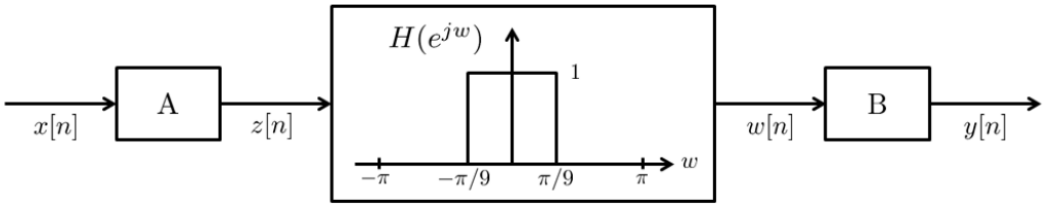
Exams

Final in 2014 – 3: Time Expansion, Down-Sampling

3. (9%) Consider the system depicted below,



where A is a process of time expansion by 3, B is a process of decimation by a factor of 2,

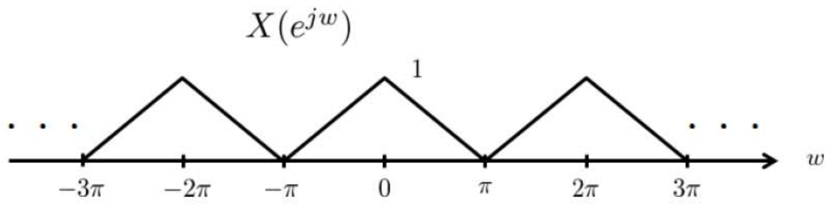
$$z[n] = x_{(3)}[n] = \begin{cases} x[n/3], & \text{if } n \text{ is a multiple of 3} \\ 0, & \text{else.} \end{cases}$$

$$y[n] = w[2n].$$

$H(e^{jw})$ is a lowpass filter,

$$H(e^{jw}) = \begin{cases} 1, & |w| \le \frac{\pi}{9}, \\ 0, & \frac{\pi}{9} < |w| \le \pi. \end{cases}$$

In this system, $x[n]$ has a discrete-time Fourier transform $X(e^{jw})$ as depicted below.



Plot the discrete-time Fourier transform $Z(e^{jw})$, $W(e^{jw})$, and $Y(e^{jw})$ of $z[n]$, $w[n]$, and $y[n]$, respectively, as well as those for some other signals needed in the intermediate steps when deriving and verifying your answer.

Final in 2014 – 4: CT -> DT: X_c(jw) -> X_d(e^jw)

4. (3%) Suppose $x_c(t)$ is a continuous-time signal, $x_d[n] = x_c(nT)$, and

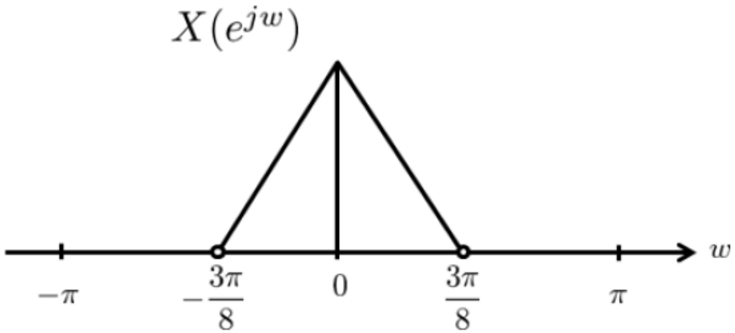
$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT).$$

The discrete-time Fourier transform of $x_d[n]$ is $X_d(e^{j\Omega})$, while the continuous-time Fourier transform of $x_p(t)$ is $X_p(jw)$. Write down and verify the relationship between $X_d(e^{j\Omega})$ and $X_p(jw)$.

Final in 2014 – 5: Sampling Without Aliasing

5. (7%) An engineer is handling a discrete-time signal $x[n]$, and finds its Fourier transform as shown,

$$|X(e^{jw})| = 0, \ \frac{3\pi}{8} \leq |w| \leq \pi.$$



He wishes to reduce the sampling frequency of $x[n]$ to minimum without introducing aliasing. Do you think this can be achieved? If yes, write it down and sketch the spectra after each process, and explain what that means. If not, explain why.

Final in 2013 – 1: Sampling without Aliasing, Reconstruct

1. (10%) Suppose a periodic signal $x(t)$ has Fourier series coefficients

$$a_k = \begin{cases} (\frac{3}{4})^k, & |k| \le 4 \\ 0, & \text{otherwise} \end{cases}.$$

The period of this signal is $T = 1$.

(a) (3%) Determine the minimum sampling interval for the signal that will prevent aliasing.

(b) (3%) The constraints of the sampling theorem can be relaxed somewhat in the case of periodic signals if we allow the reconstructed signal to be a time-scaled version of the original. Suppose we choose a sampling interval $T_s = \frac{20}{19}$ and use a reconstructed filter

$$H_r(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}.$$

Show that the reconstructed signal is a time-scaled version of $x(t)$, and identify the scaling factor.

(c) (4%) Find the constraints in the sampling interval T_s so that the use of $H_r(j\omega)$ in Part (b) results in the reconstructed filter being a time-scaled version of $x(t)$, and determine the relationship between the scaling factor and T_s.

Final in 2012 – 6: Sampling Theorem

6. (10%) The sampling theorem *in the frequency domain* states that if a real signal $x(t)$ is a duration-limited signal such that $x(t)=0,\ \forall |t|>t_M$, then its Fourier transform $X(j\omega)$ can be uniquely determined from its values $X(j\frac{n\pi}{t_M})$ at a series of equidistant points spaced π/t_M apart. In fact, $X(j\omega)$ is given by

$$X(j\omega)=\sum_{n=-\infty}^{\infty} X(j\frac{n\pi}{t_M})\frac{\sin(\omega t_M-n\pi)}{\omega t_M-n\pi}.$$

Verify the above sampling theorem in the frequency domain.

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8. (20%) Compact discs (CDs) work by converting analog, continuous-time music $x(t)$ into a digital, discrete-time signal $x[n]$. The sampling frequency is typically 44kHz. Suppose the music $x(t)$ is uniformly sampled by the sampling frequency 44kHz. Please answer the following questions.

(a) (5%) According to the sampling theorem, what would be the maximum frequency f_{max} of the analog music $x(t)$? In practice, why it is difficult, if not impossible, to perfectly reconstruct a signal that has significant energy up to f_{max}?

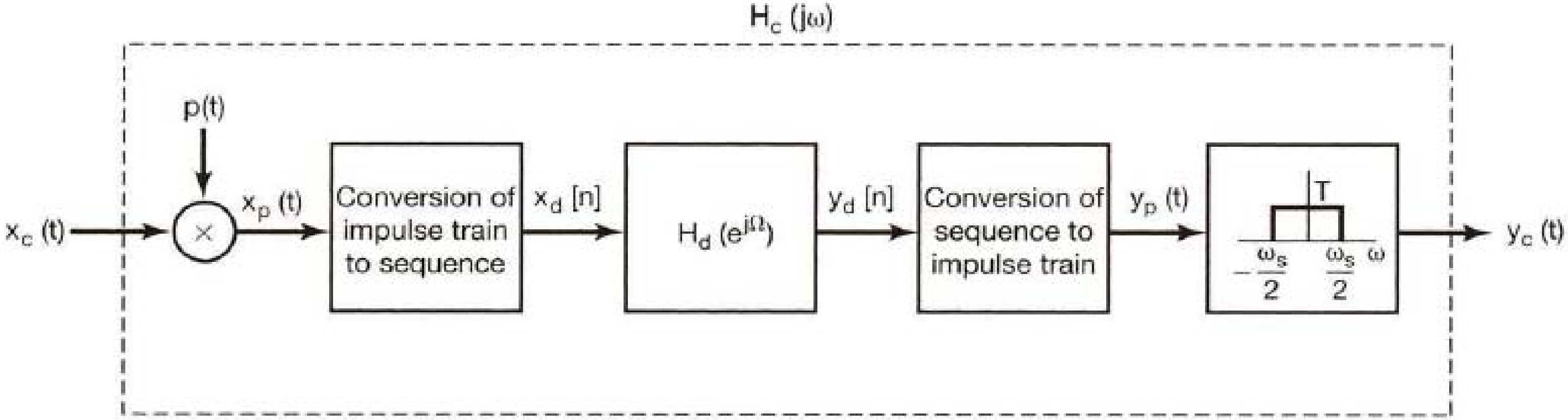
(b) (5%) Due to the constraint stated in (a), an "anti-alias filter" $A(j\omega)$ is used to significantly attenuate the energy of $x(t)$ from 0.9 f_{max} to f_{max}. Specifically, it is desired that $A(j\omega)$ has passband gain of 0dB and stopband attenuation of at least -40dB. Plot $A(j\omega)$ as precisely as you can. How many poles are needed for such a filter $A(j\omega)$?

(c) (5%) The CD player performs D/A conversion such that the digital signal $x[n]$ is converted back to an analog signal for playout. Based on what you have learned from class, draw the time and frequency domain representation of an ideal interpolation filter. Why is it difficult to use such an ideal interpolation filter in practice?

(d) (5%) To relax the design requirements of the interpolation filter, oversampling is often performed to convert the digital signal $x[n]$ to $x_{os}[n]$ before interpolation. For a CD player with "8X oversampling," draw the spectrum of $x_{os}[n]$. Why does this relax the requirements of the interpolation filter?

Final in 2011 – 8: Sampling, xc-xd-yd-yc by Hd-Hc

8. [15] Consider the following system for filtering a continuous-time signal using a discrete-time filter, where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ denotes the sampling function with sampling period T and sampling frequency $\omega_s = 2\pi / T$. Let the input signal $x_c(t) = \sin(\pi t/T)/\pi t$ and $y_c(t) = dx_c(t)/dt$



(a) Find $x_d[n]$. [5]

(b) Find $y_d[n]$. [5]

(c) Find the impulse response $h_d[n]$ of the filter $H_d(e^{j\Omega})$. [5]