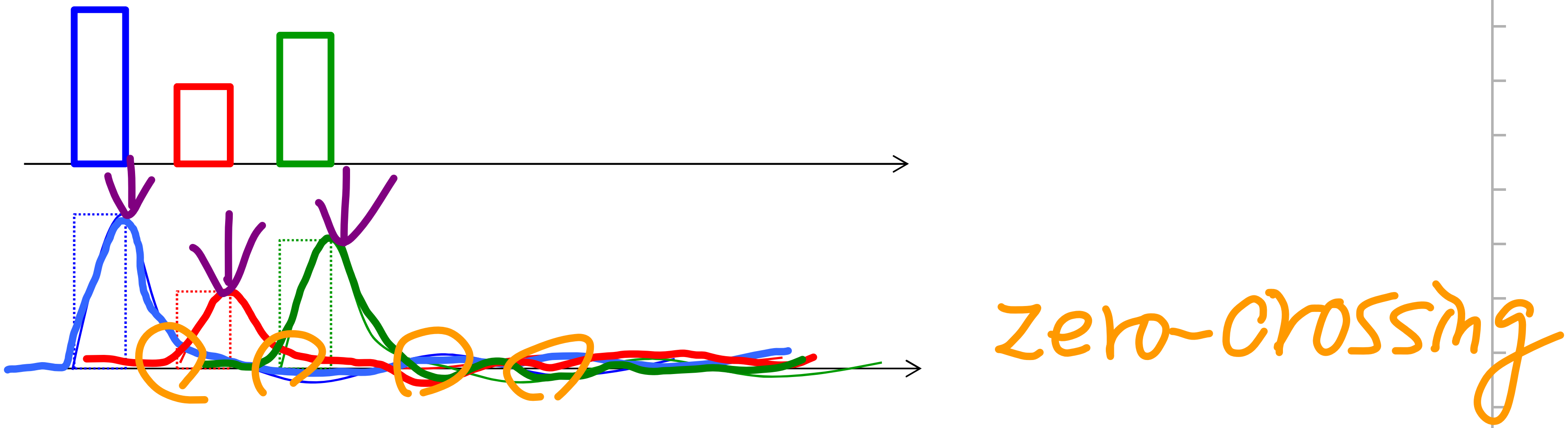


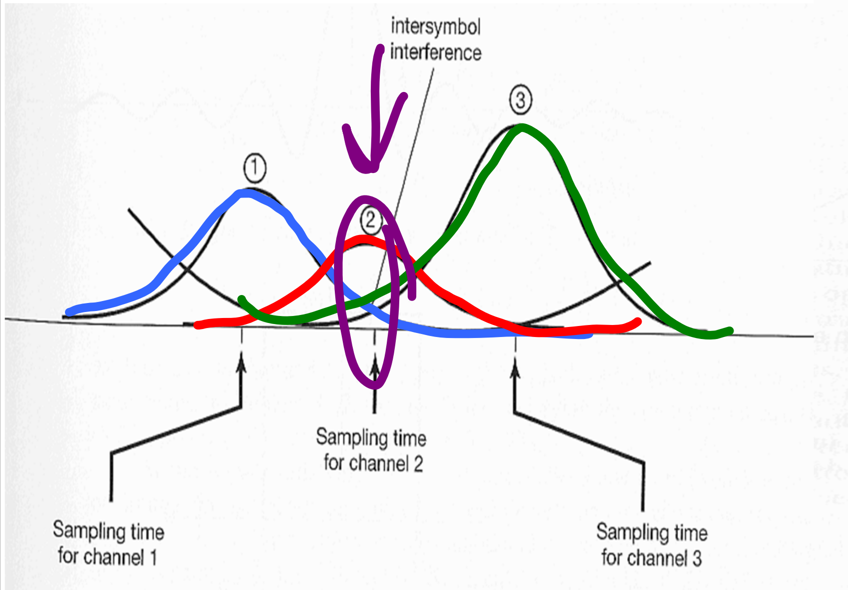
- TDM-PAM:

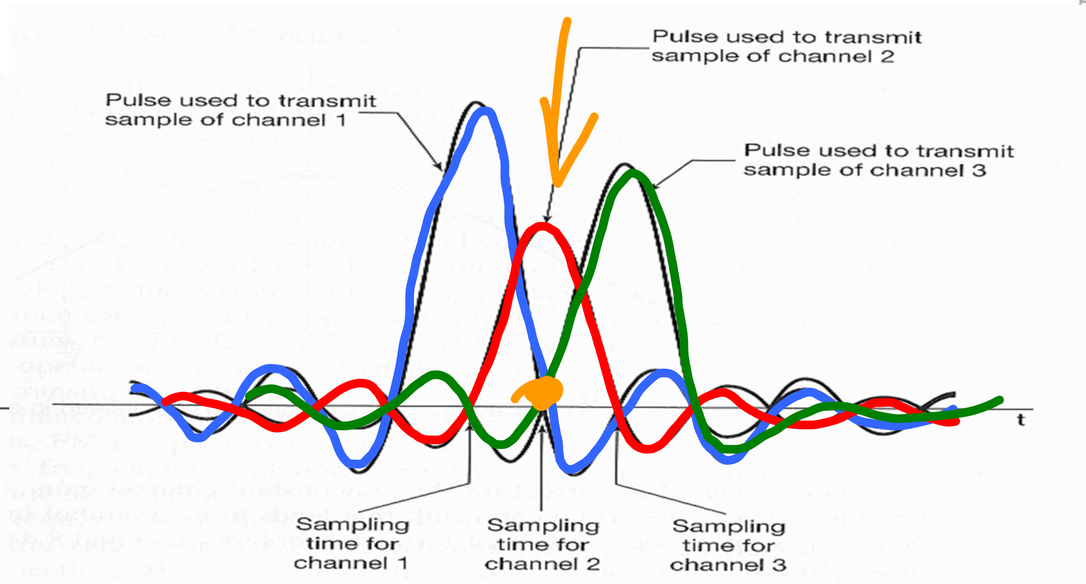
$$y(t) = \begin{cases} A\,x_1(t), & t = 0, \pm 3T_1, \cdots, \\ A\,x_2(t), & t = T_1, T_1 \pm 3T_1, \cdots, \\ A\,x_3(t), & t = 2T_1, 2T_1 \pm 3T_1, \cdots, \end{cases}$$



- Intersymbol Interference in PAM Systems:







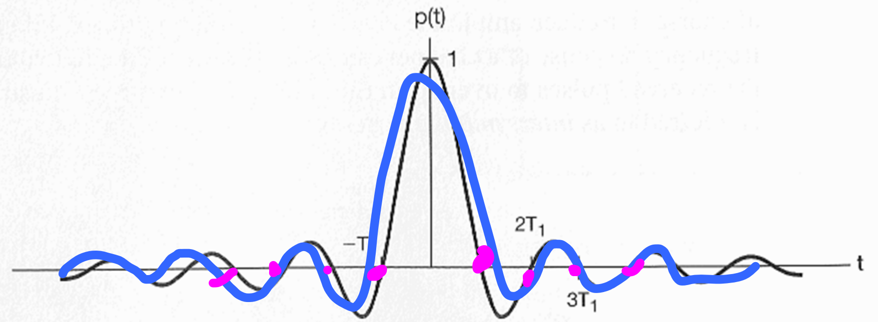
Avoiding Intersymbol Interference in PAM Systems:

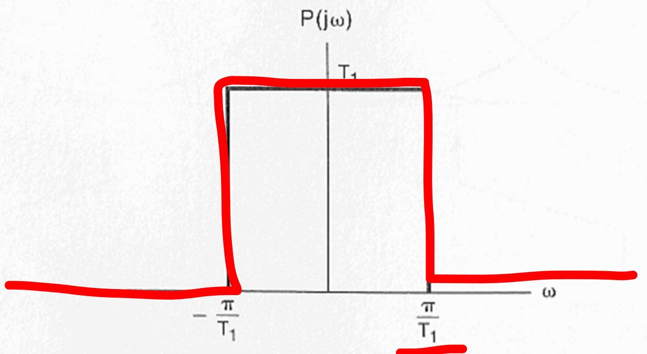
$$\boxed{p(t) = \frac{T_1 \sin(\pi\, t\, T_1)}{\pi t}}$$

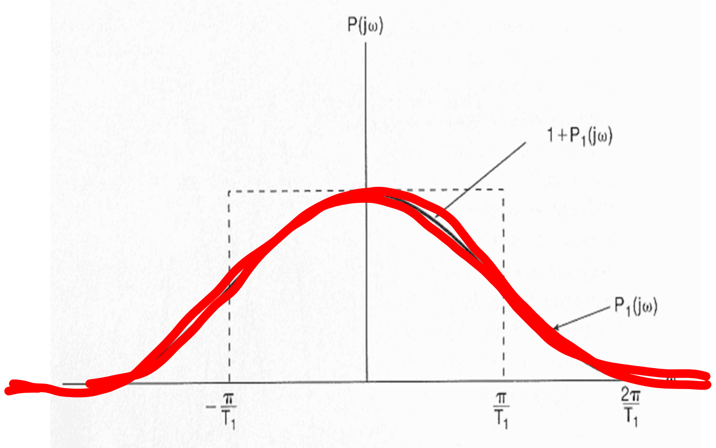
sinc

$p(\pm T_1) = 0,\ p(\pm 2T_1) = 0,\ p(\pm 3T_1) = 0, \cdots$

Zero-Crossing at kT_1







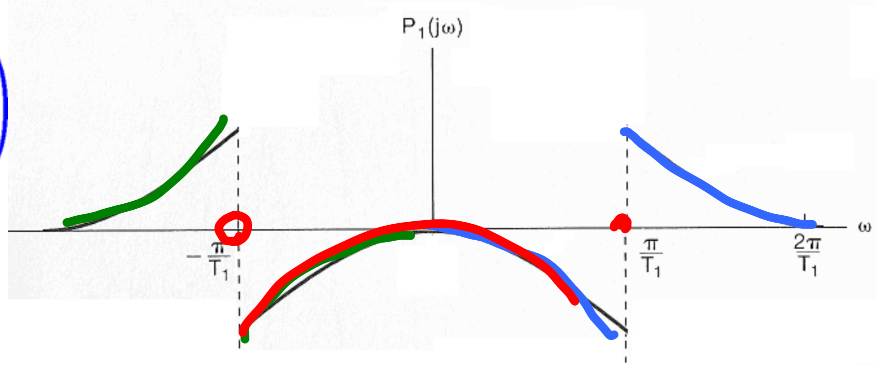
- General Form of Band-Limited Pulses with Time-Domain Zero-Crossing at kT_1, $k \in Z$:

Problem 8.42

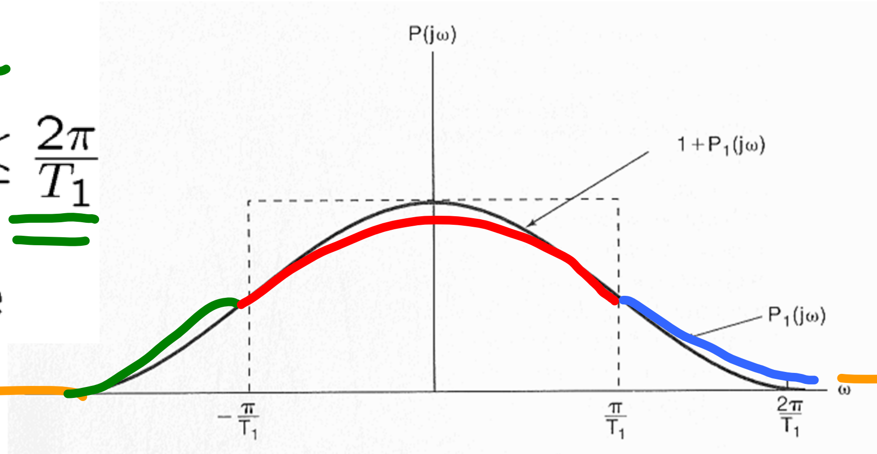
$P_1(jw)$: odd symmetry around π/T_1

$$P_1\left(-jw + j\frac{\pi}{T_1}\right) = -P_1\left(jw + j\frac{\pi}{T_1}\right)$$

$$0 \leq w \leq \frac{\pi}{T_1}$$

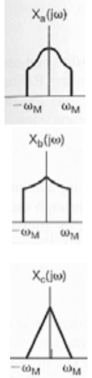


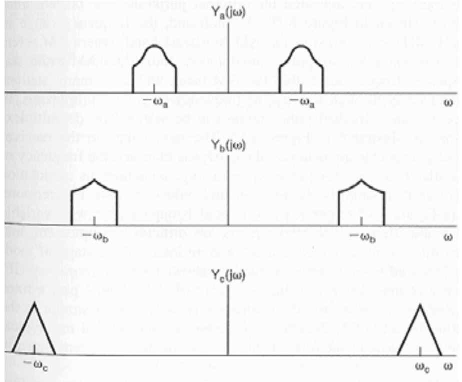
$$P(jw) = \begin{cases} 1 + P_1(jw) & |w| \leq \frac{\pi}{T_1} \\ P_1(jw) & \frac{\pi}{T_1} < |w| \leq \frac{2\pi}{T_1} \\ 0 & \text{otherwise} \end{cases}$$

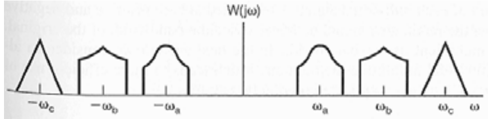


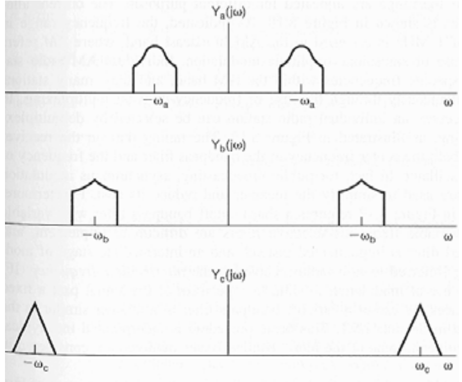
$\Rightarrow$ $p(t)$ has zero crossing at $\pm T_1, \pm 2T_1, \cdots$ i.e., $p(\pm kT_1) = 0$

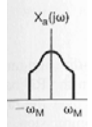
(De)Modulation and (De)Multiplexing

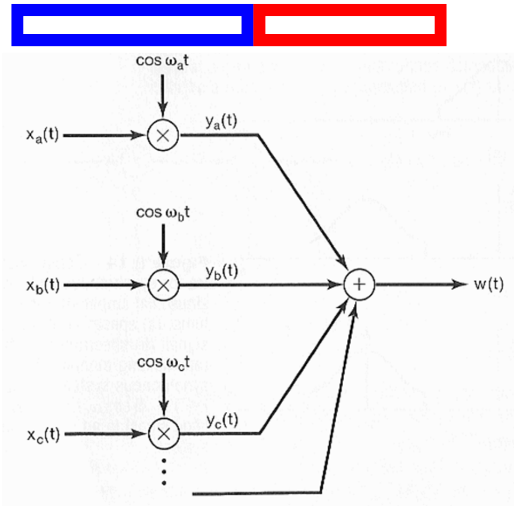


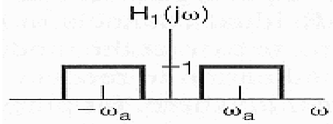


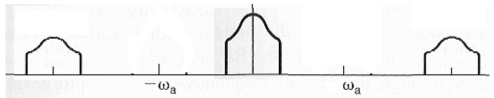


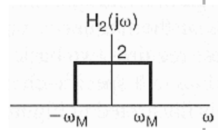


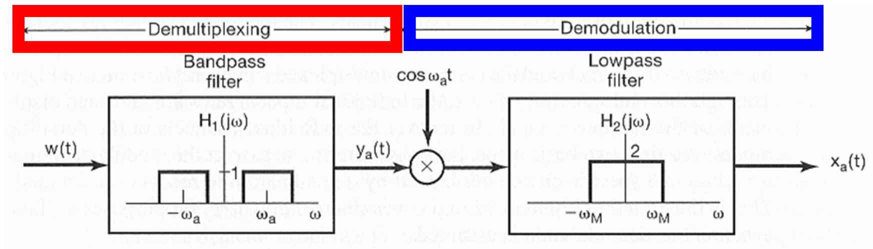


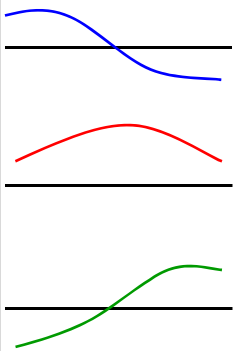


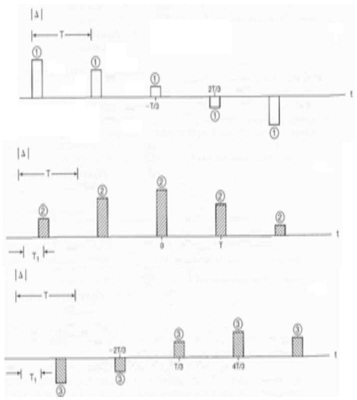


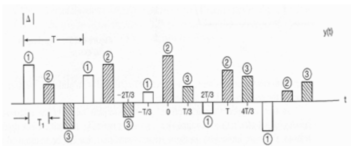


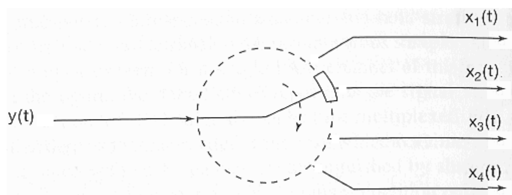












AM

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- Sinusoidal Frequency Modulation FM
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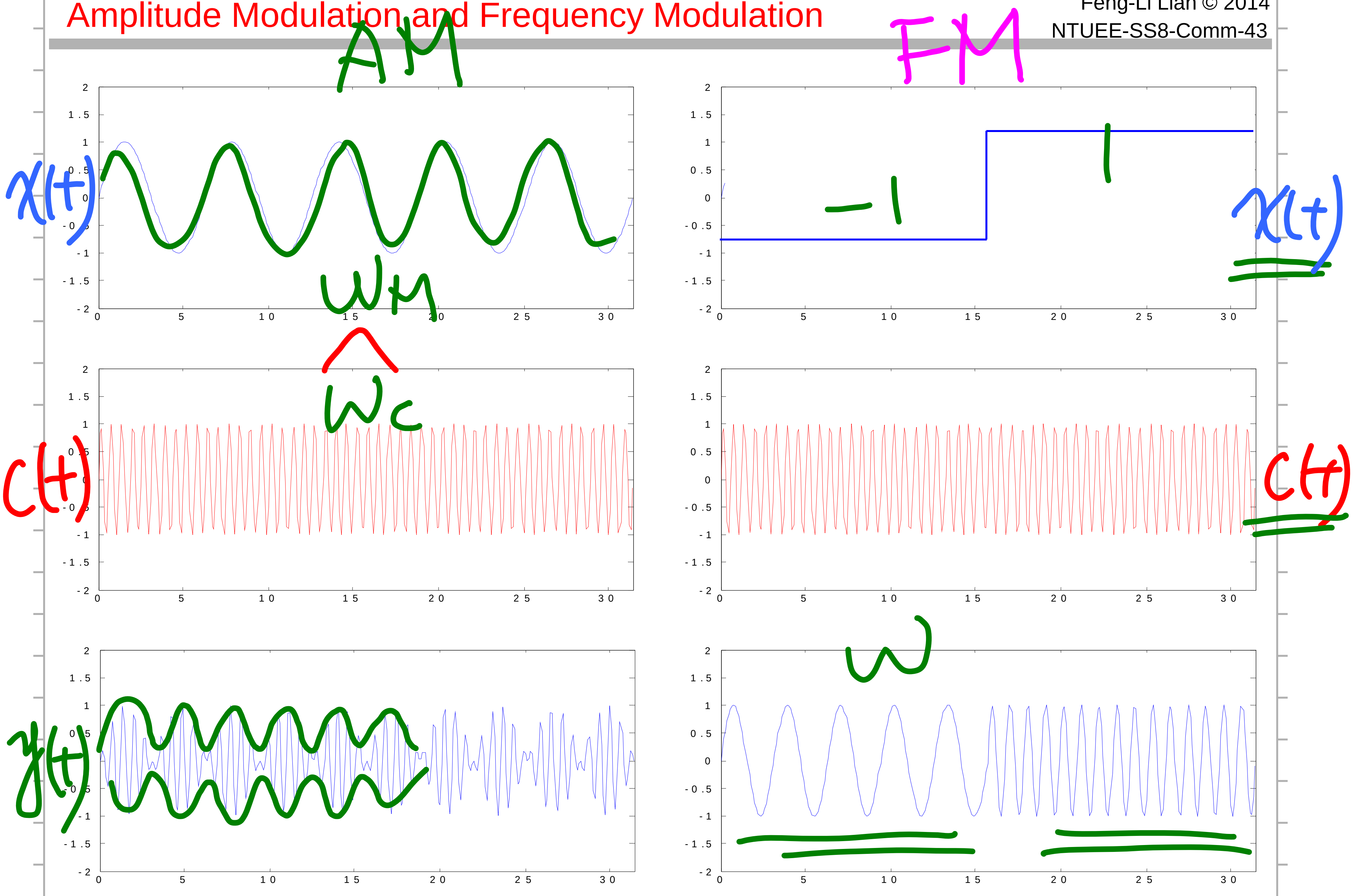
Amplitude Modulation and Frequency Modulation



Frequency Modulation (FM):

- The modulating signals is used
  to control the frequency of a sinusoidal carrier

- With sinusoidal AM, the peak amplitude of the envelope of the carrier
  directly depends on the amplitude of the modulating signal $x(t)$,
  which can have a large dynamic range.

- With FM, the envelope of the carrier is constant

- An FM transmitter can always operate at peak power and
  amplitude variations introduced over a transmission channel
  due to additive disturbances or fading
  can be eliminated at the receiver

- FM generally requires greater bandwidth than does sinusoidal AM

- **Angle Modulation:**

$$c(t) = A\cos(w_c t + \theta_c) = A\cos\theta(t)$$

- Phase Modulation:
  - Use the modulating signal x(t) to vary the phase θ_c

$$y(t) = A\cos(\theta(t)) = A\cos(w_c t + \theta_c(t))$$

x(t)

$$\theta_c(t) = \theta_0 + k_p x(t)$$

$$\frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

- Frequency Modulation:
  - Use the modulating signal x(t) to vary the derivative of the angle

$$y(t) = A\cos(\theta(t)) \qquad \frac{d\theta(t)}{dt} = w_c + k_f x(t)$$

- Phase & Frequency Modulation:

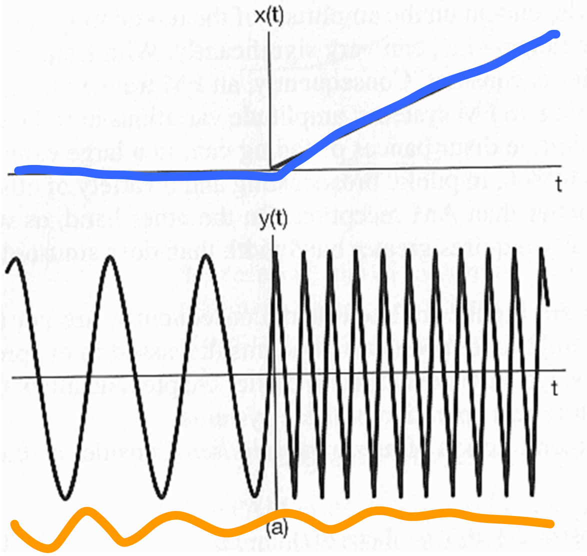
$$\frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

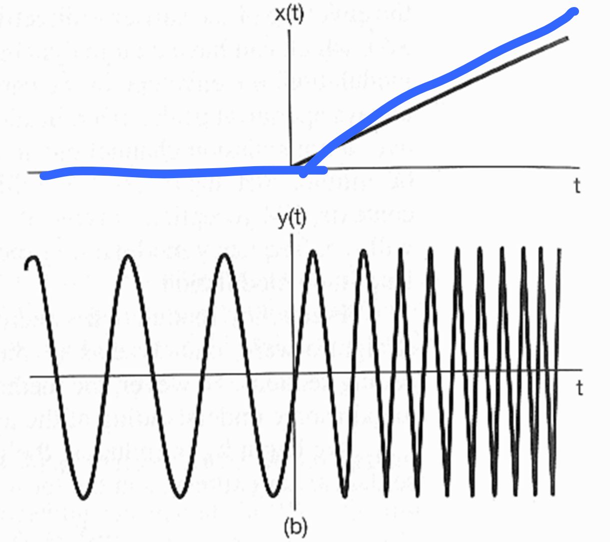
$$\frac{d\theta(t)}{dt} = w_c + k_f \, x(t)$$

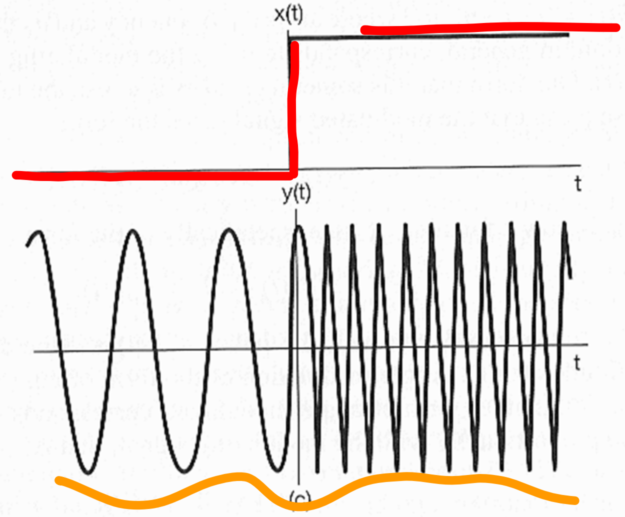
phase modulation

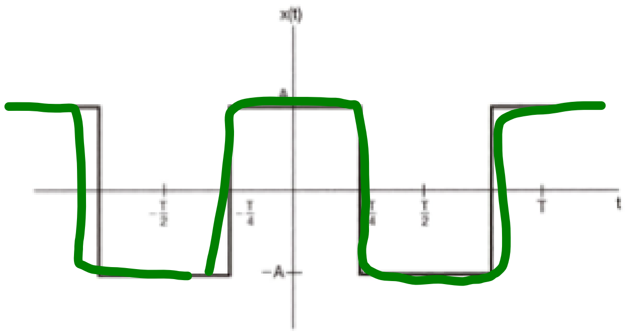
frequency modulation

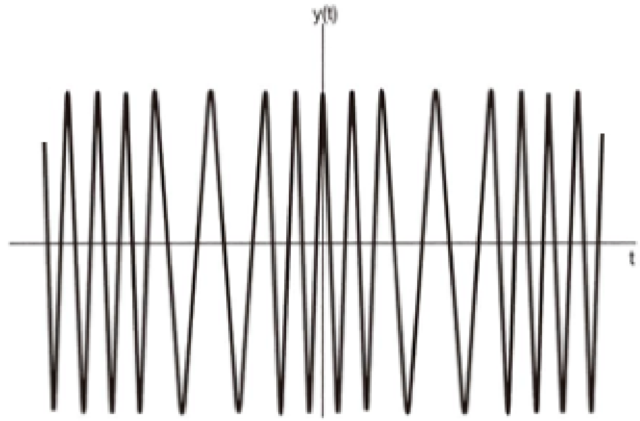
frequency modulation











Instantaneous Frequency:

$$y(t) = A \cos(\theta(t)) \quad \Rightarrow \quad w_i = \frac{d\theta(t)}{dt}$$

- If y(t) is truly sinusoidal:

$$\theta(t) = w_c t + \theta_0 \qquad w_i = w_c$$

- Phase Modulation:

$$w_i = \frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

- Frequency Modulation:

$$w_i = \frac{d\theta(t)}{dt} = w_c + k_f \, x(t)$$

- **Wideband FM:** $c(t) = A\cos\left(w_c t + \theta_c\right) = A\cos\theta(t)$
- **Narrowband FM:** $x(t) = A\cos\left(w_m t\right)$
  - Frequency Modulation with $w_i = \dfrac{d\theta(t)}{dt} = w_c + k_f\, x(t)$
  - Instantaneous Frequency:

AM

$$w_i(t) = \frac{d\theta(t)}{dt} = w_c + k_f A \cos\left(w_m t\right)$$

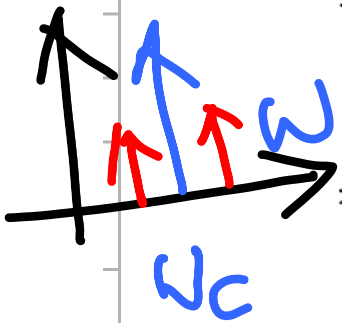
±1

$w_c \pm w_m$

$$\Rightarrow\ w_c - k_f A \le w_i(t) \le w_c + k_f A$$

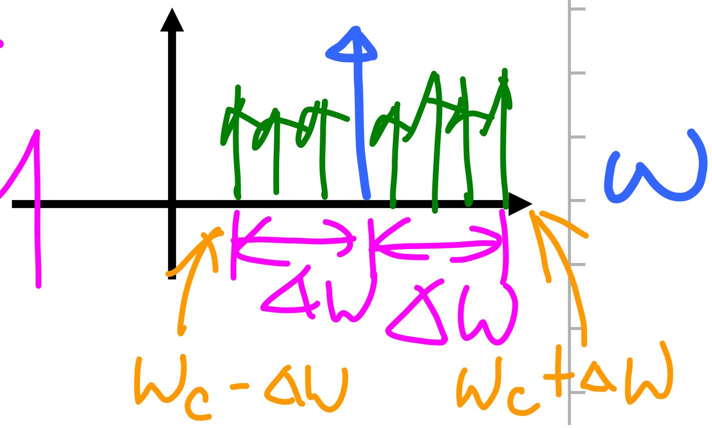
2Δw

w_c



$$\Rightarrow\ \Delta w \triangleq \boxed{k_f A}$$

FM



$$\Rightarrow\ w_i(t) = w_c + \Delta w \cos\left(w_m t\right)$$

- Wideband FM:
- Narrowband FM:

$$x(t) = A\cos(w_m t)$$

$$y(t) = \cos(\theta(t)) = \cos(w_c t + \theta_c(t))$$

$$\frac{d\theta(t)}{dt} = w_c + k_f\, x(t)$$

$$\Delta w \triangleq k_f\, A$$

$$\Rightarrow y(t) = \cos\left(w_c t + k_f \int x(t)dt\right)$$

$$= \cos\left(w_c t + k_f \frac{A}{w_m}\sin(w_m t) + \cancel{\theta_0}\right)$$

let $\theta_0 = 0$

$$= \cos\left(w_c t + \boxed{\frac{\Delta w}{w_m}}\sin(w_m t)\right)$$

- Modulation Index for FM:

$$m \triangleq \boxed{\frac{\Delta w}{w_m}}$$

MI for AM

$$m = \frac{K}{A}$$

- Which m is small → narrowband FM

- **Narrowband FM:**

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\Rightarrow y(t) = \cos\left(w_c t + m\sin(w_m t)\right)$$

$$\text{or} \quad y(t) = \cos\left(w_c t\right)\boxed{\cos\left(m\sin(w_m t)\right)} - \sin\left(w_c t\right)\boxed{\sin\left(m\sin(w_m t)\right)}$$

≈ 1 $\qquad \approx$

- When m is sufficiently small (<< π/2)

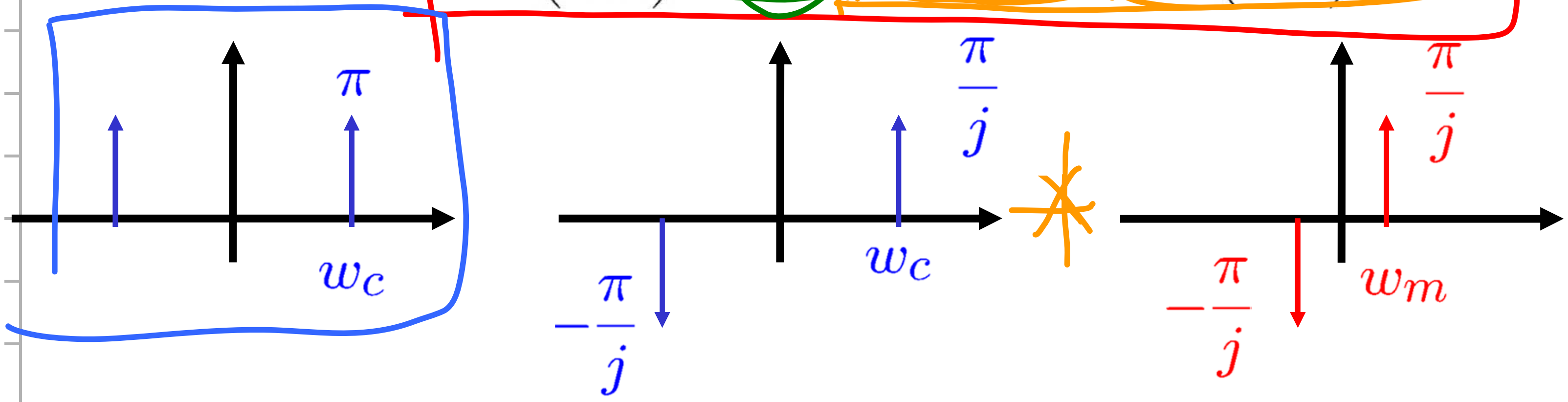
if $0 < \theta << 1$

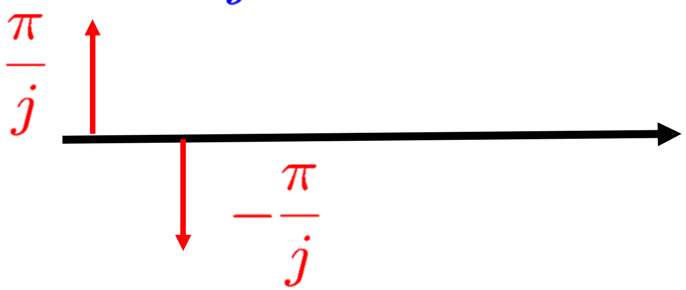
$$\Rightarrow \begin{array}{l} \boxed{\cos\left(m\sin(w_m t)\right)} \approx 1 \\ \boxed{\sin\left(m\sin(w_m t)\right)} \approx m\sin(w_m t) \end{array} \qquad \Rightarrow \begin{array}{l} \cos(\theta) \approx 1 \\ \sin(\theta) \approx \theta \end{array}$$

$$\Rightarrow y(t) \approx \cos\left(w_c t\right) - m\sin(w_m t)\sin\left(w_c t\right)$$

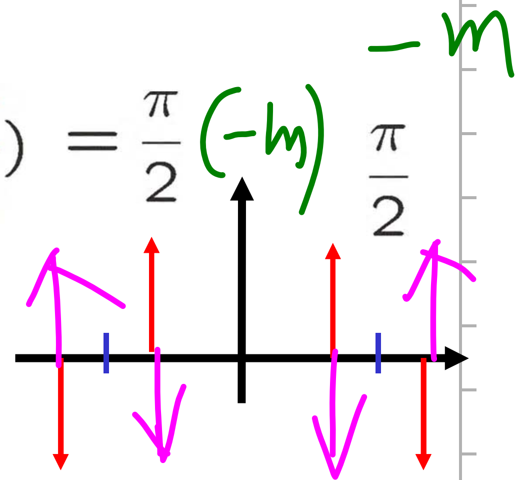
- **Narrowband FM:**

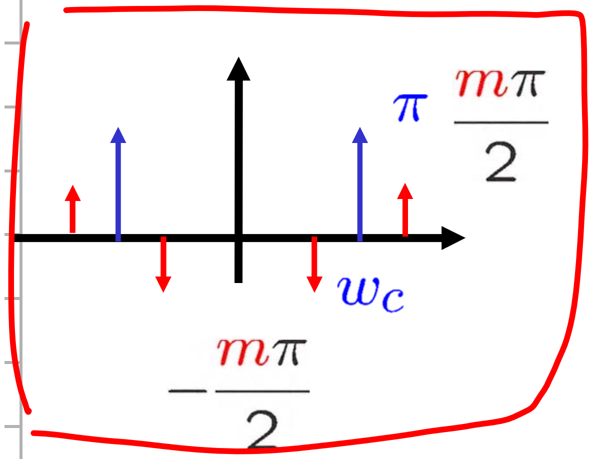
$$\Rightarrow y(t) \approx \cos(w_c t) - m \sin(w_m t) \sin(w_c t)$$





$$\frac{1}{2\pi}\left(-\frac{\pi}{j}\right)\left(\frac{\pi}{j}\right) = \frac{\pi}{2}(-m)$$



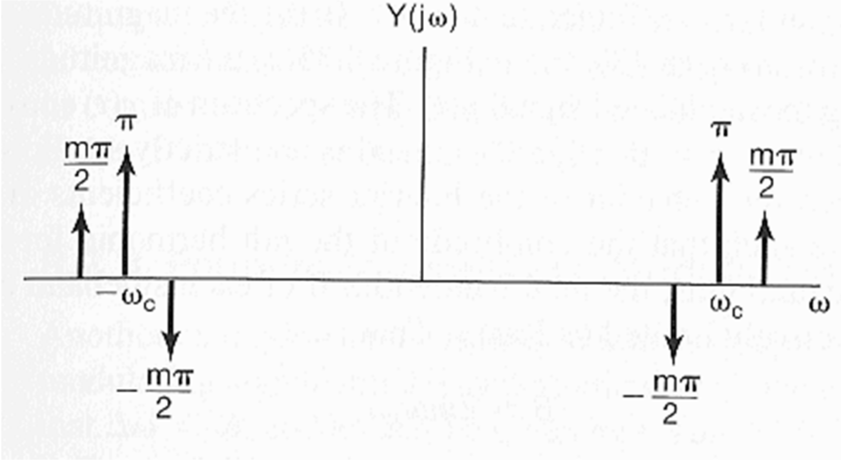


$$(-m)\left(-\frac{\pi}{2}\right)$$

$$\frac{1}{2\pi}\left(-\frac{\pi}{j}\right)\left(-\frac{\pi}{j}\right) = -\frac{\pi}{2}(-m)$$

- **Narrowband FM:**

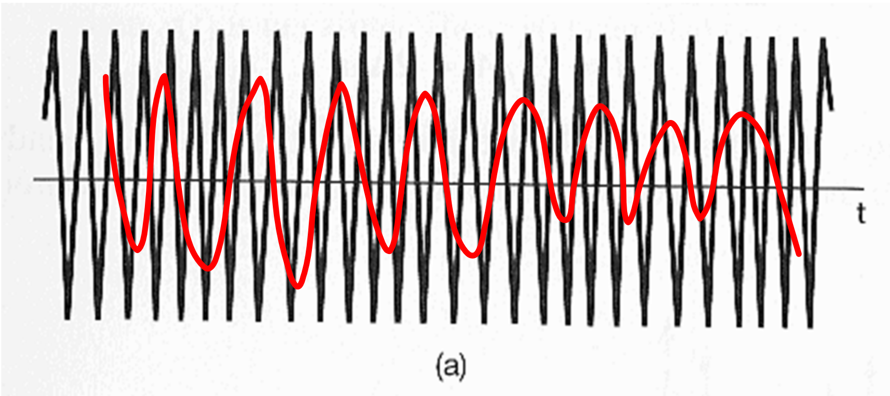
$$x(t) = A\cos(w_m t)$$



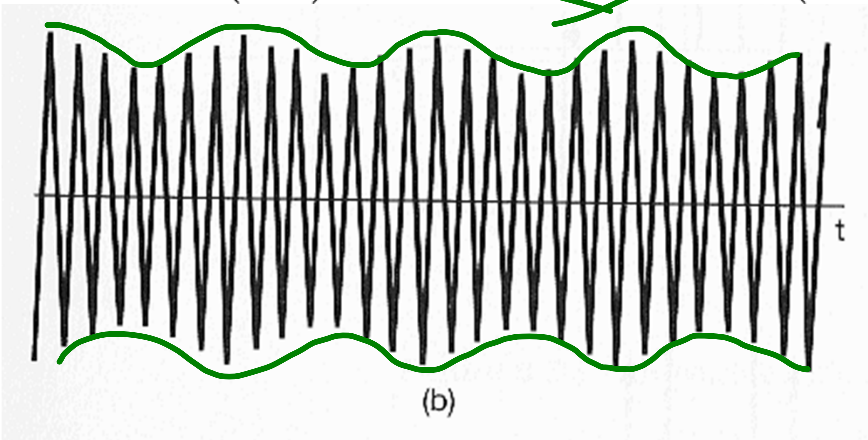
Approximate spectrum for narrowband FM

$$y(t) \approx \cos(w_c t) - m\sin(w_m t)\sin(w_c t)$$

$$y_2(t) = \cos(w_c t) + m\cos(w_m t)\cos(w_c t)$$



Narrowband FM



AM-Double Sideband/with carrier

Wideband FM:

$$m \triangleq \frac{\Delta w}{w_m}$$

- When m is large

$$y(t) = \cos(w_c t)\cos(m\sin(w_m t)) - \sin(w_c t)\sin(m\sin(w_m t))$$

+

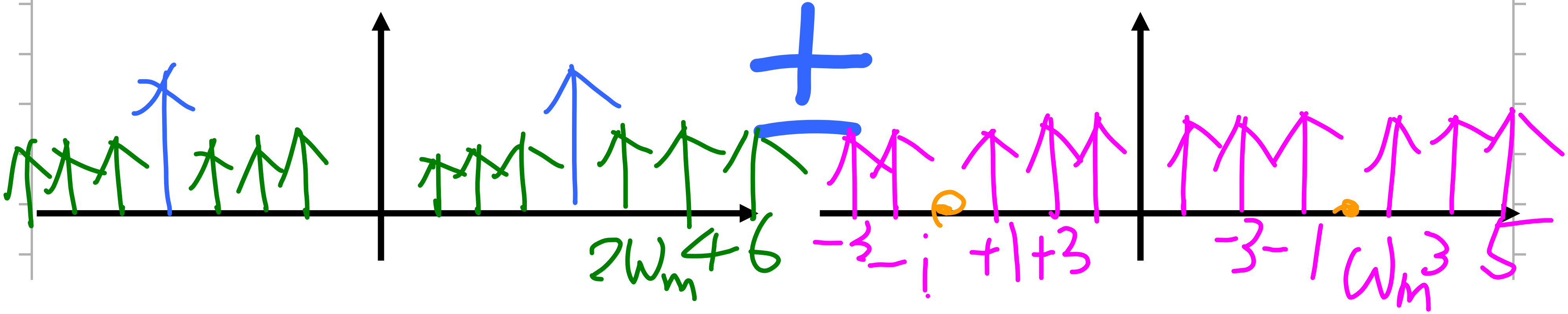
Periodic signals with fundamental frequency ω_m

2 4 6 8

$$\cos(m\sin(w_m t)) = J_0(m) + \sum_{n \text{ even}}^{\infty} 2J_n(m)\cos(nw_m t)$$

$$\sin(m\sin(w_m t)) = \sum_{n \text{ odd}}^{\infty} 2J_n(m)\sin(nw_m t)$$

1 3 5 7



Magnitude of Spectrum of Wideband FM:

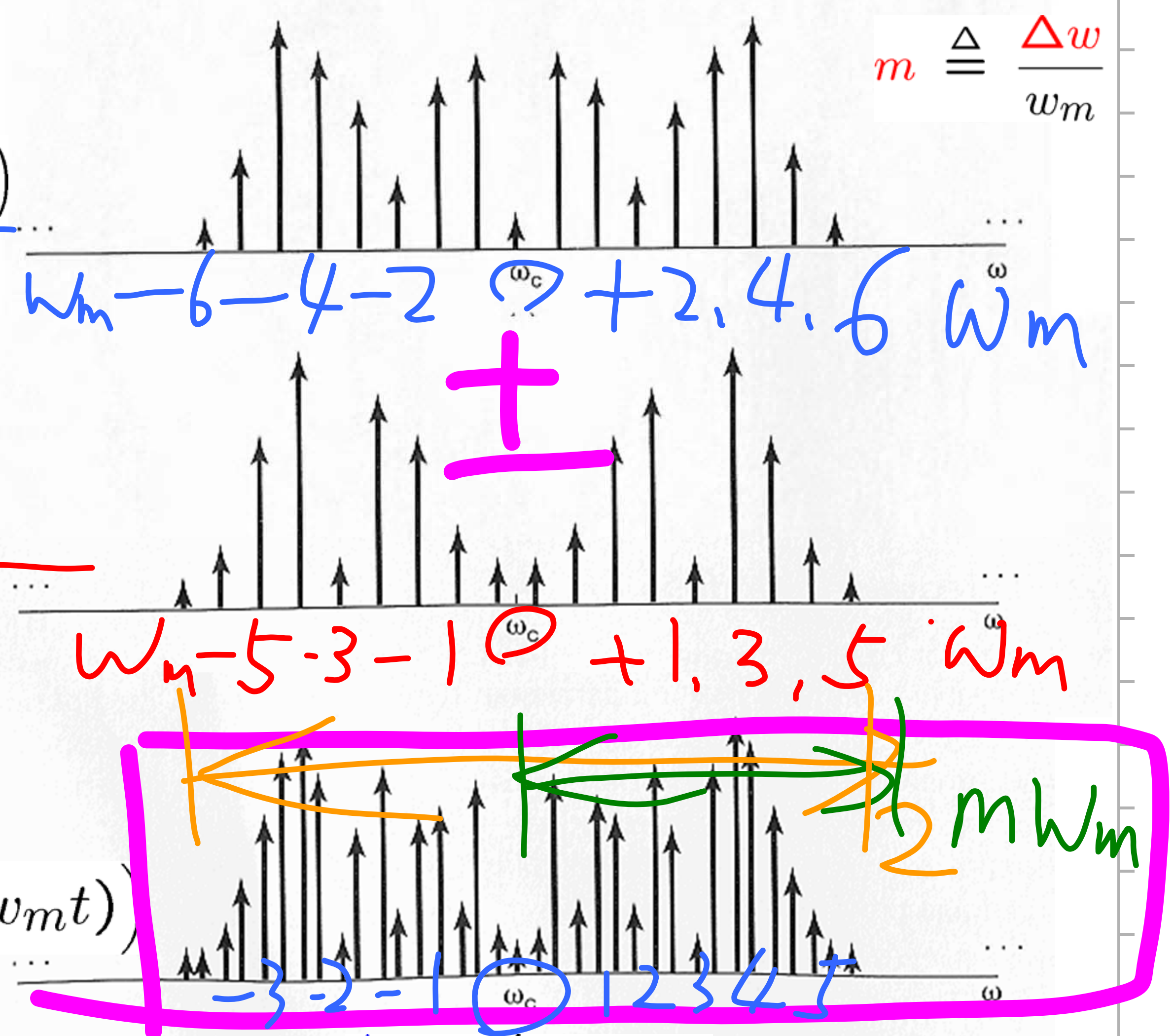
$\Delta w \triangleq k_f A$

$m \triangleq \frac{\Delta w}{w_m}$

$\cos(w_c t) \cos(m \sin(w_m t))$

$\sin(w_c t) \sin(m \sin(w_m t))$

$y(t) = \cos(w_c t + m \sin(w_m t))$



$\pm w_c + n\, w_m$

$\Rightarrow B \approx 2\, m\, w_m = 2\, k_f\, A = 2\, \Delta w$

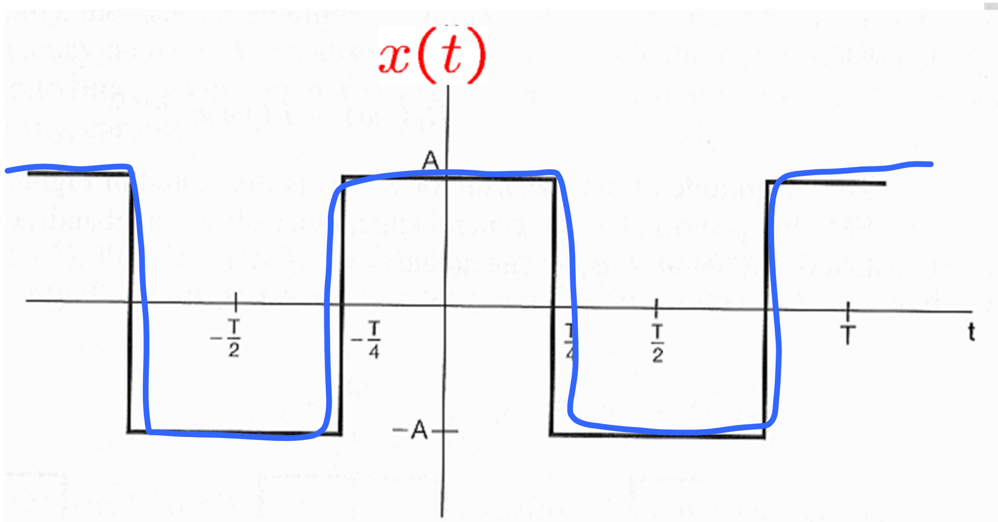
Periodic Square-Wave Modulating Signal:

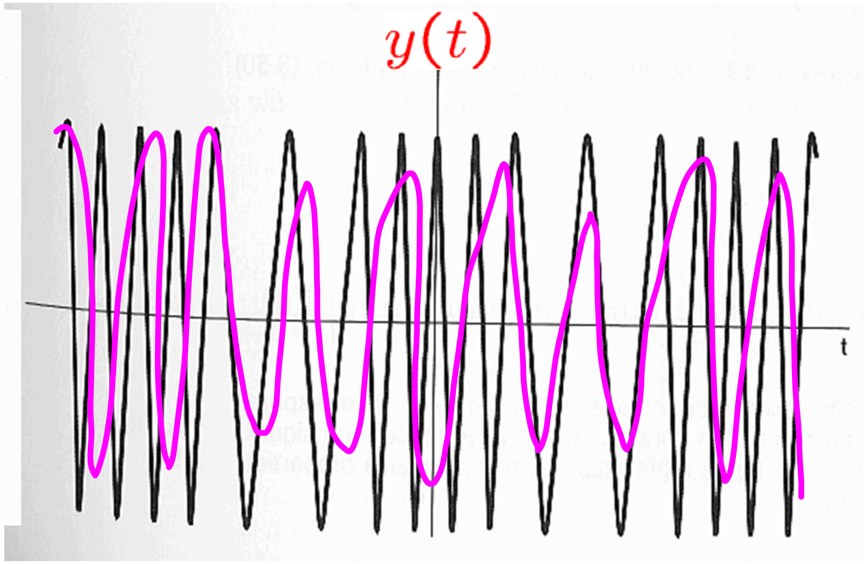
$$\Delta w \triangleq k_f A$$

$$m \triangleq \frac{\Delta w}{w_m}$$

$$w_i(t) = w_c + k_f x(t) \qquad k_f = 1 \Rightarrow \Delta w = A$$

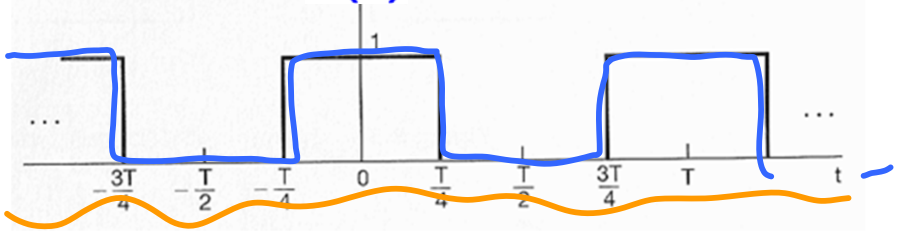
- When $x(t) > 0$, $w_i(t) = w_c + \Delta w$
- When $x(t) < 0$, $w_i(t) = w_c - \Delta w$



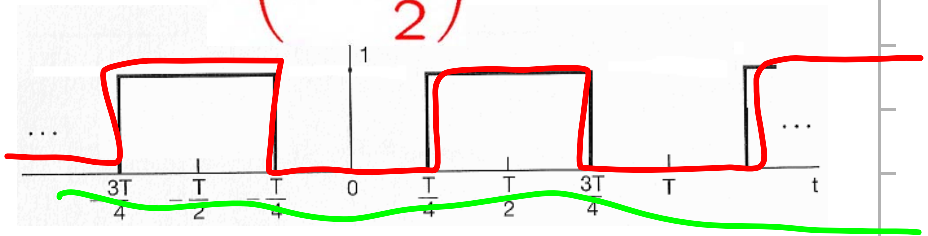


Sinusoidal Frequency Modulation

$r(t)$



$r\left(t-\frac{T}{2}\right)$



$$\Rightarrow \; y(t) = r(t)\cos\left((w_c+\Delta w)t\right) + r\left(t-\frac{T}{2}\right)\cos\left((w_c-\Delta w)t\right)$$

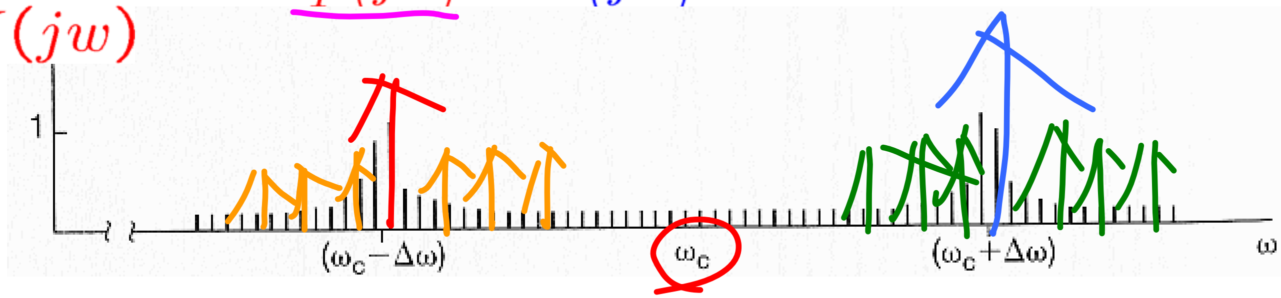
$$\Rightarrow \; Y(jw) = \frac{1}{2}\left[R\left(jw+jw_c+j\Delta w\right) + R\left(jw-jw_c-j\Delta w\right)\right]$$
$$+\frac{1}{2}\left[R_T\left(jw+jw_c-j\Delta w\right) + R_T\left(jw-jw_c+j\Delta w\right)\right]$$

Ex 4.6

$$R(jw) = \sum_{k=-\infty}^{\infty} \frac{2}{2k+1}(-1)^k \delta\left(w - \frac{2\pi(2k+1)}{T}\right) + \pi\delta(w)$$

$$R_T(jw) = R(jw)e^{-jwT/2}$$

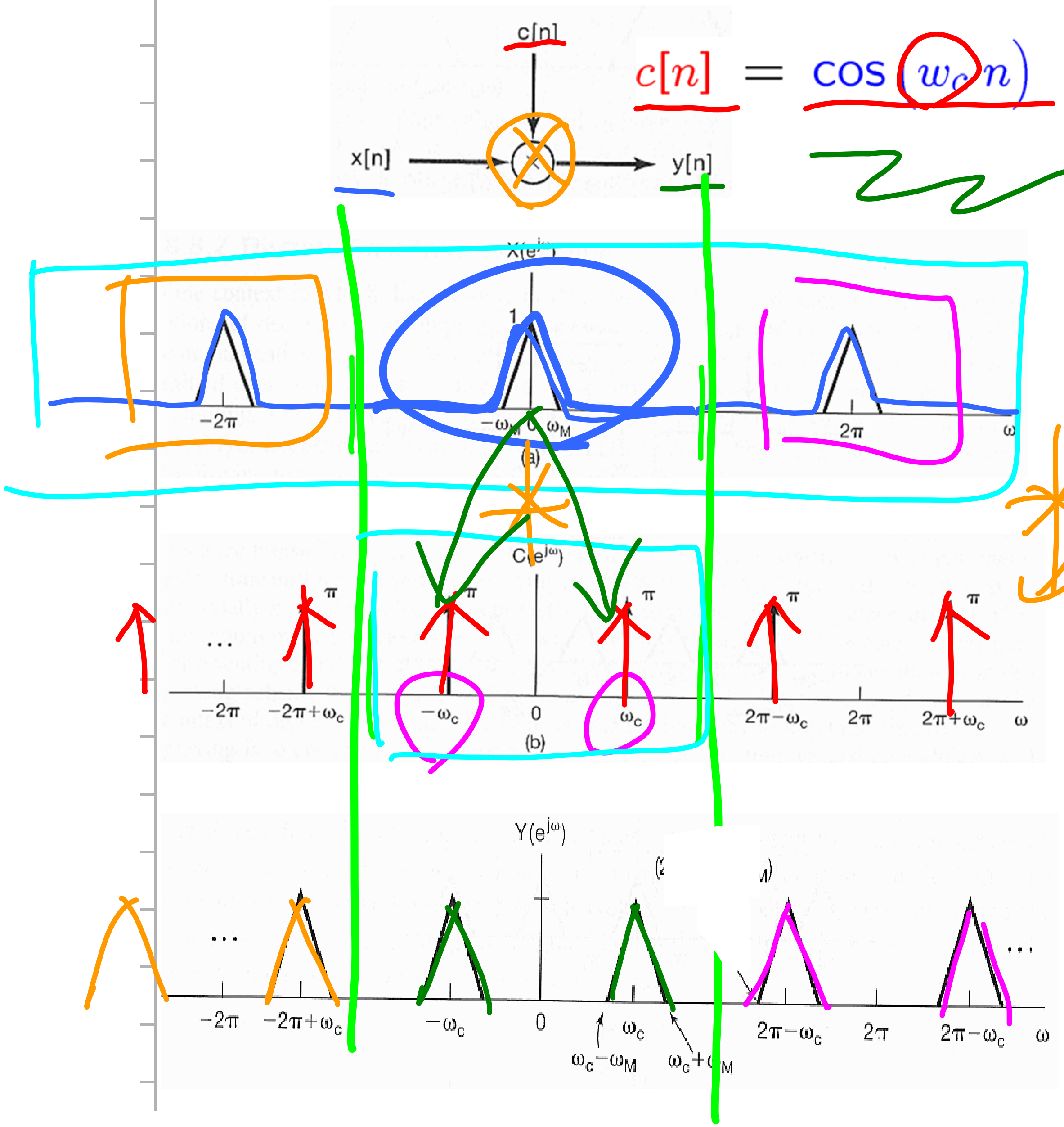
$Y(jw)$

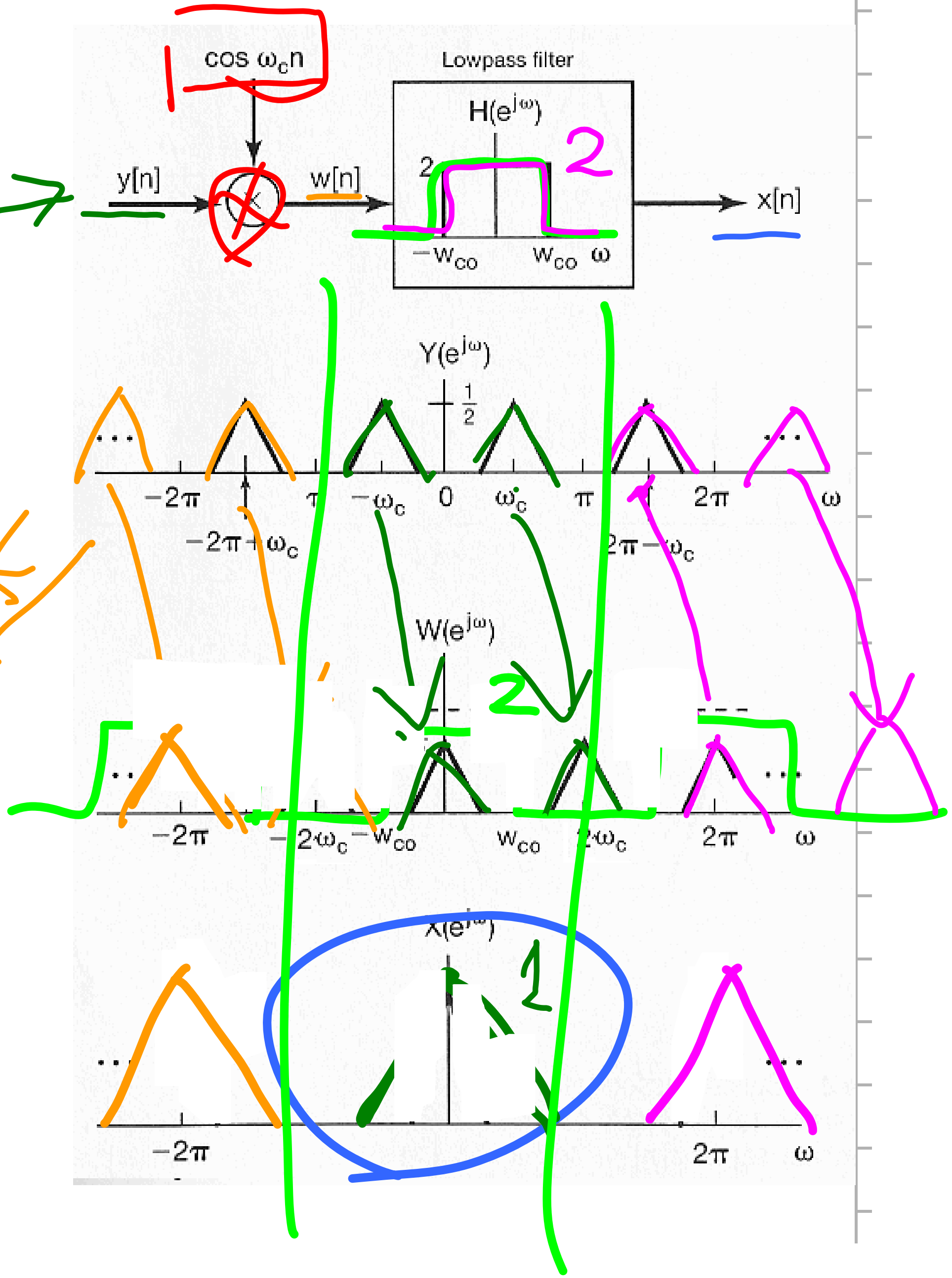


- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
  - » Time-Division Multiplexing
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

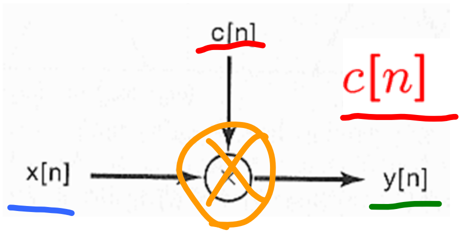
DT Sinusoidal AM:

$c[n] = \cos(w_c n)$

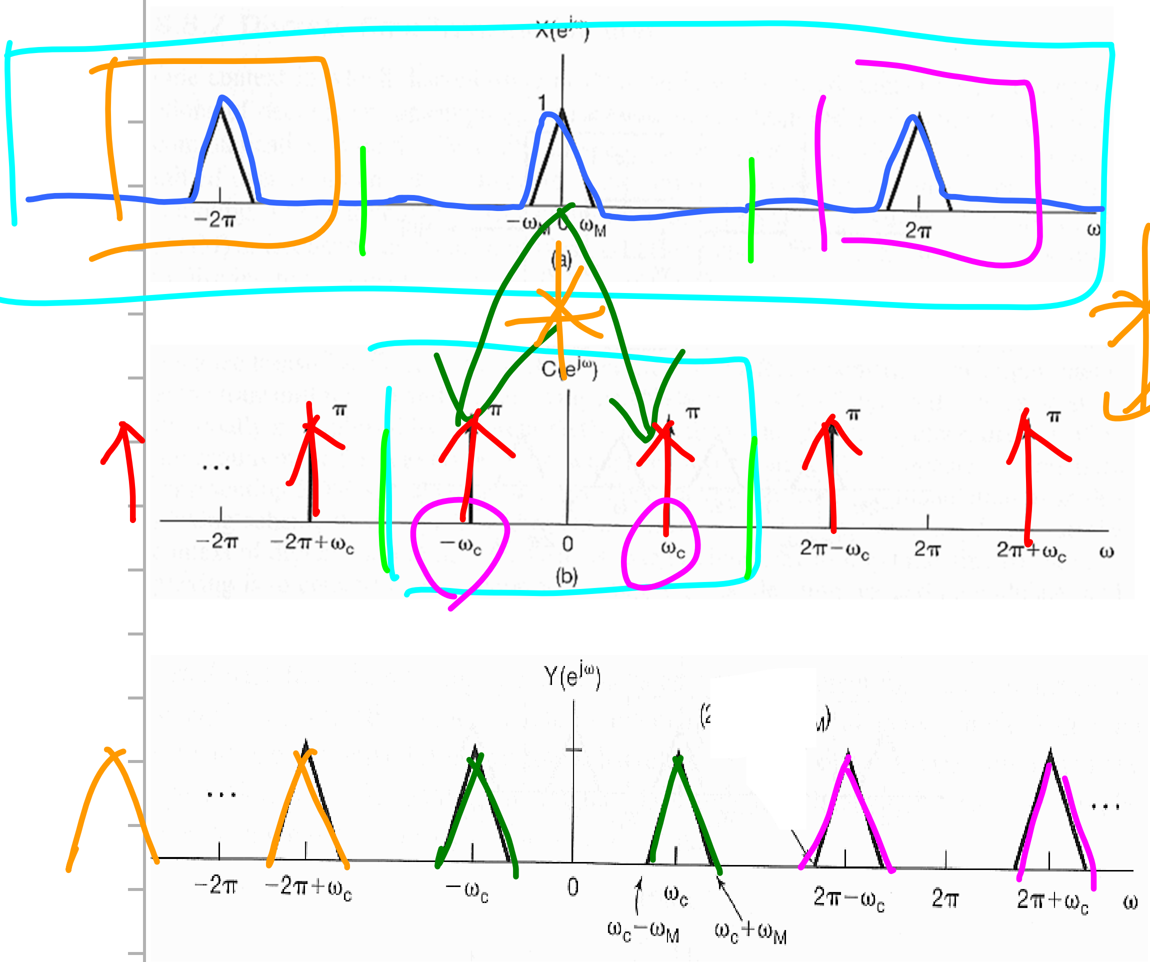


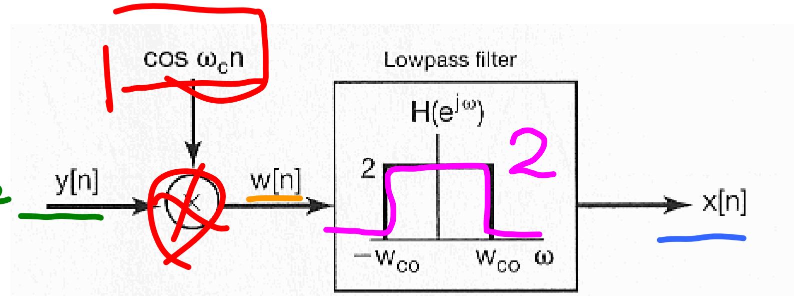


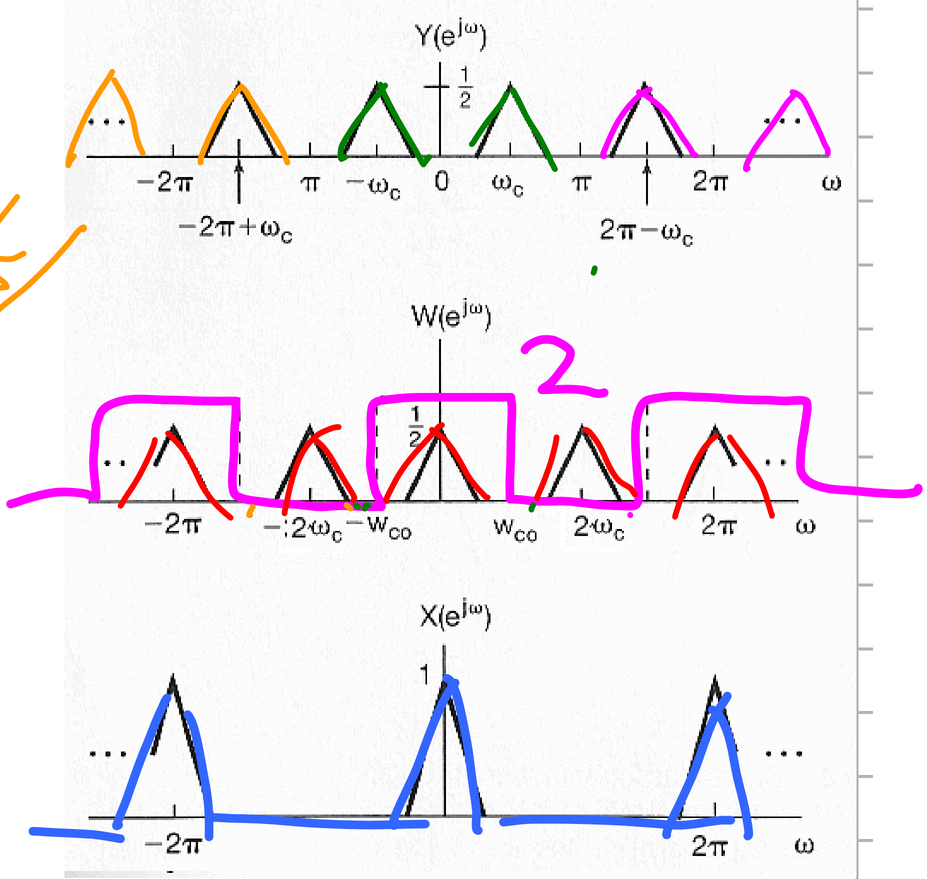
DT Sinusoidal AM:



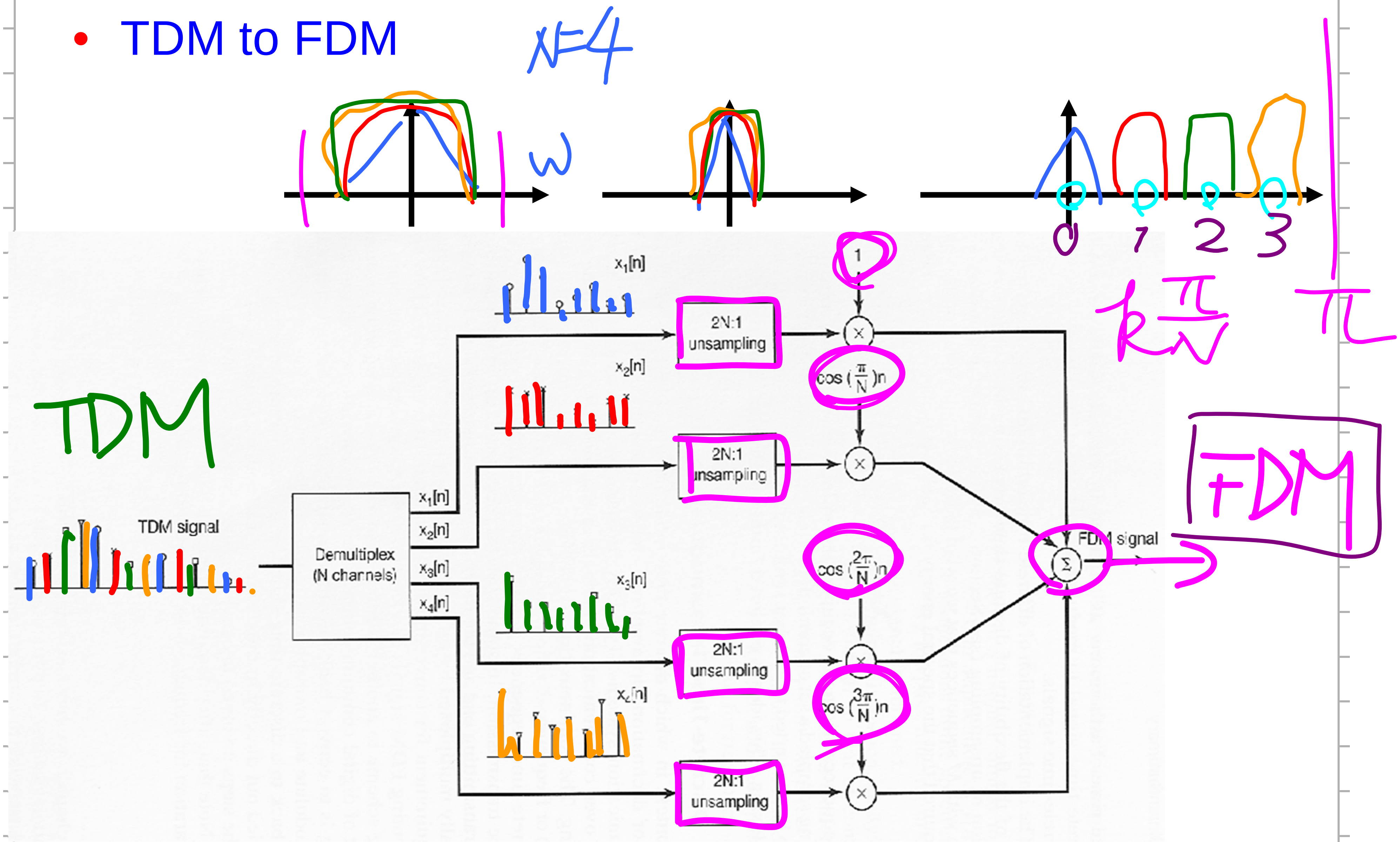
$$c[n] = \cos(\omega_c n)$$







- **Transmodulation or Transmultiplexing:**
  - TDM to FDM



Higher Equivalent Sampling Rate: Up-sampling