

- Representation of of a CT Signal by Its Samples:
 - The Sampling Theorem
- Reconstruction of of a Signal from Its Samples
 - Using exact interpolation
 - Using zero-order hold
 - Using higher-order hold
- The Effect of Under-sampling
 - Overlapping in Frequency-Domain
 - Aliasing
- DT Processing of CT Signals
- Sampling of Discrete-Time Signals
 - Down-sampling
 - Up-sampling

7.2: Sampling Rate - Reconstruction

7.2. A continuous-time signal $x(t)$ is obtained at the output of an ideal lowpass filter with cutoff frequency $\omega_c = 1,000\pi$. If impulse-train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate lowpass filter?

(a) $T = 0.5 \times 10^{-3}$

(b) $T = 2 \times 10^{-3}$

(c) $T = 10^{-4}$

7.4: Nyquist Rate

7.4. Let $x(t)$ be a signal with Nyquist rate ω_0 . Determine the Nyquist rate for each of the following signals:

(a) $x(t) + x(t - 1)$

(b) $\frac{dx(t)}{dt}$

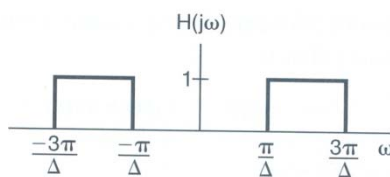
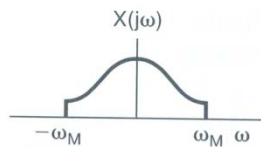
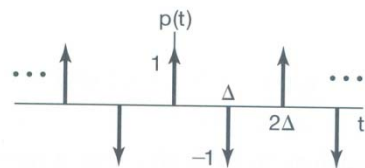
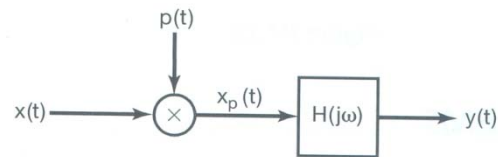
(c) $x^2(t)$

(d) $x(t) \cos \omega_0 t$

7.23: Sampling and Recover

7.23. Shown in Figure P7.23 is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

- (a) For $\Delta < \pi/(2\omega_M)$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.
- (b) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.
- (c) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $y(t)$.
- (d) What is the *maximum* value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$?



7.40: Sampling of Sinusoid – Sampling Theorem

7.40. Consider a disc on which four cycles of a sinusoid are painted. The disc is rotated at approximately 15 revolutions per second, so that the sinusoid, when viewed through a narrow slit, has a frequency of 60 Hz.

The arrangement is indicated in Figure P7.40. Let $v(t)$ denote the position of the line seen through the slit. Then

$$v(t) = A \cos(\omega_0 t + \phi), \quad \omega_0 = 120\pi.$$

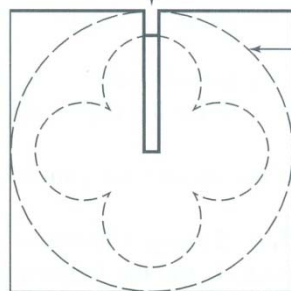
For notational convenience, we will normalize $v(t)$ so that $A = 1$. At 60 Hz, the eye is not able to follow $v(t)$, and we will assume that this effect can be explained by modeling the eye as an ideal lowpass filter with cutoff frequency 20 Hz.

Sampling of the sinusoid can be accomplished by illuminating the disc with a strobe light. Thus, the illumination can be represented by an impulse train; that is,

$$i(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT),$$

where $1/T$ is the strobe frequency in hertz. The resulting sampled signal is the product $r(t) = v(t)i(t)$. Let $R(j\omega)$, $V(j\omega)$, and $I(j\omega)$ denote the Fourier transforms of $r(t)$, $v(t)$, and $i(t)$, respectively.

Position of line varies sinusoidally
at 60 cycles per second



Disk rotating at
15 rps

7.40: Sampling of Sinusoid – Sampling Theorem

- (a) Sketch $V(j\omega)$, indicating clearly the effect of the parameters ϕ and ω_0 .
- (b) Sketch $\hat{I}(j\omega)$, indicating the effect of T .
- (c) According to the sampling theorem, there is a maximum value for T in terms of ω_0 such that $v(t)$ can be recovered from $r(t)$ using a lowpass filter. Determine this value of T and the cutoff frequency of the lowpass filter. Sketch $R(j\omega)$ when T is slightly less than the maximum value.

If the sampling period T is made greater than the value determined in part (c), aliasing of the spectrum occurs. As a result of this aliasing, we perceive a lower frequency sinusoid.

- (d) Suppose that $2\pi/T = \omega_0 + 20\pi$. Sketch $R(j\omega)$ for $|\omega| < 40\pi$. Denote by $v_a(t)$ the apparent position of the line as we perceive it. Assuming that the eye behaves as an ideal lowpass filter with 20-Hz cutoff and unity gain, express $v_a(t)$ in the form

$$v_a(t) = A_a \cos(\omega_a t + \phi_a),$$

where A_a is the apparent amplitude, ω_a the apparent frequency, and ϕ_a the apparent phase of $v_a(t)$.

- (e) Repeat part (d) for $2\pi/T = \omega_0 - 20\pi$.