

Spring 2013

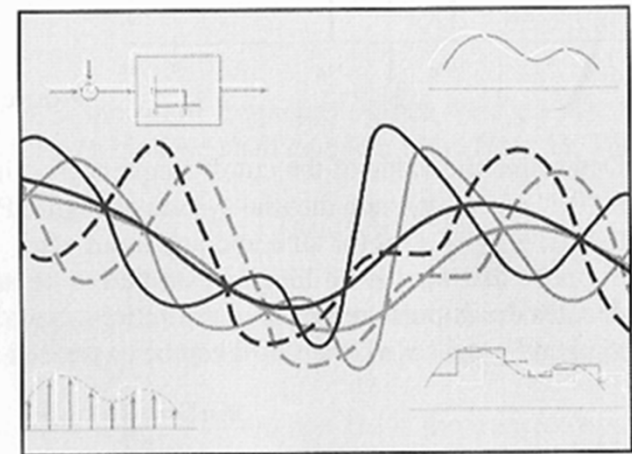
信號與系統 Signals and Systems

Chapter SS-7 Sampling

Feng-Li Lian

NTU-EE

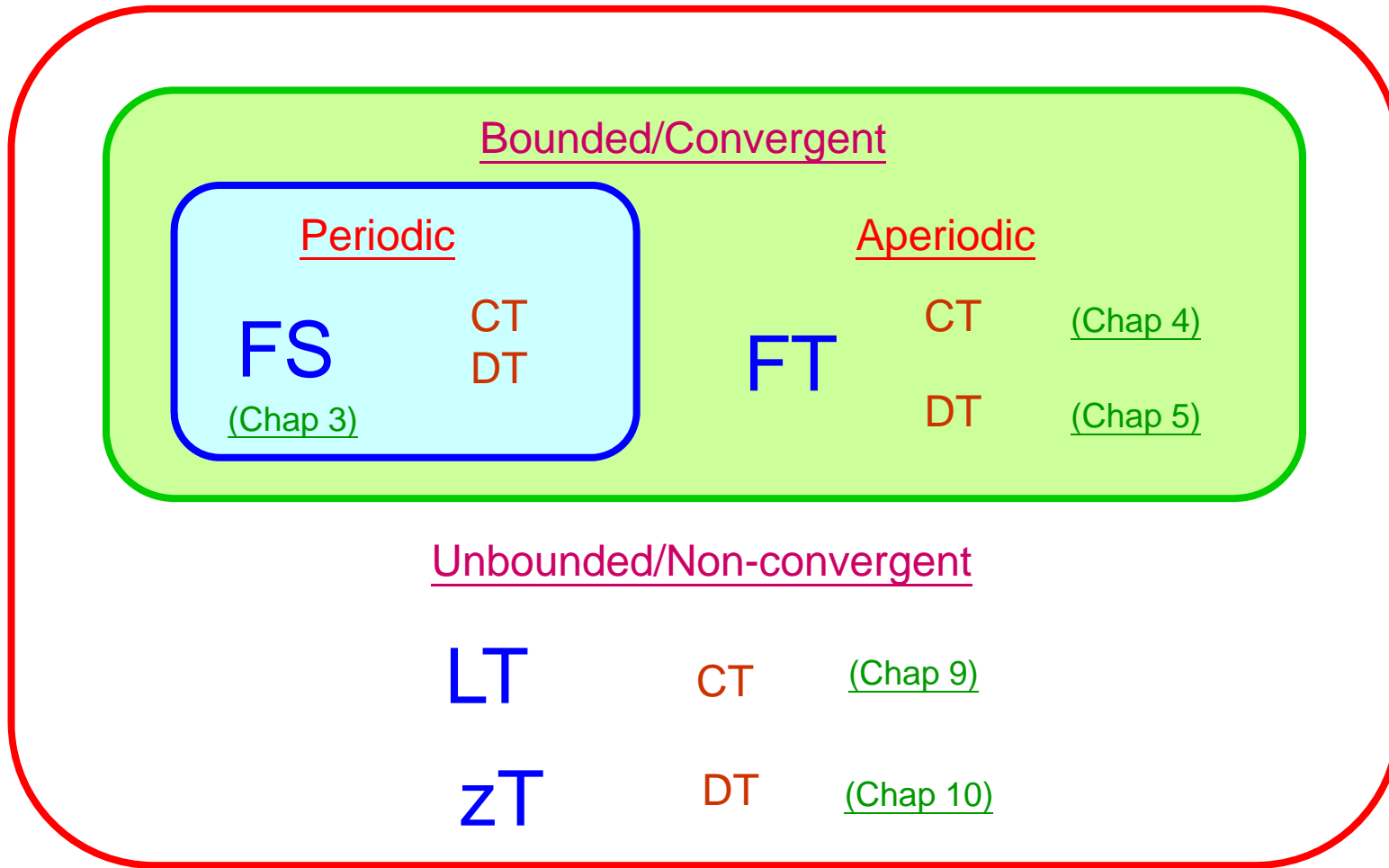
Feb13 – Jun13



Figures and images used in these lecture notes are adopted from
“Signals & Systems” by Alan V. Oppenheim and Alan S. Willsky, 1997

Introduction (Chap 1)

LTI & Convolution (Chap 2)



Time-Frequency (Chap 6)

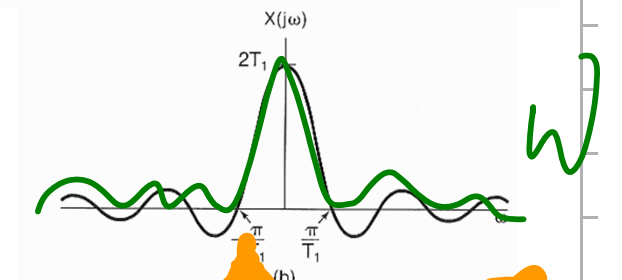
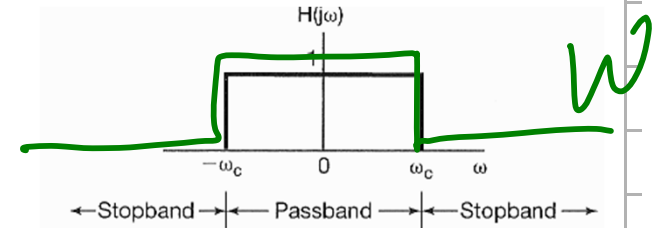
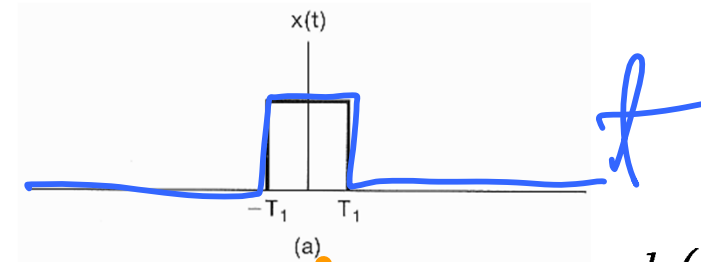
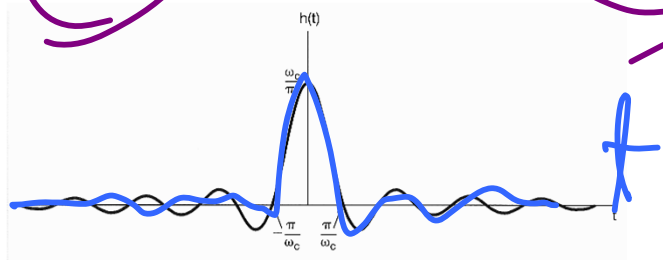
Communication (Chap 8)

Digital Signal Processing (dsp-8)

CT-DT (Chap 7)

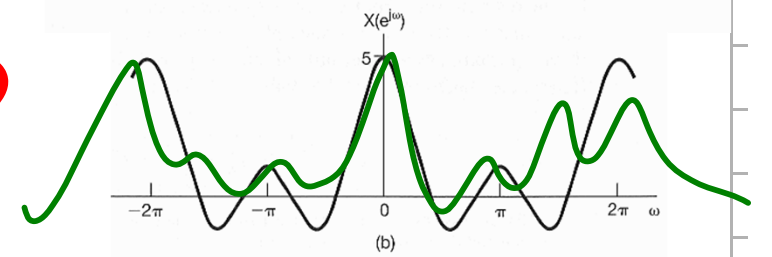
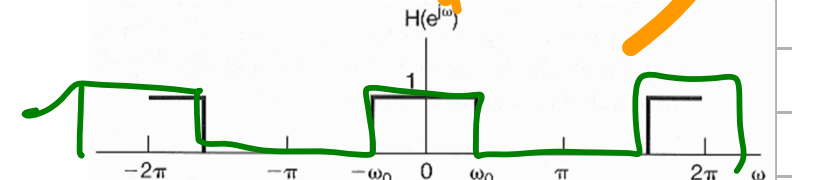
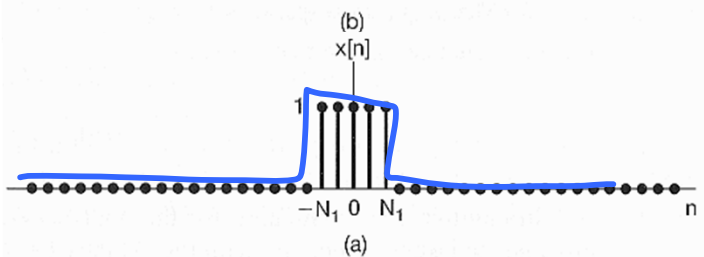
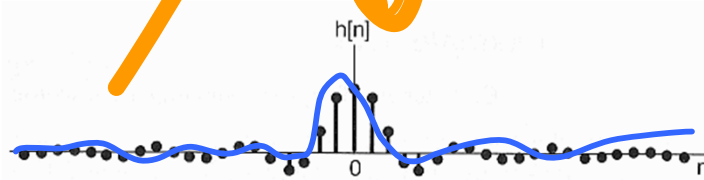
Control (Chap 11)

Time-Domain & Frequency-Domain Characterization:



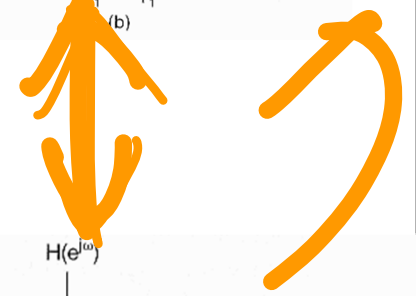
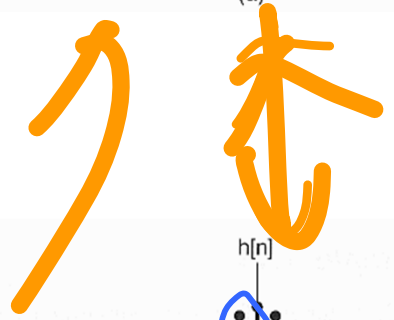
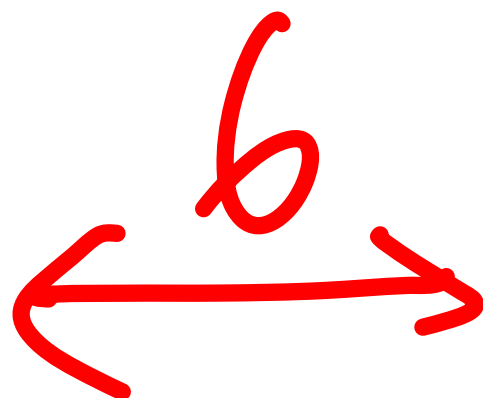
$$h(t) \xleftrightarrow{\mathcal{F}} H(j\omega)$$

$$h[n] \xleftrightarrow{\mathcal{F}} H(e^{j\omega})$$



CT

DT



Fourier Series, Fourier Transform, Laplace Transform, z-Transform

	CT		DT	
	time	frequency	time	frequency
FS				
FT	 	 	 	
LT/zT	 		 	

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

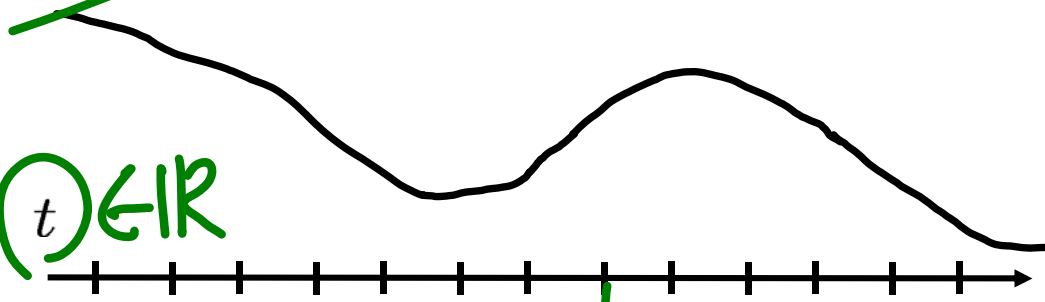
C.T.



Representation of CT Signals by its Samples

$x_1(t)$
=

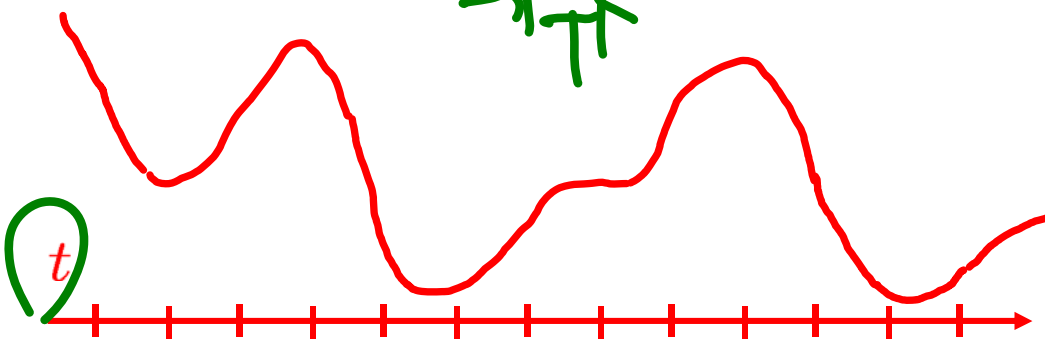
$t \in \mathbb{R}$



$\rightarrow \frac{1}{T}$

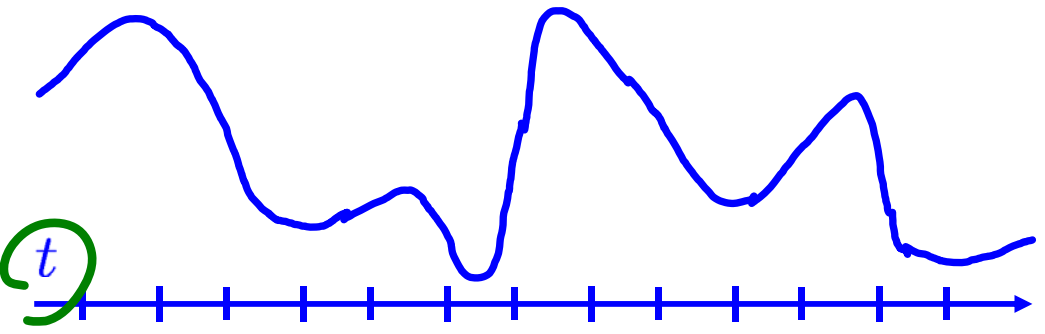
$x_2(t)$
=

t

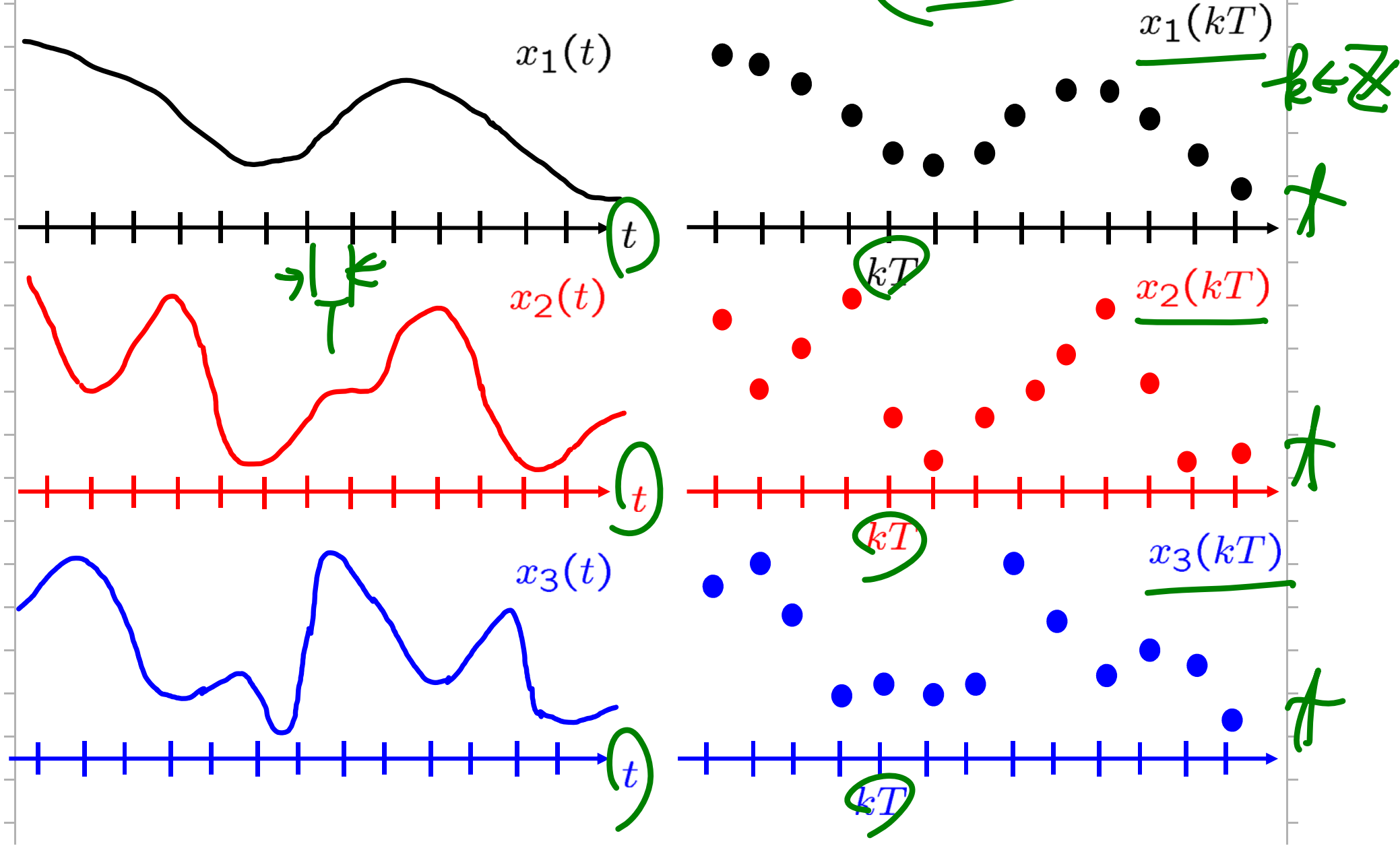


$x_3(t)$
=

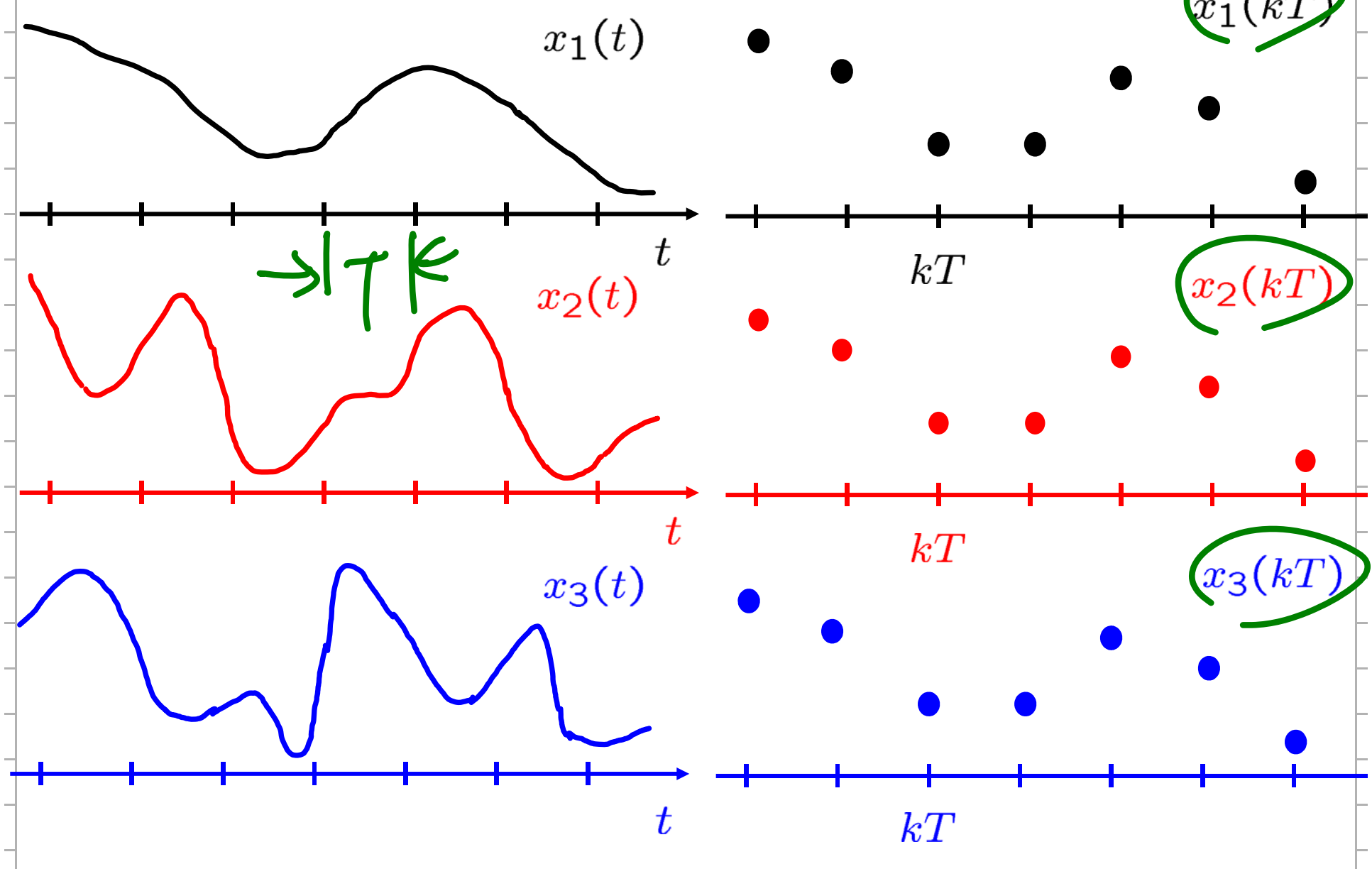
t



Representation of CT Signals by its Samples



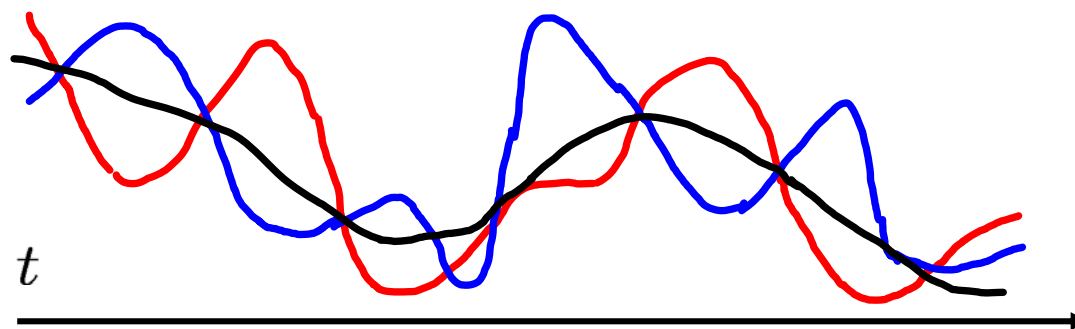
Representation of CT Signals by its Samples



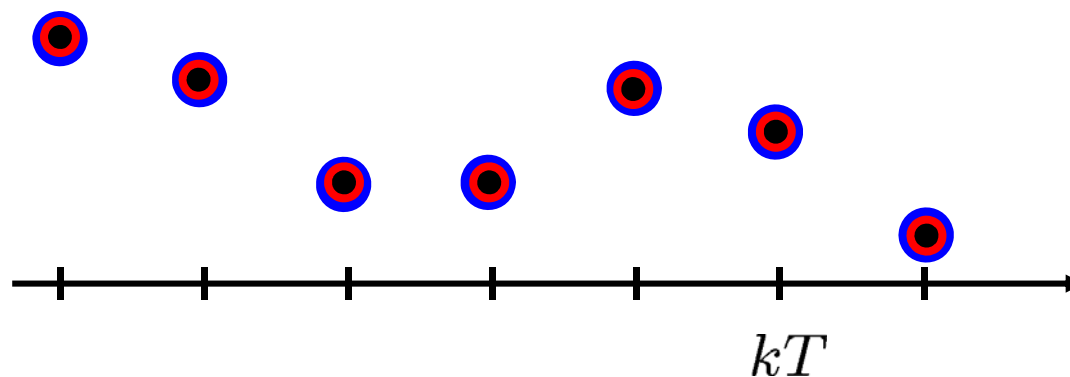
The Sampling Theorem

Representation of CT Signals by its Samples

$$x_1(t) \neq x_2(t) \neq x_3(t)$$



$$x_1(kT) = x_2(kT) = x_3(kT)$$



Impulse-Train Sampling:

$p(t)$: sampling function

T : sampling period

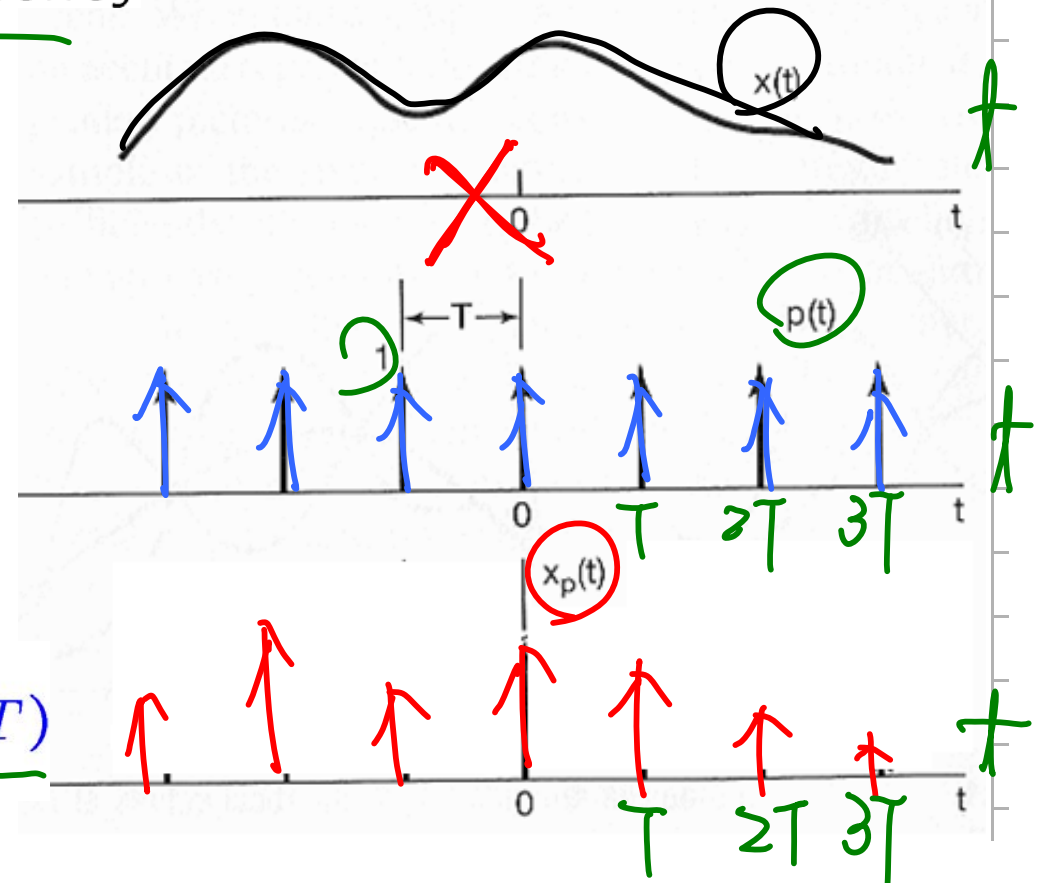
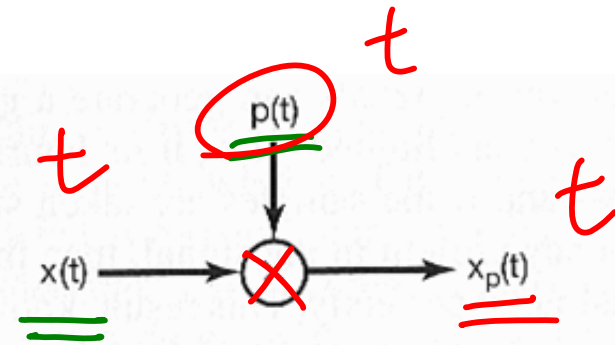
$w_s = \frac{2\pi}{T}$: sampling frequency

$$\Rightarrow x_p(t) = x(t) p(t)$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t - nT)$$

$t = nT$



Impulse-Train Sampling:

$$x_p(t) = x(t) p(t) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) P(j(\omega - \theta)) d\theta$$

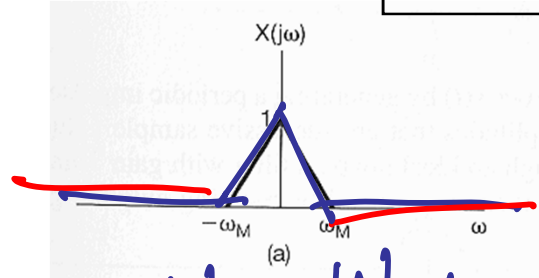
Eq 4.70, p. 322

From multiplication property,

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

Ex 4.21, p. 323

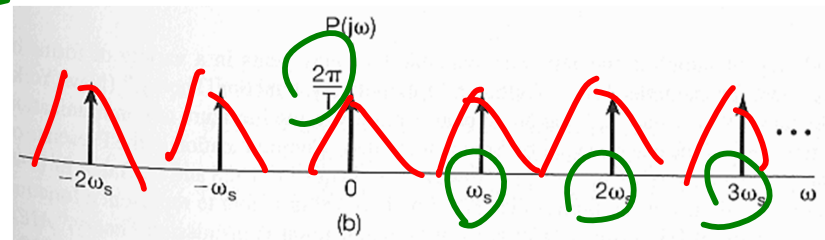
$$\underline{x(t)} \xleftrightarrow{\mathcal{F}} \underline{X(j\omega)}$$



$$\underline{p(t)} \xleftrightarrow{\mathcal{F}} \underline{P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s)}$$

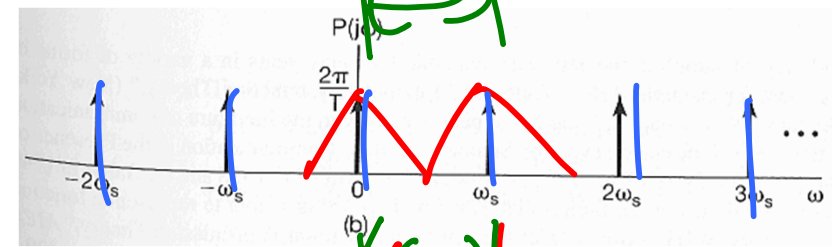
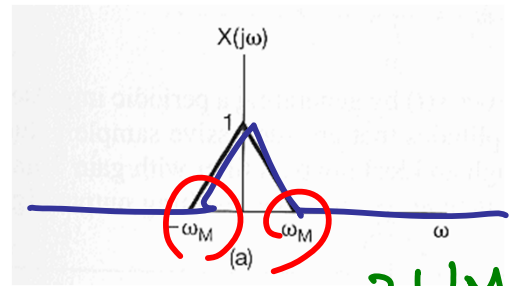
$-\omega_M$ ω_M
*

Ex 4.8, pp. 299-300

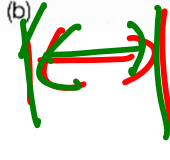


Impulse-Train Sampling:

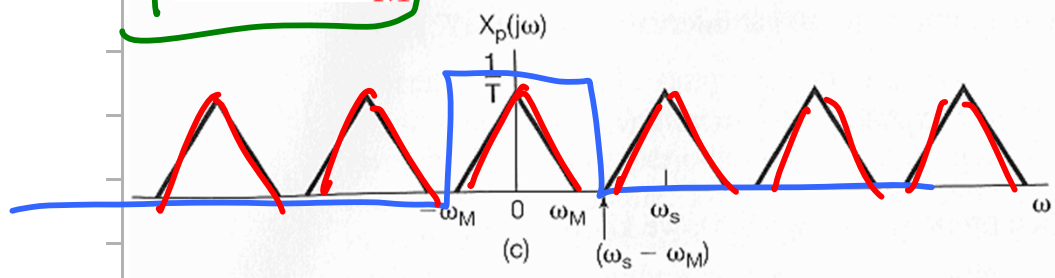
Ex 4.21, 4.22, pp. 323-4



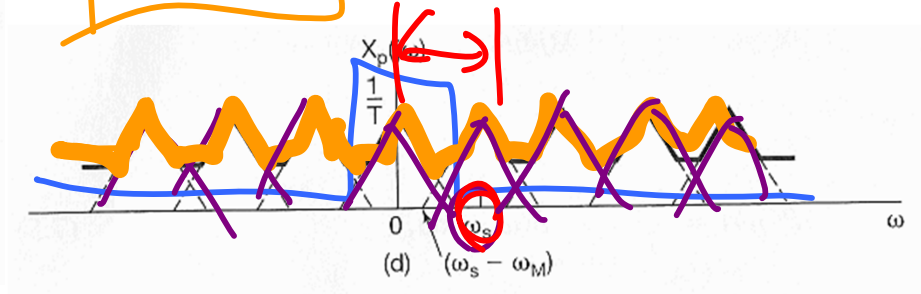
$2\omega_M$



$\omega_s > 2\omega_M$



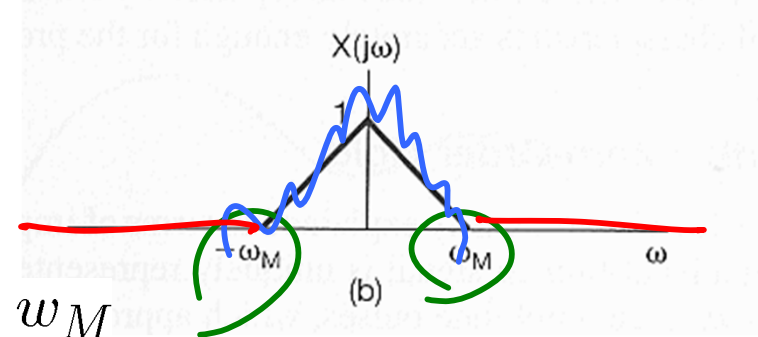
$\omega_s < 2\omega_M$



■ The Sampling Theorem:

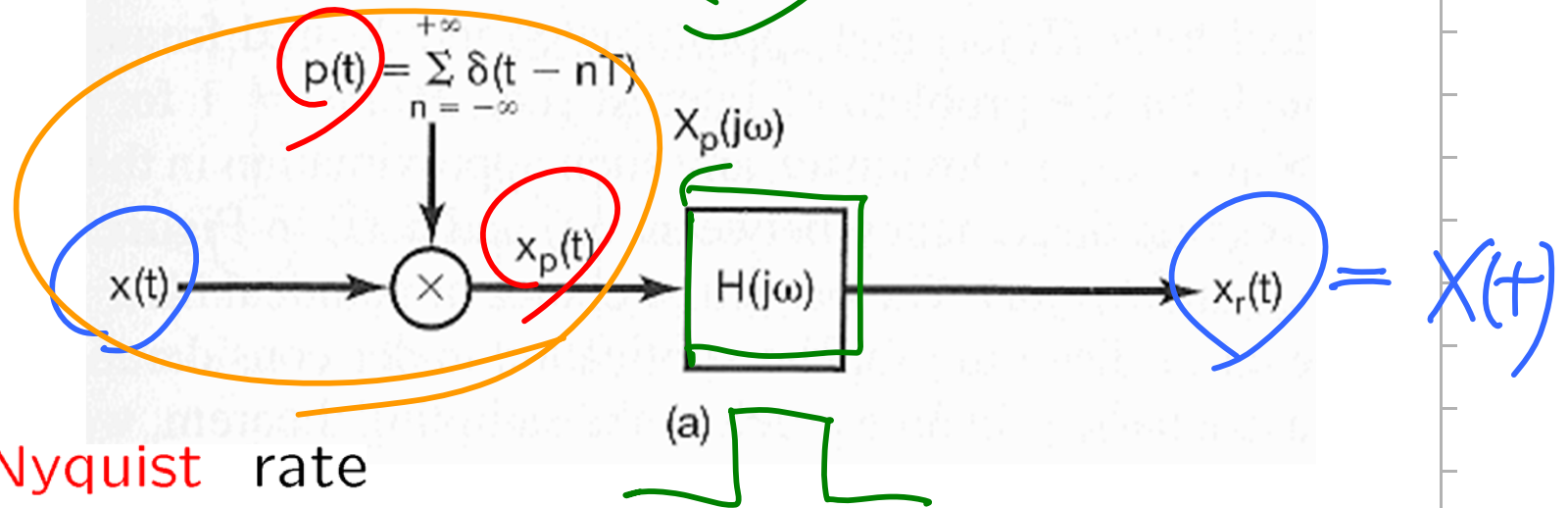
$x(t)$: a band-limited signal

with $X(j\omega) = 0$ for $|\omega| > \omega_M$



if $\omega_s > 2\omega_M$ where $\omega_s = \frac{2\pi}{T}$

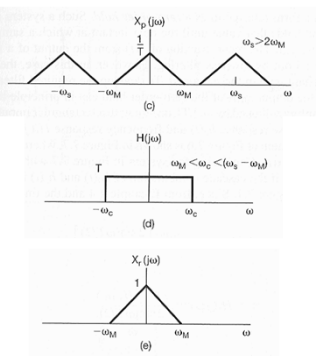
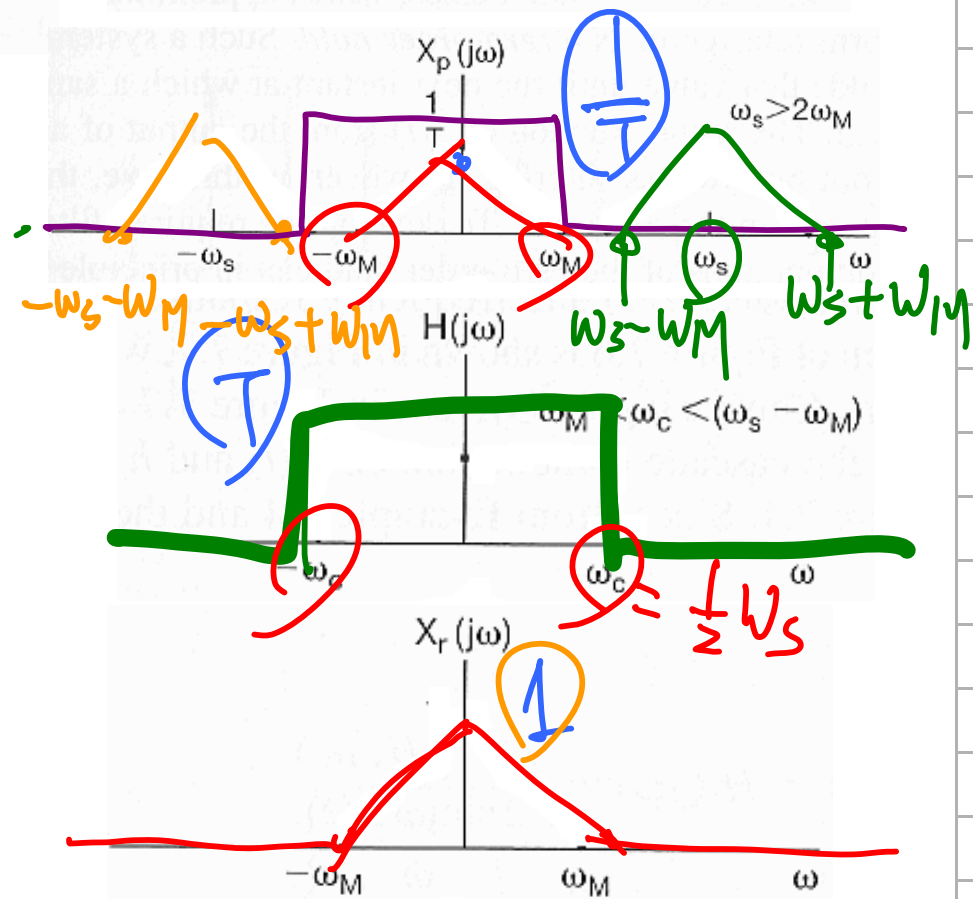
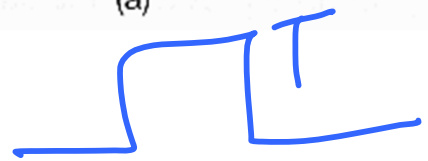
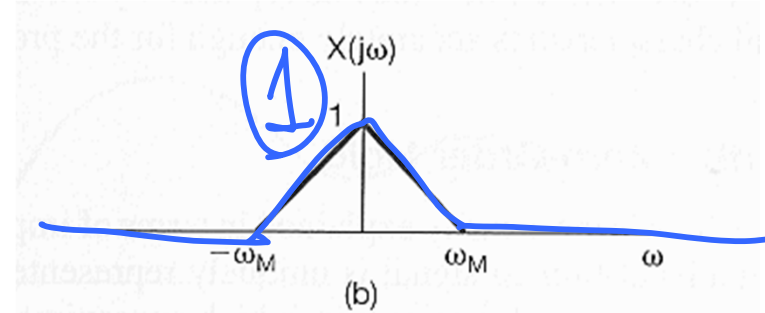
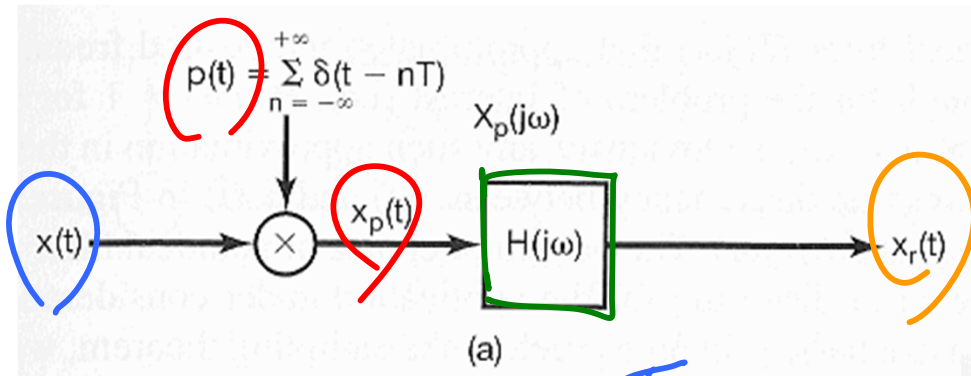
\Rightarrow $x(t)$ is uniquely determined by $x(nT)$, $n = 0, \pm 1, \pm 2, \dots$,



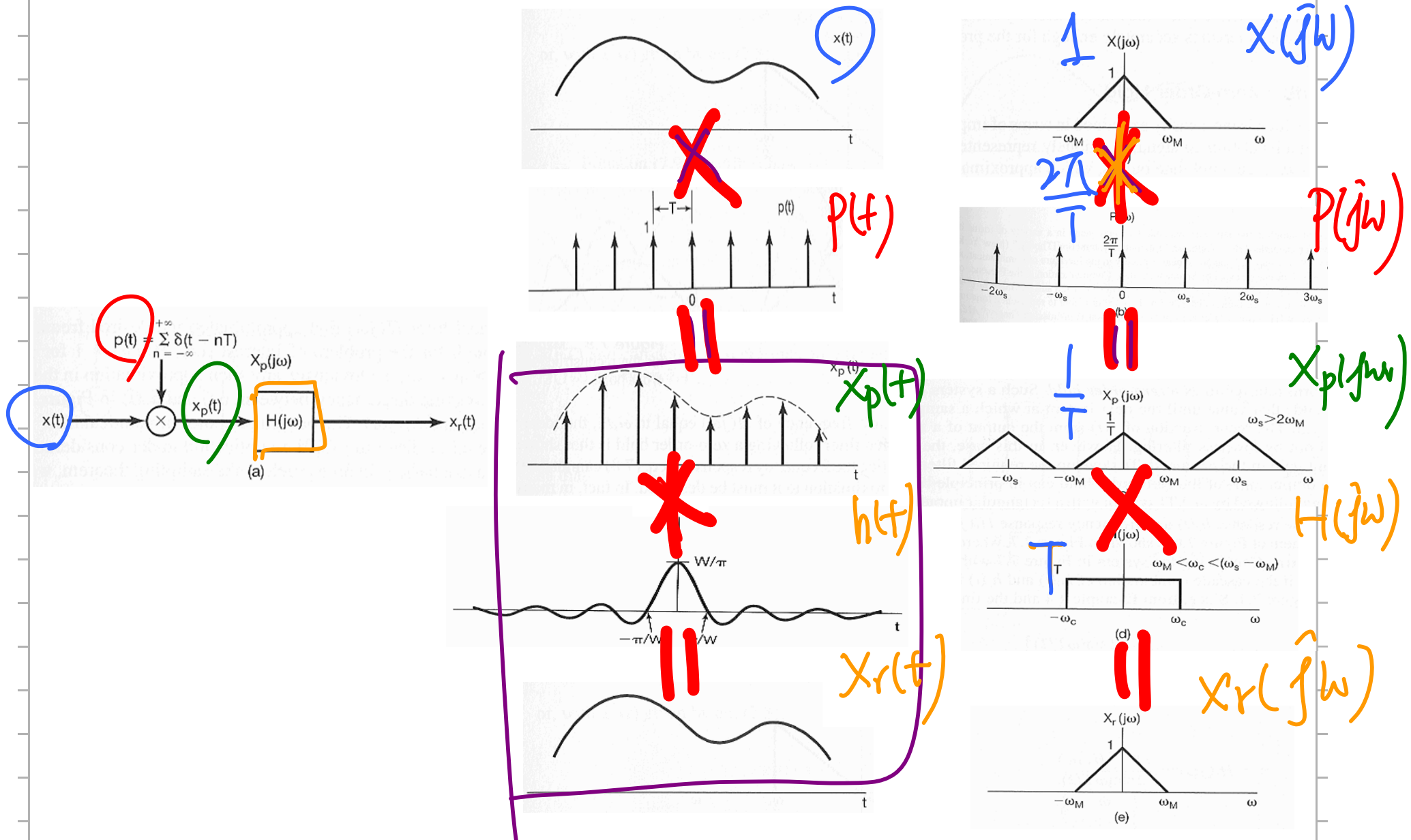
\Rightarrow $2\omega_M$: Nyquist rate

ω_M : Nyquist frequency

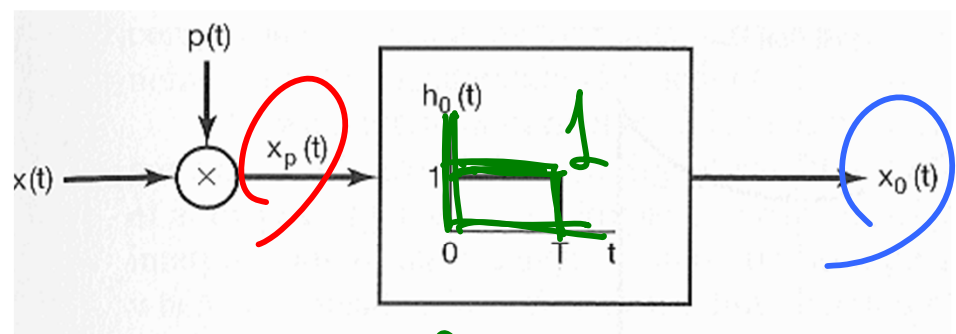
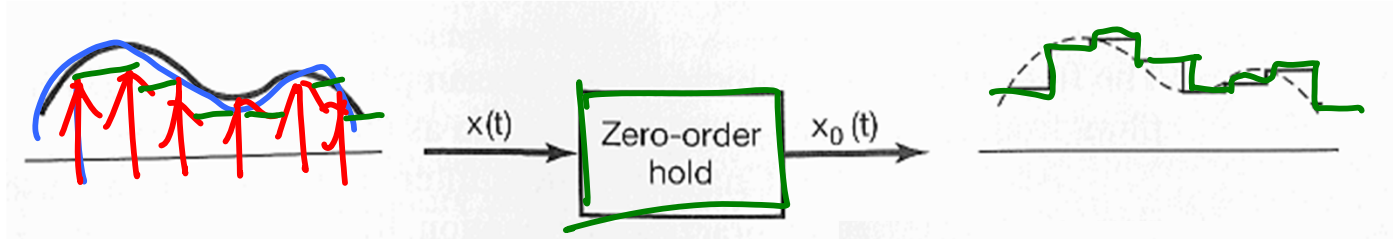
Exact Recovery by an Ideal Lowpass Filter:



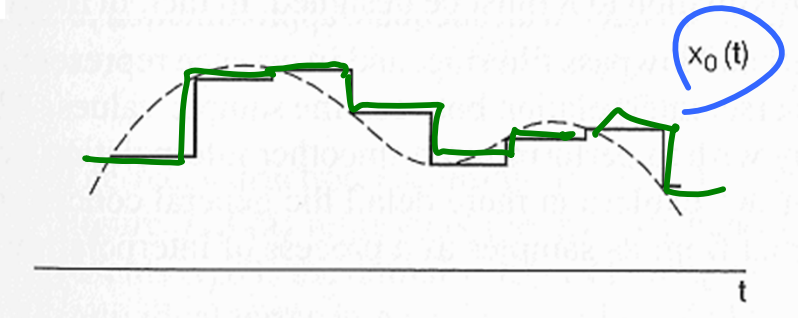
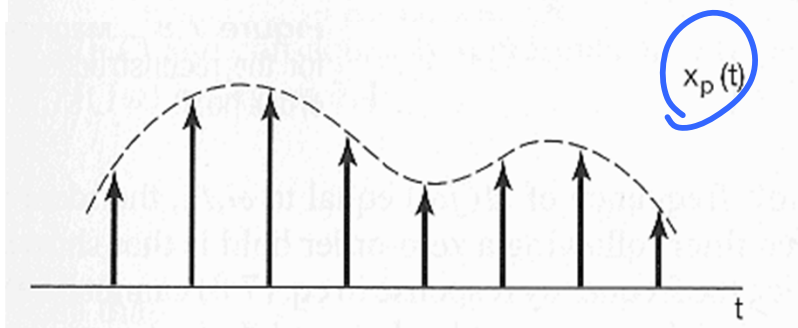
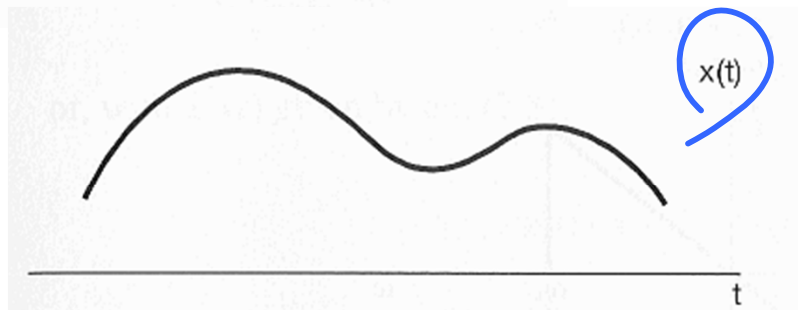
Exact Recovery by an Ideal Lowpass Filter:



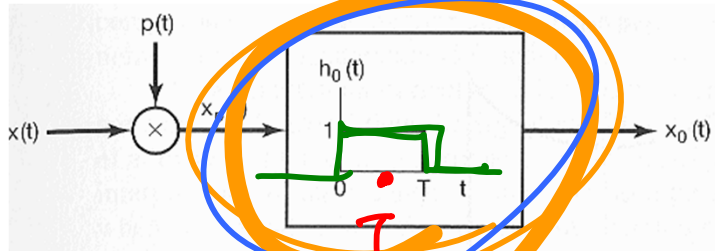
■ Sampling with Zero-Order Hold:



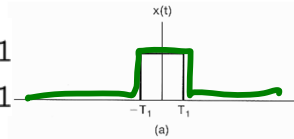
0 T



Sampling with Zero-Order Hold:

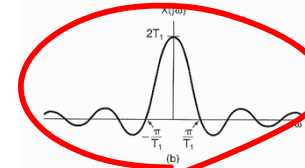


$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



Ex 4.4, p. 293

$$X(j\omega) = 2 \frac{\sin(\omega T_1)}{\omega}$$

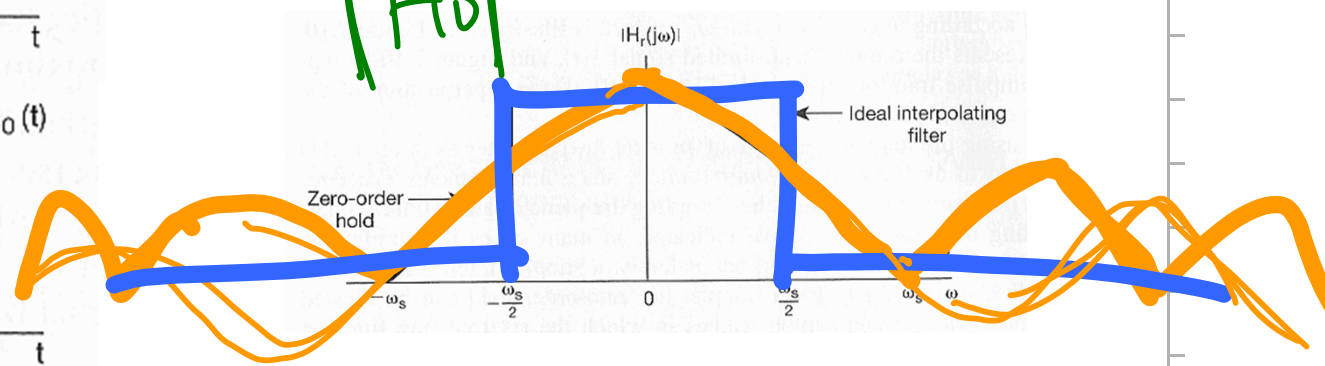
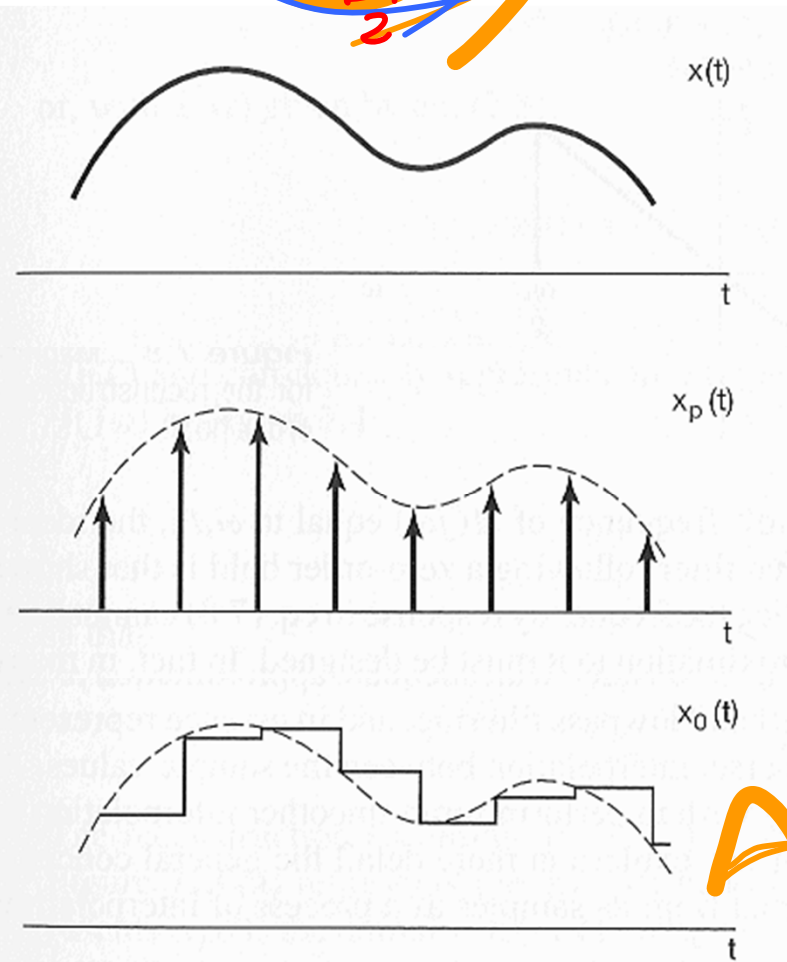


$$x(t - t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0} X(j\omega)$$

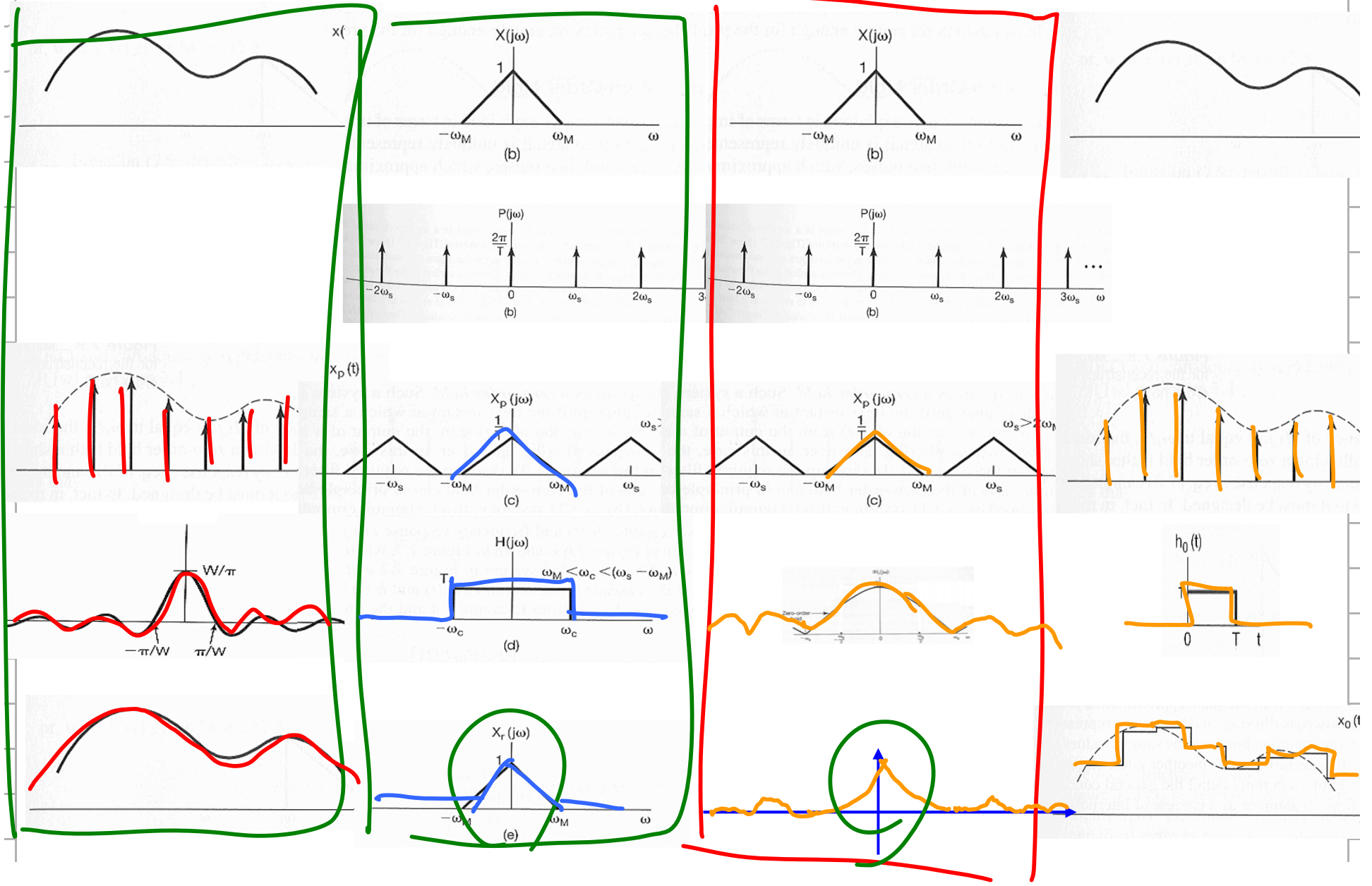
Eq 4.27, p. 301

$$H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right]$$

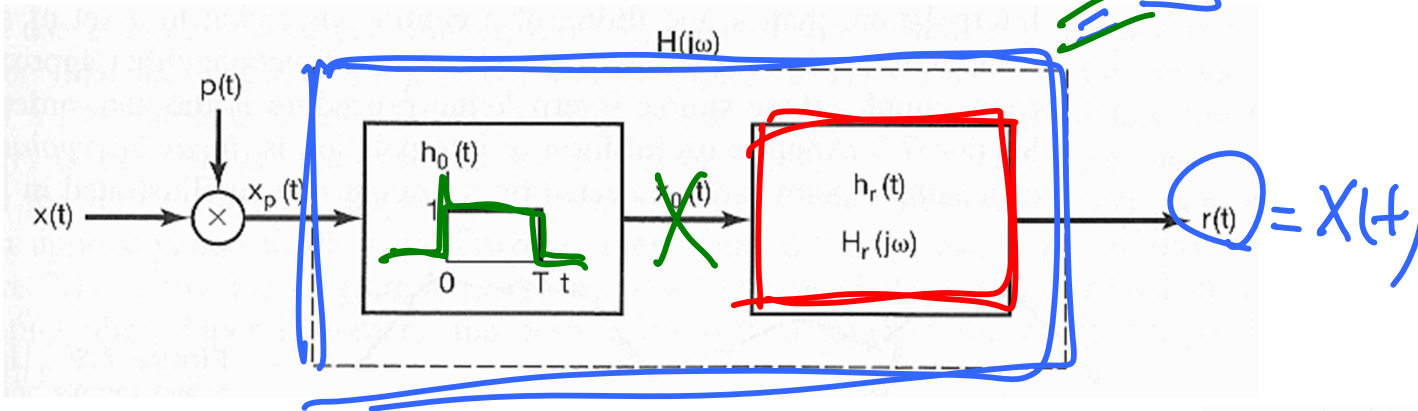
$|H_0|$



With Ideal Lowpass Filter & with Zero-Order Hold:



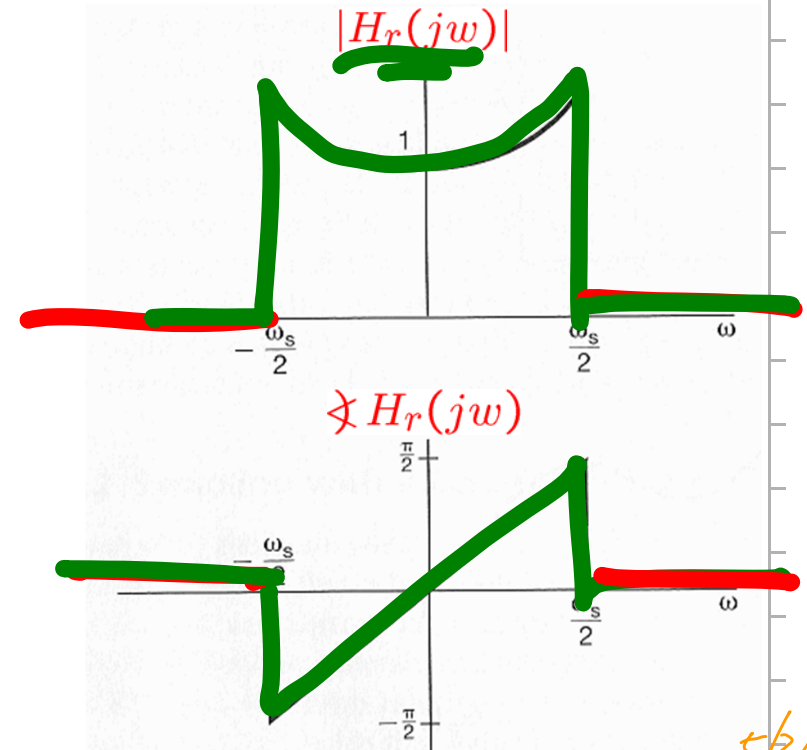
Sampling with Zero-Order Hold:



$$H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right]$$

$$H(j\omega) = H_0(j\omega) H_r(j\omega)$$

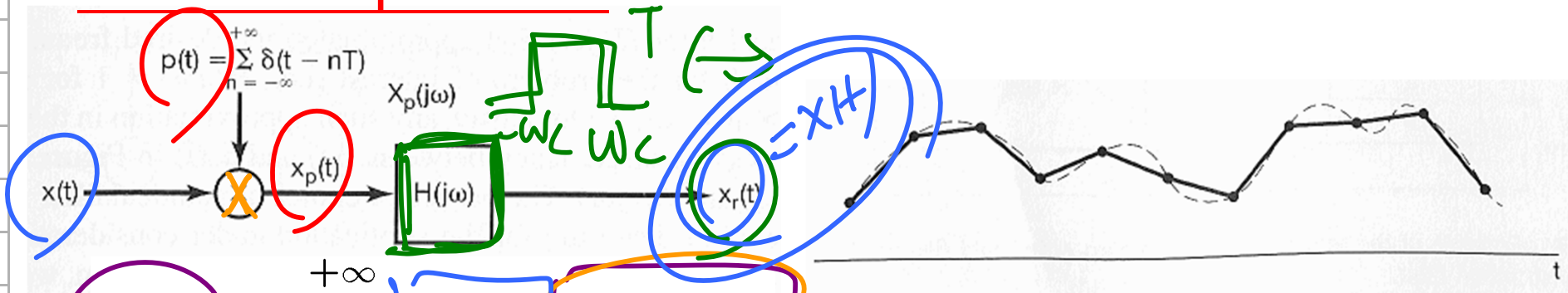
$$\Rightarrow H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{2 \sin(\omega T/2)}$$



5/2/13
2:15 PM

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

Exact Interpolation:



$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t - nT)$$

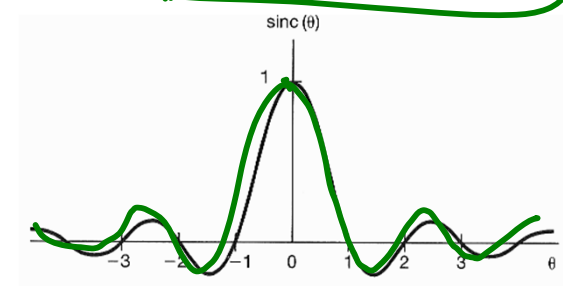
$$x_r(t) = x_p(t) * h(t)$$

ideal lowpass filter
with a magnitude of T

$$h(t) = T \frac{\omega_c}{\pi} \frac{\sin(\omega_c t)}{\omega_c t}$$

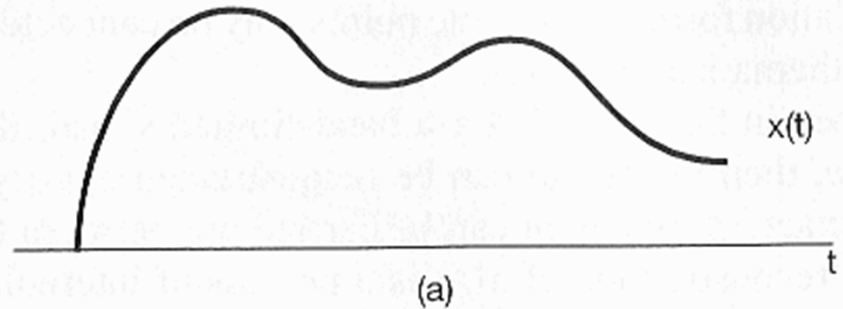
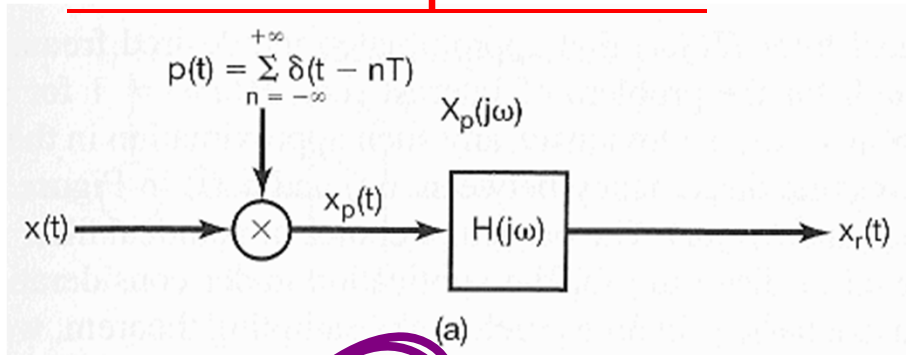
Ex 2.11, p. 110 $x(t - t_0) = x(t) * \delta(t - t_0)$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) h(t - nT)$$



$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{\omega_c T}{\pi} \frac{\sin(\omega_c(t - nT))}{\omega_c(t - nT)} = x(t)$$

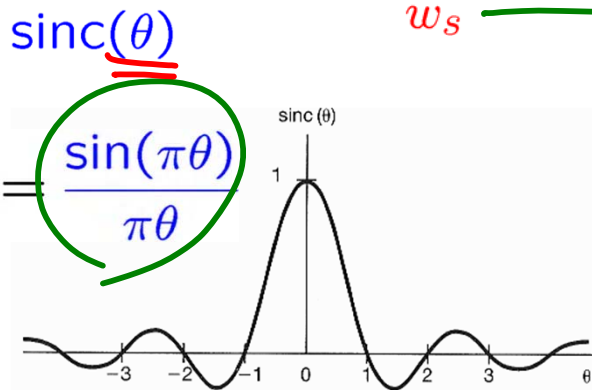
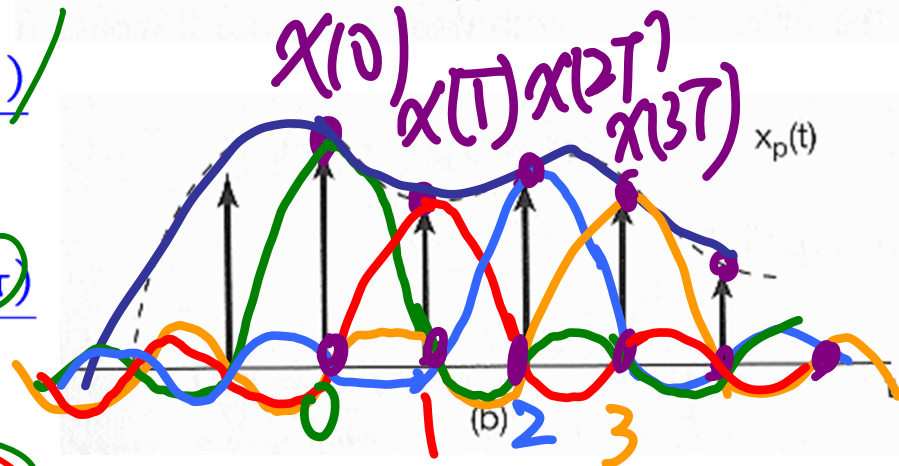
Exact Interpolation:



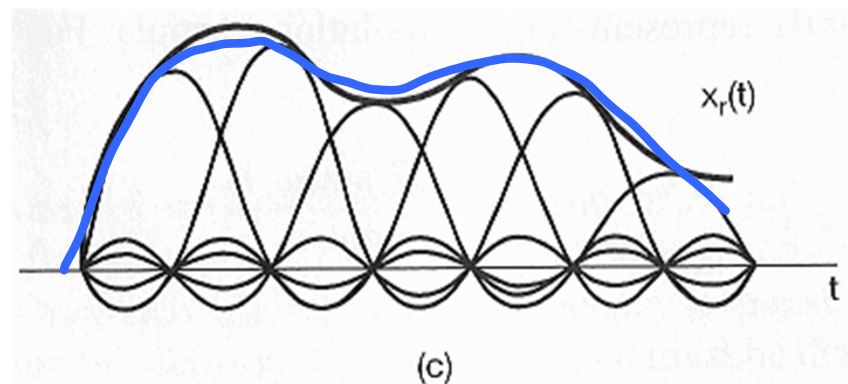
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{\omega_c T \sin(\omega_c(t - nT))}{\pi \omega_c(t - nT)}$$

$$\frac{\omega_c}{\pi} \frac{2\pi}{\omega_s} \frac{\sin \pi(\omega_c(t - nT)/\pi)}{\pi \omega_c(t - nT)/\pi}$$

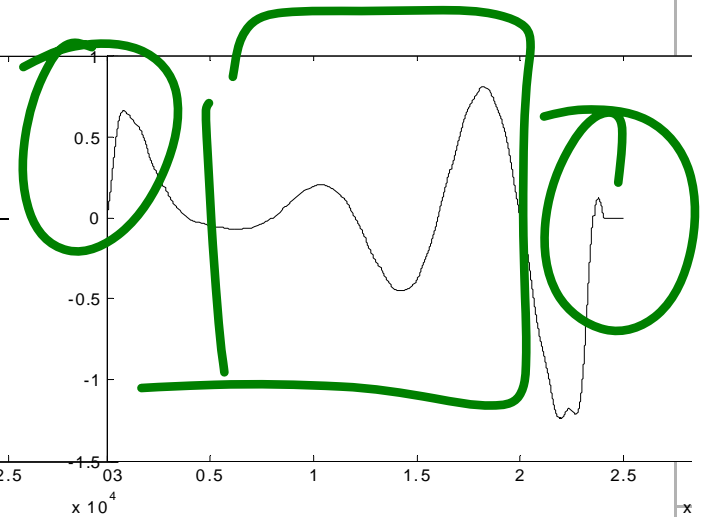
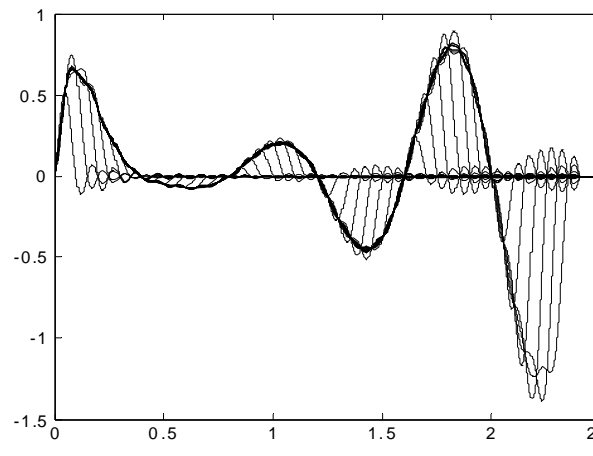
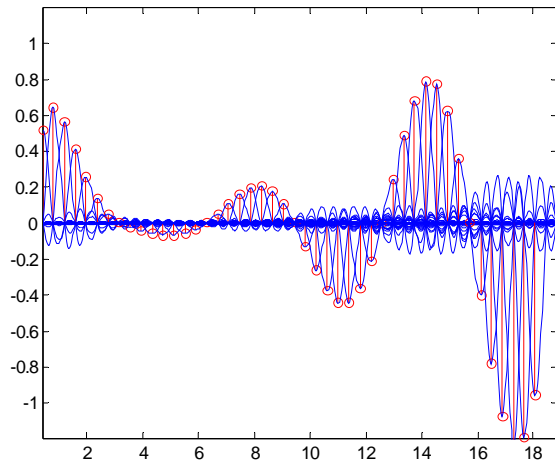
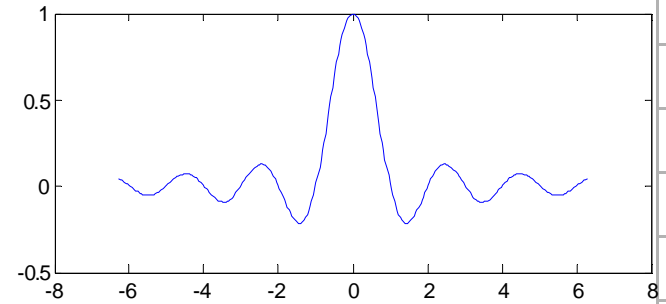
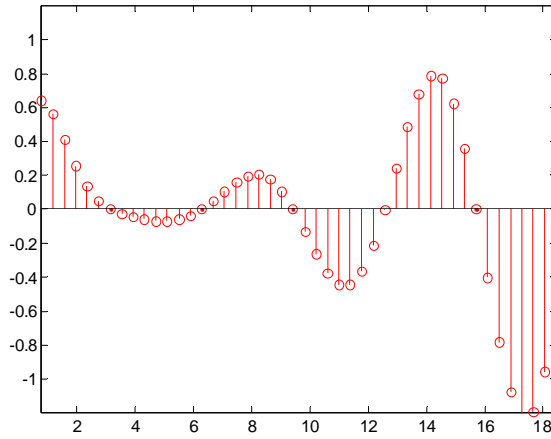
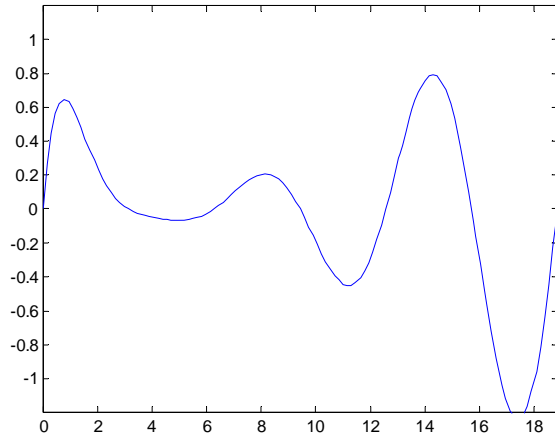
$$\frac{2\omega_c}{\omega_s} \text{sinc}(\omega_c(t - nT)/\pi)$$



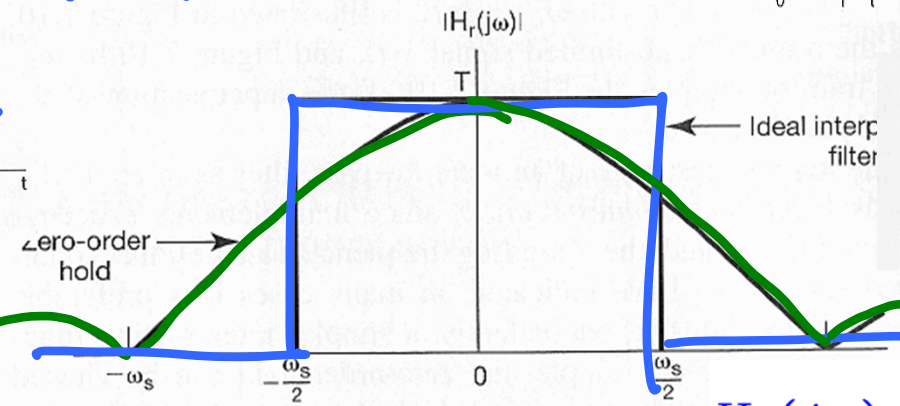
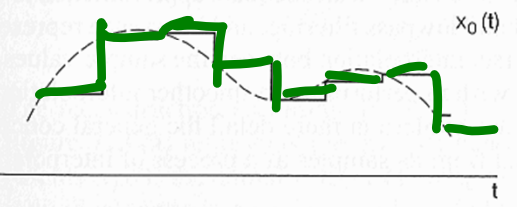
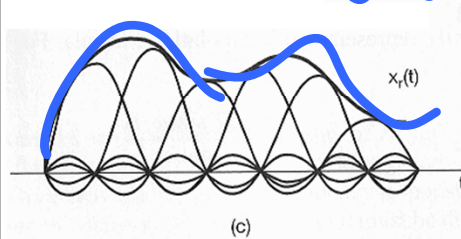
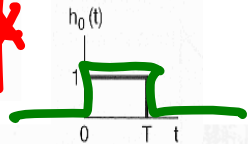
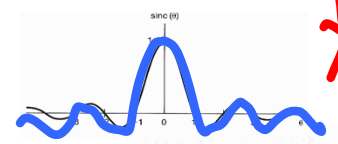
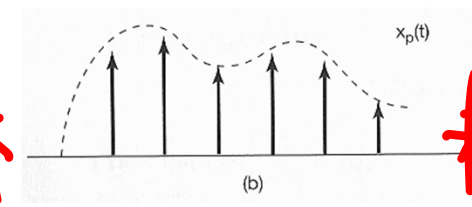
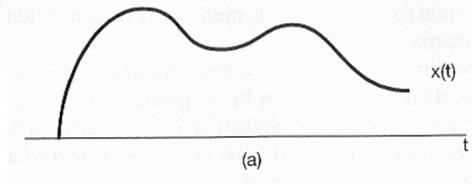
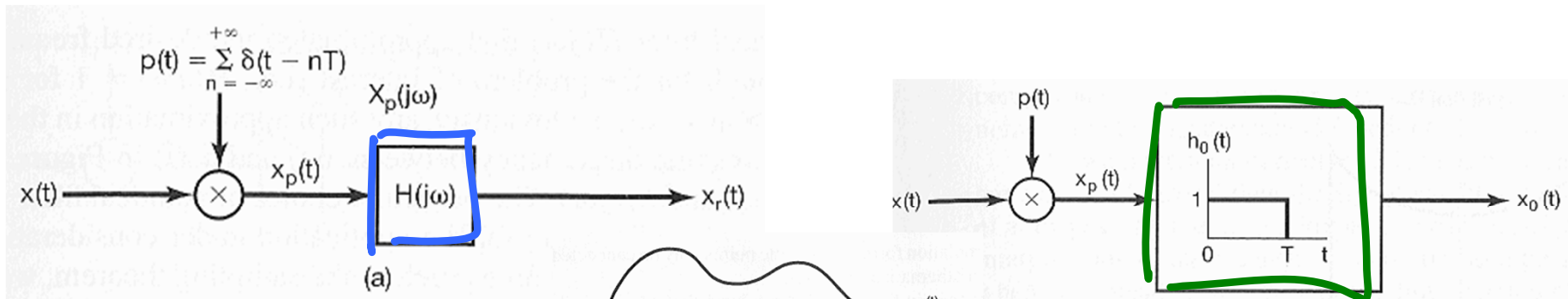
$n=0$
 $n=1$
 $n=2$
 $n=3$



Reconstruction of a Signal from its Samples Using Interpolation



Ideal Interpolating Filter & The Zero-Order Hold:



$$H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right]$$

■ Sampling & Interpolation of Images:

original image

impulse sampling

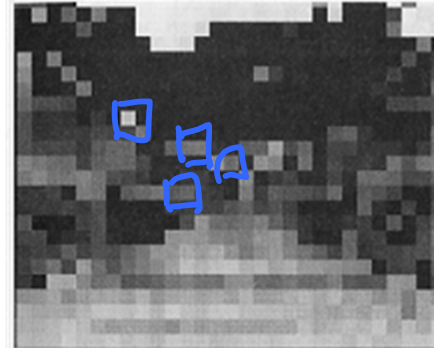
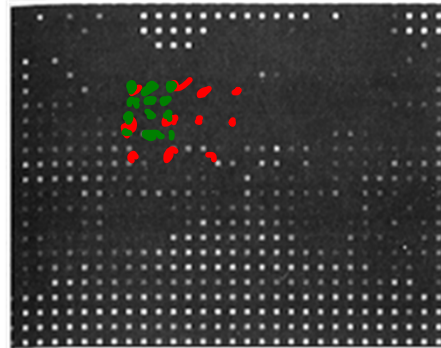
zero-order hold

zero-order hold

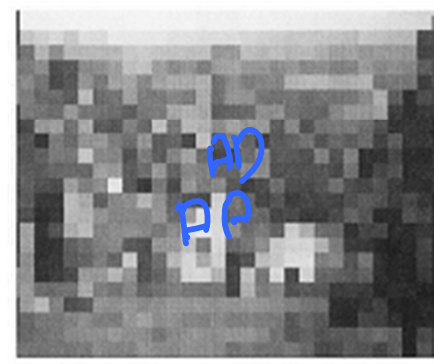
4 : 1



(a)

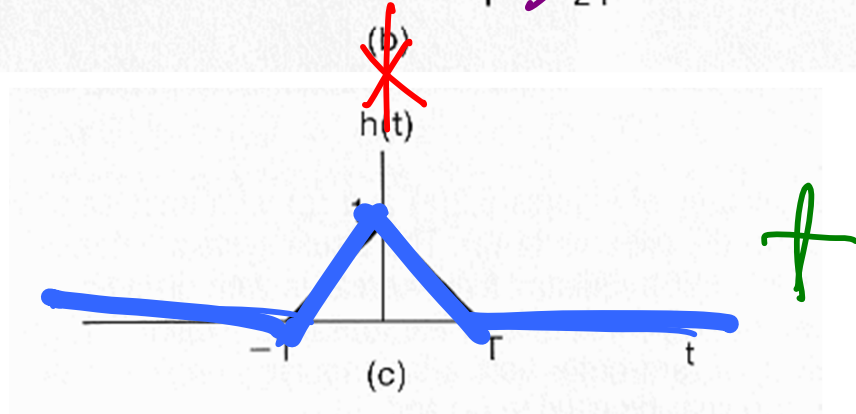
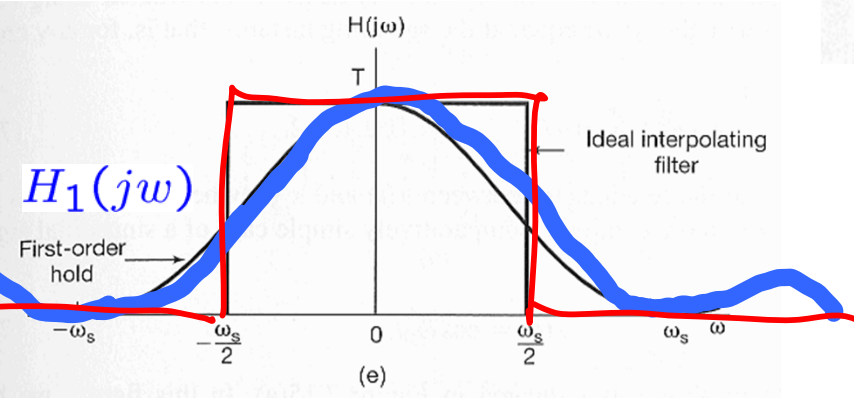
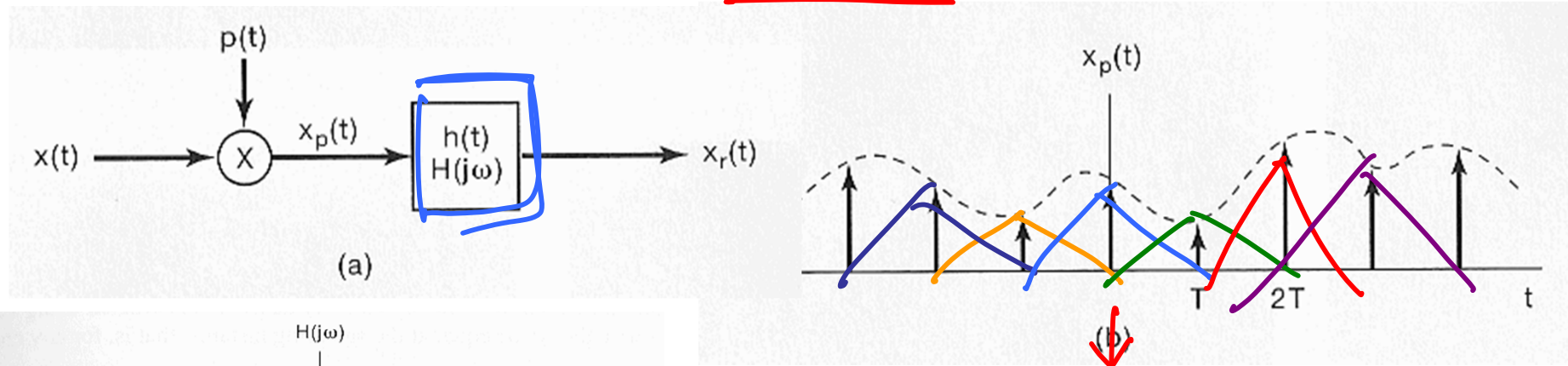


(g)

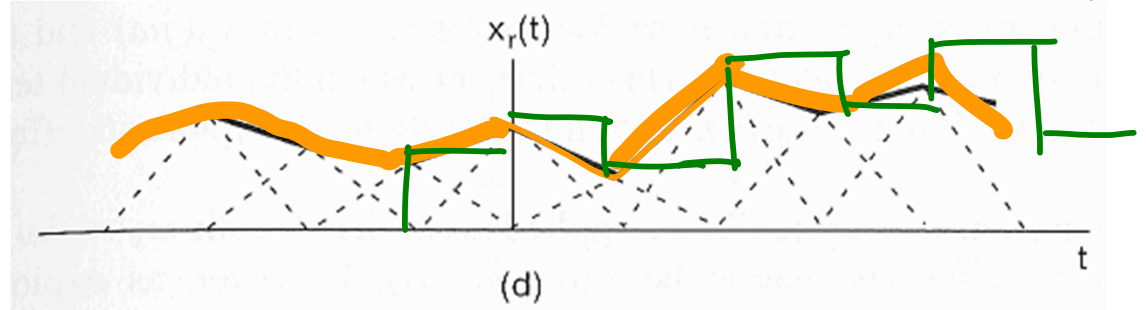


Reconstruction of a Signal from its Samples Using Interpolation

- Higher-Order Holds: 1st-order



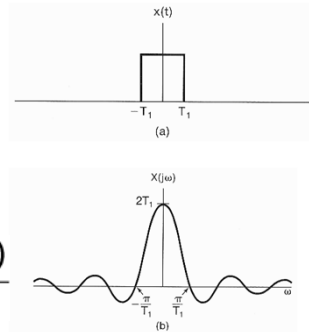
$$H_1(j\omega) = \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2$$



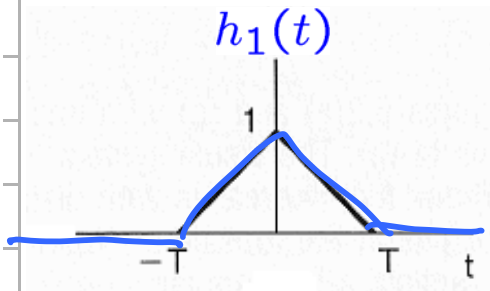
Higher-Order Holds:

Ex 4.4, p. 293

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



$$X(jw) = 2 \frac{\sin(wT_1)}{w}$$



$H_1(jw)$

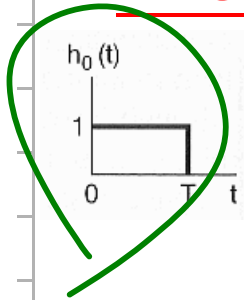
$$= \frac{1}{T} \left(h_{sq}(t) * h_{sq}(t) \right)$$

$$= \frac{1}{T} \left(2 \frac{\sin(wT/2)}{w} \times 2 \frac{\sin(wT/2)}{w} \right)$$

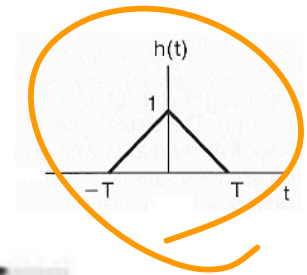
~~X~~

$$= \frac{1}{T} \left[\frac{\sin(wT/2)}{w/2} \right]^2$$

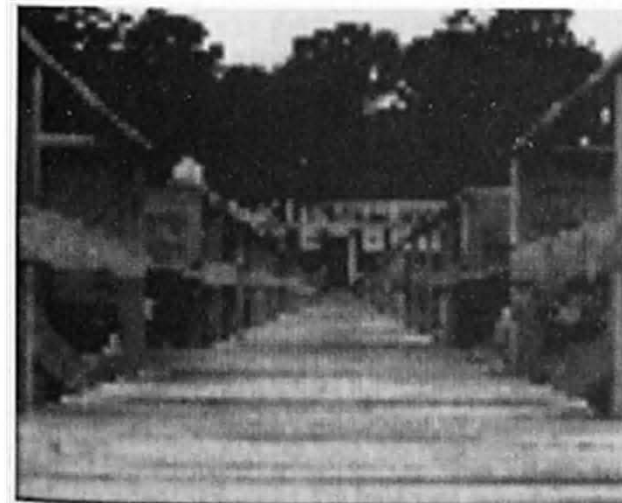
■ First-Order Hold on Image Processing:



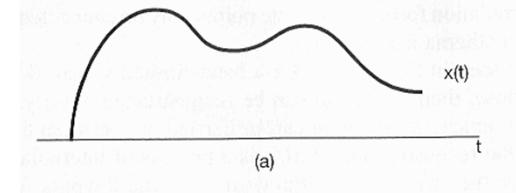
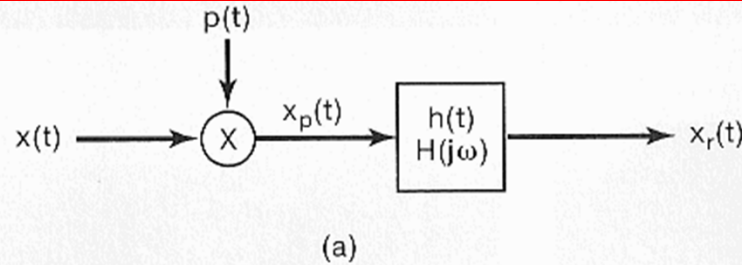
zero-order hold



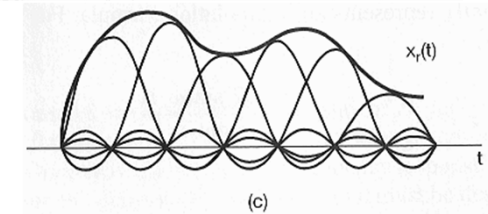
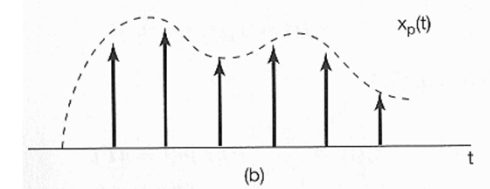
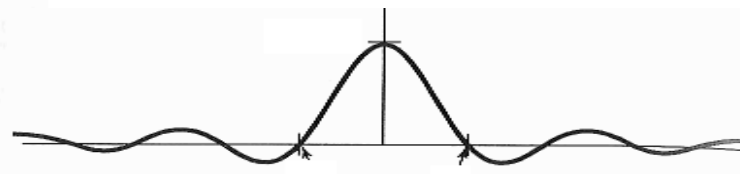
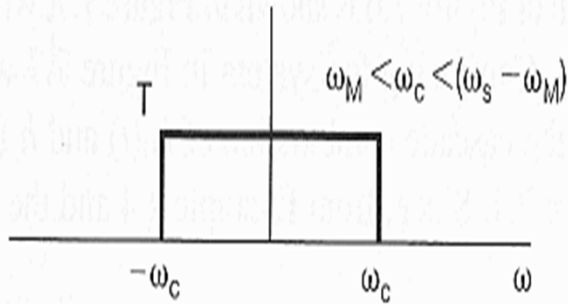
first-order hold



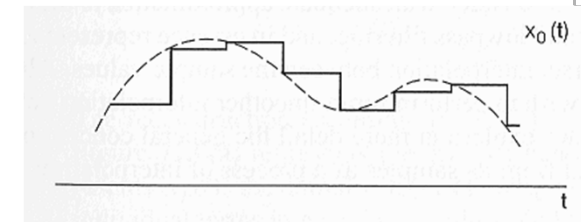
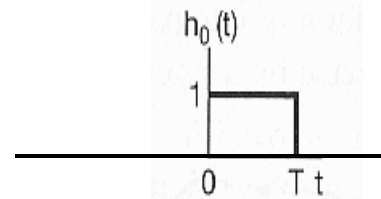
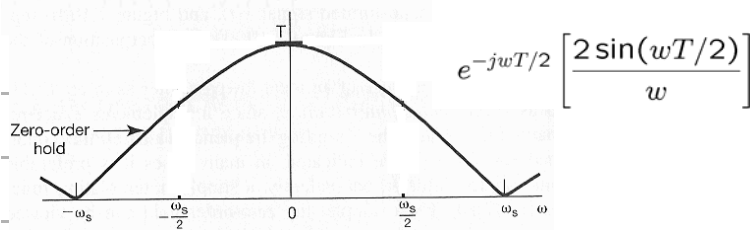
Three Filters: Ideal Lowpass, Zero-Order, First-Order



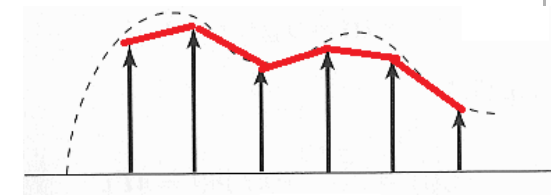
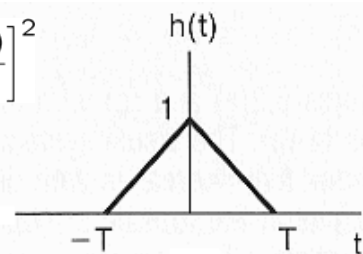
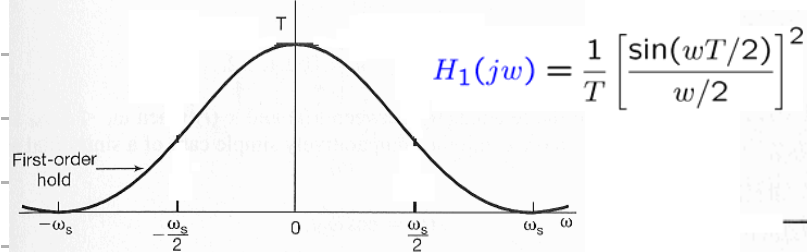
ideal lowpass



zero-order hold $H_0(j\omega) =$

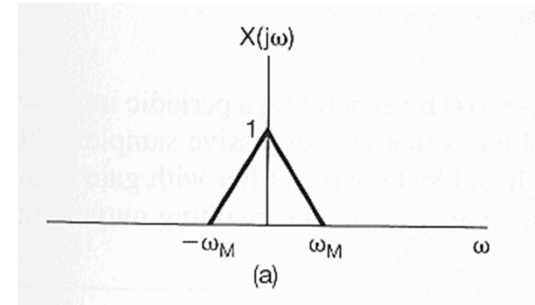
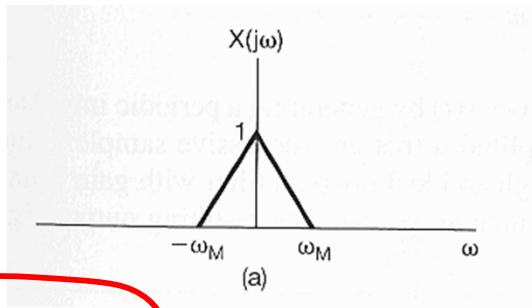


first-order hold



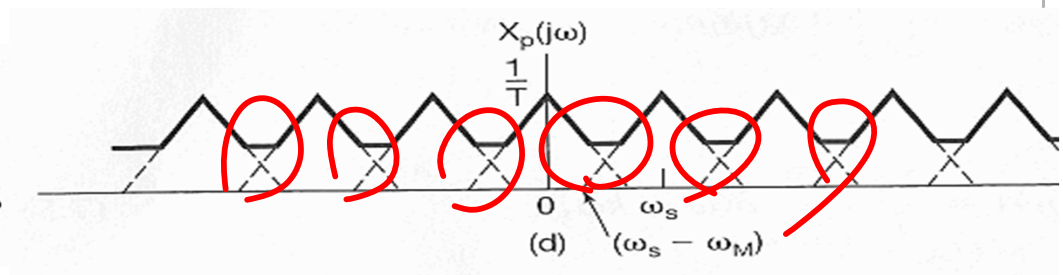
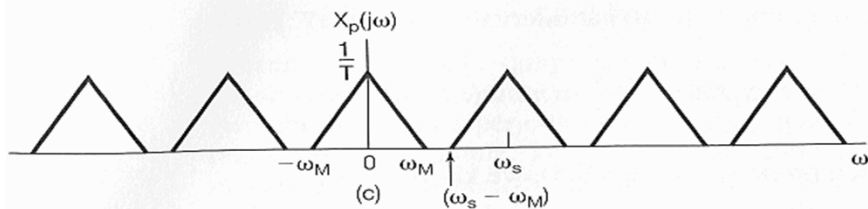
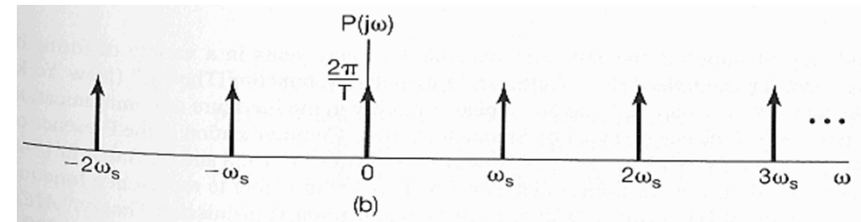
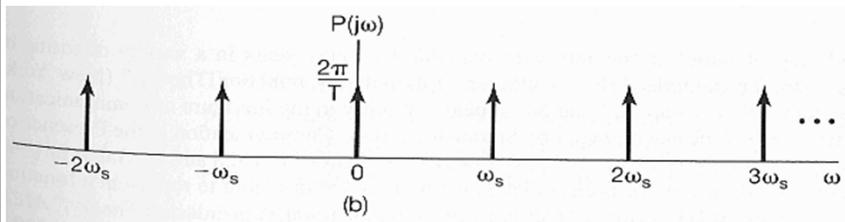
- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

Overlapping in Frequency-Domain: Aliasing



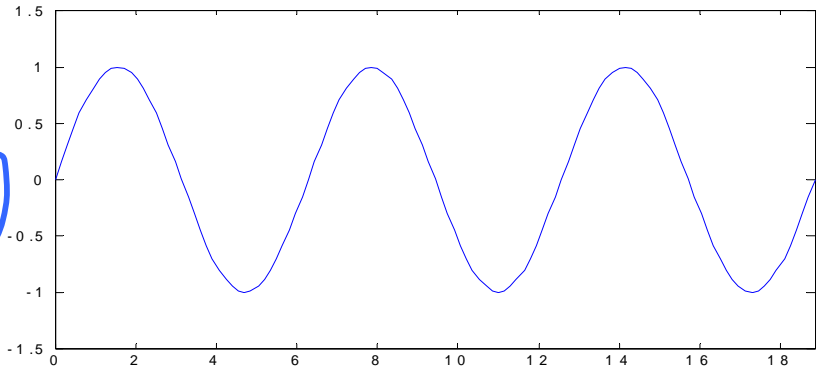
$\omega_s > 2\omega_M$

$\omega_s < 2\omega_M$

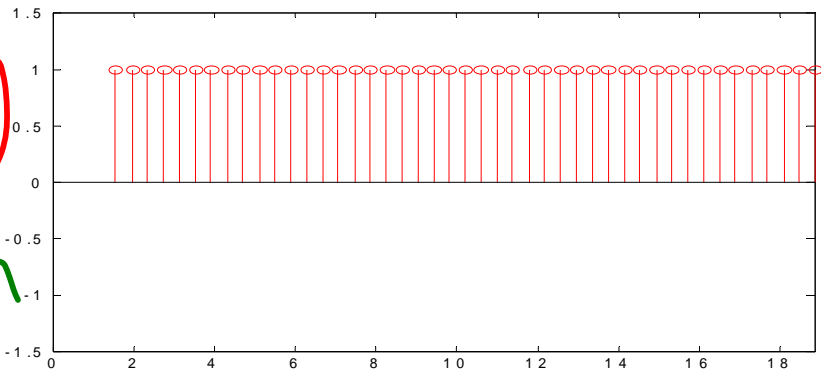


Overlapping in Frequency-Domain: Aliasing

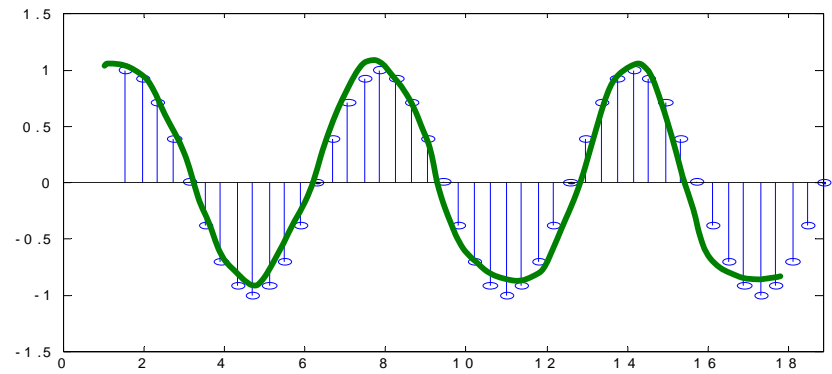
$X(t)$



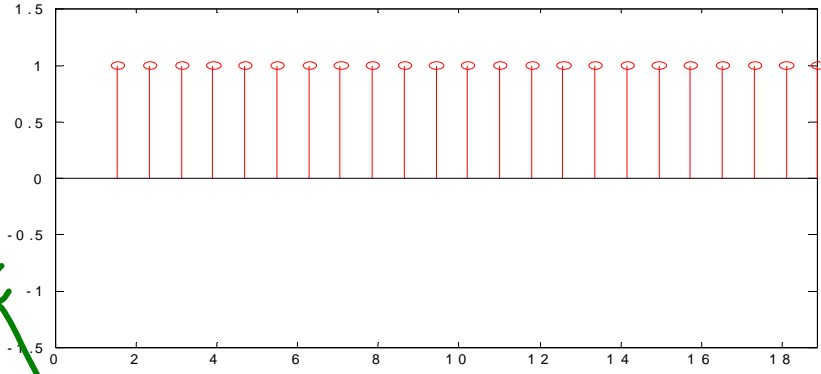
$P(t)$



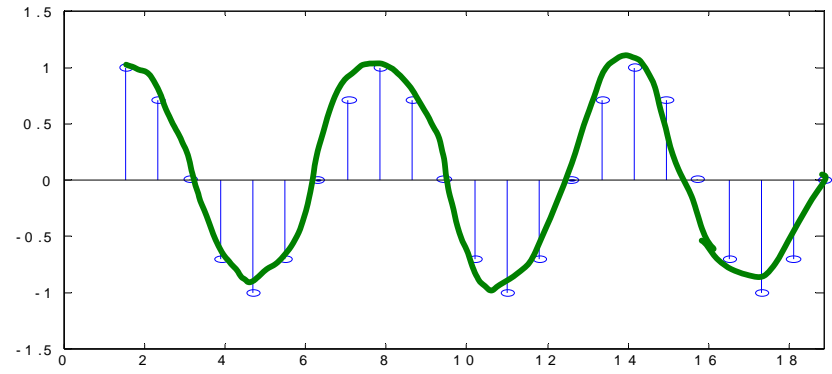
T_{11}



$P(t)$

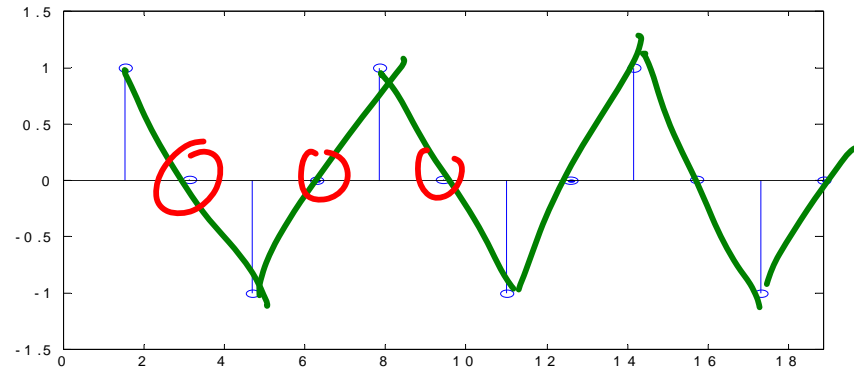
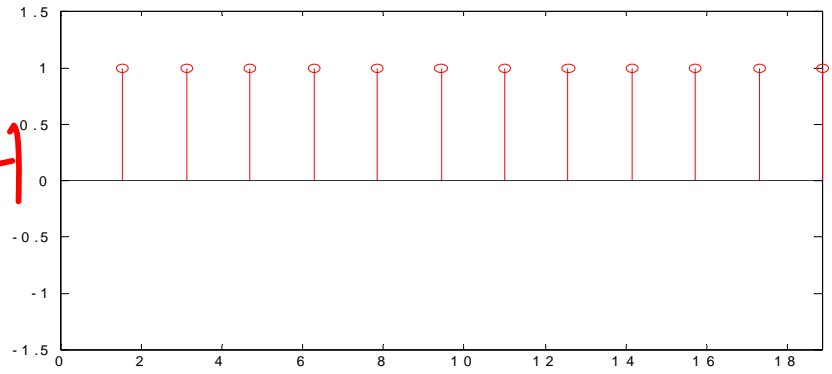


T_K

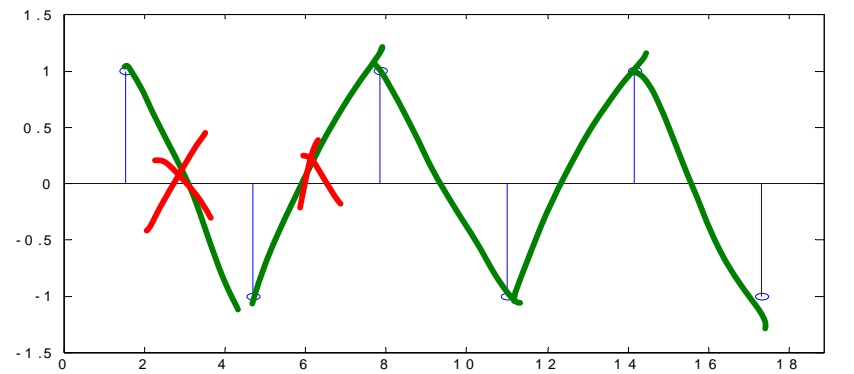
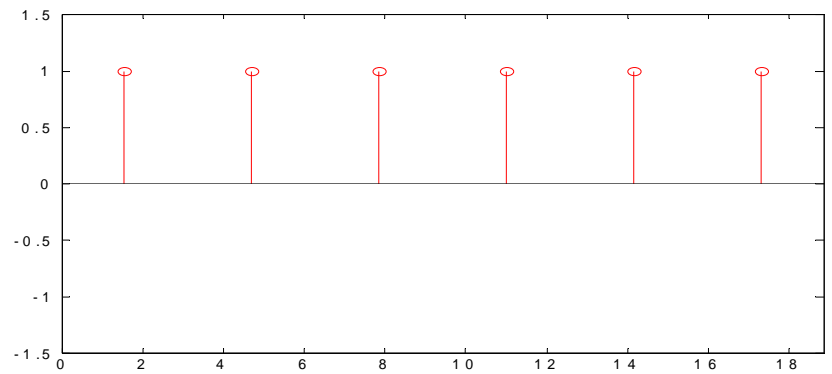


Overlapping in Frequency-Domain: Aliasing

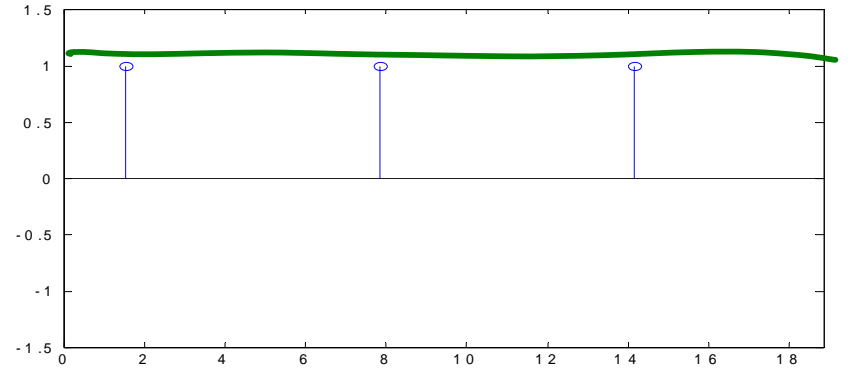
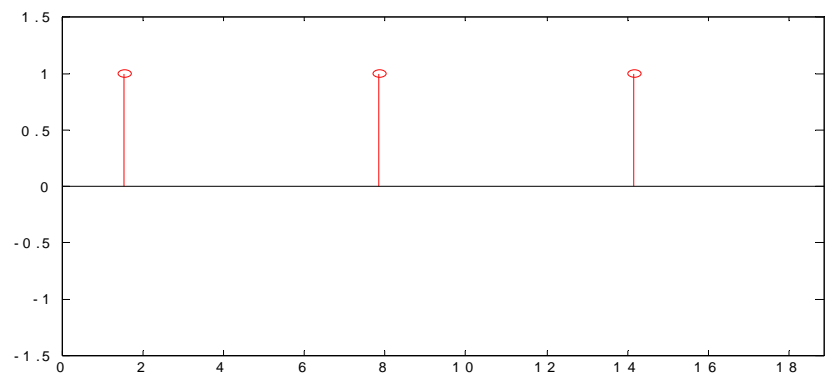
$P(t)$



$\frac{1}{2}$

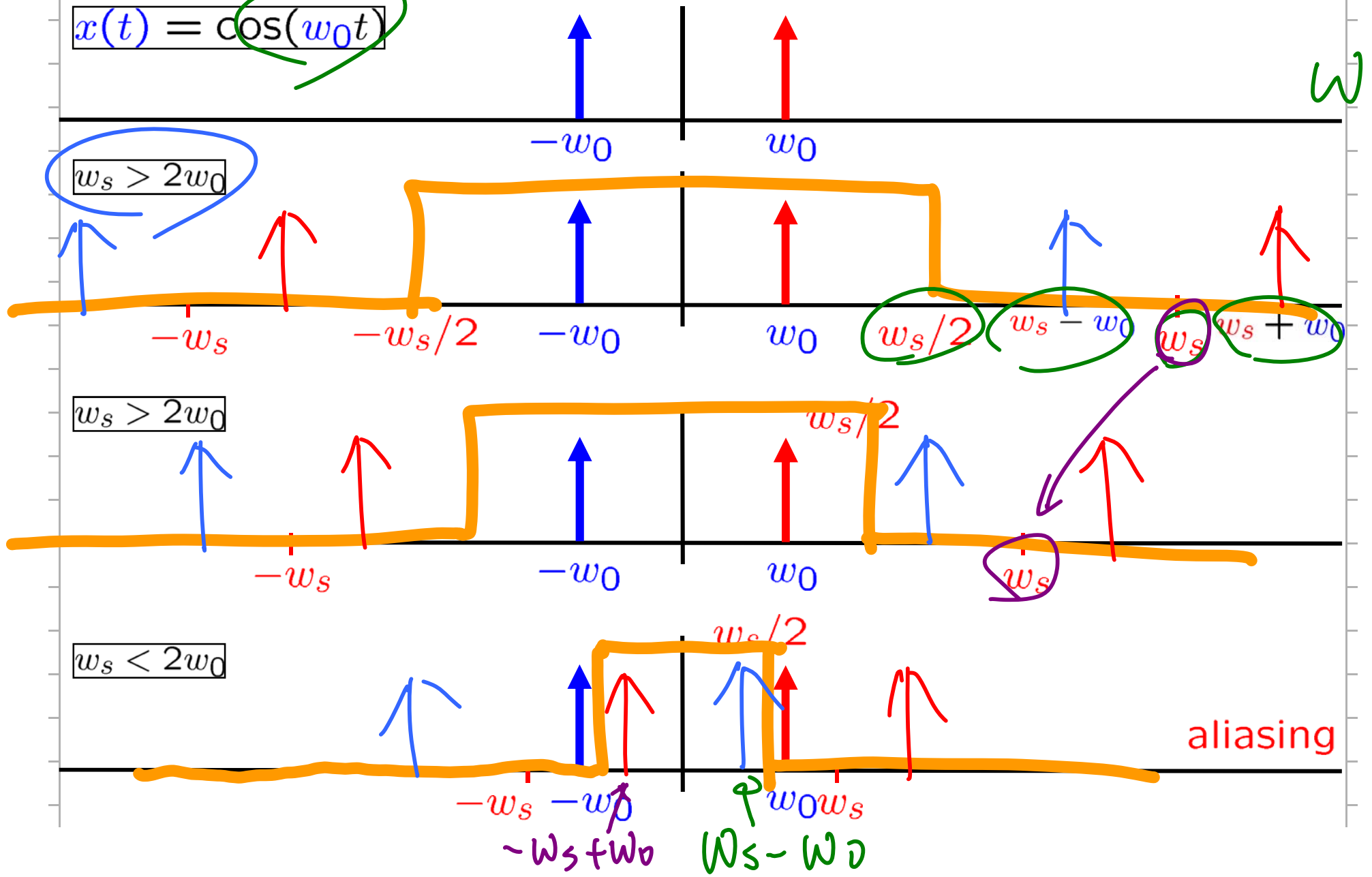


$\frac{1}{4}$



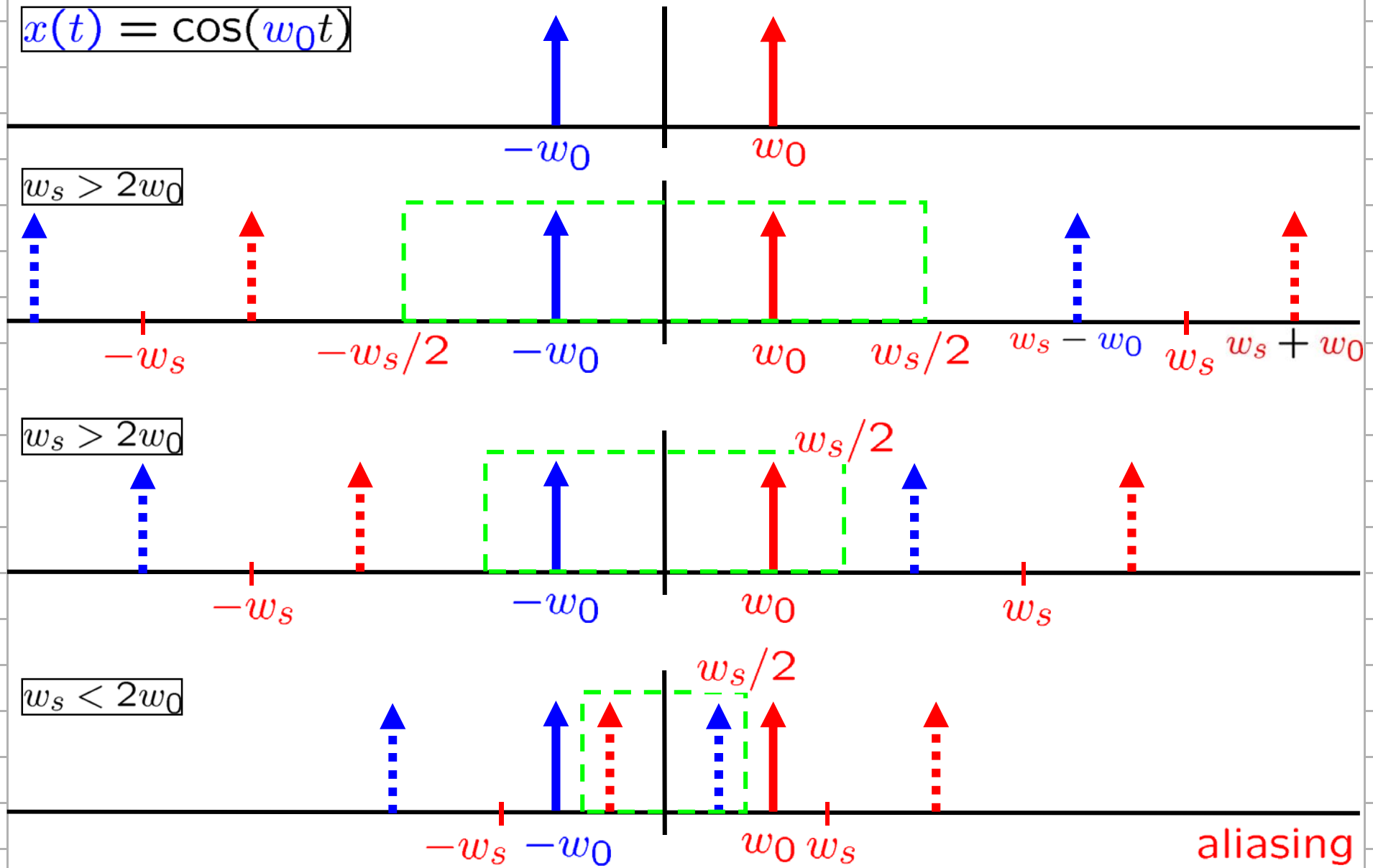
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$



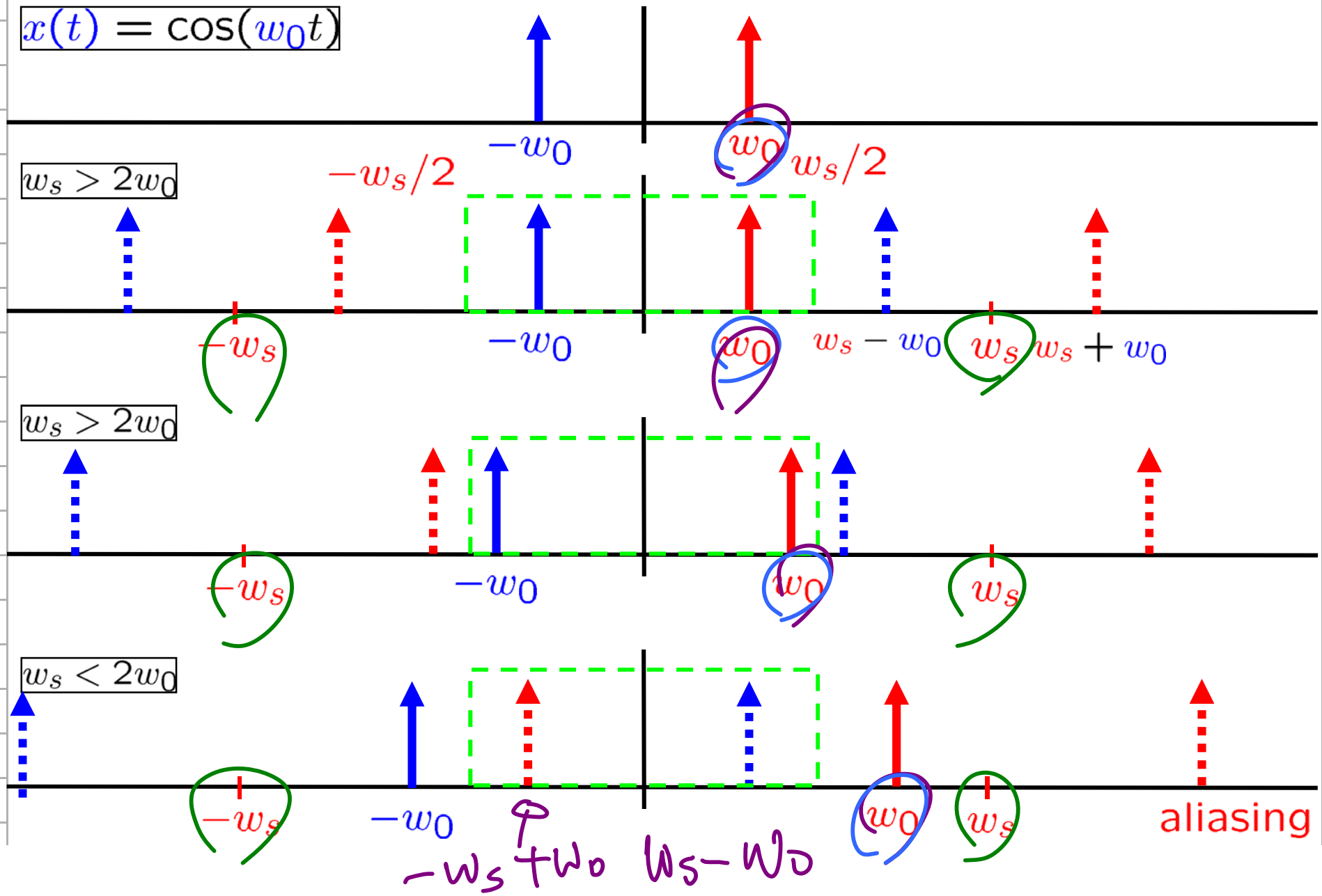
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$



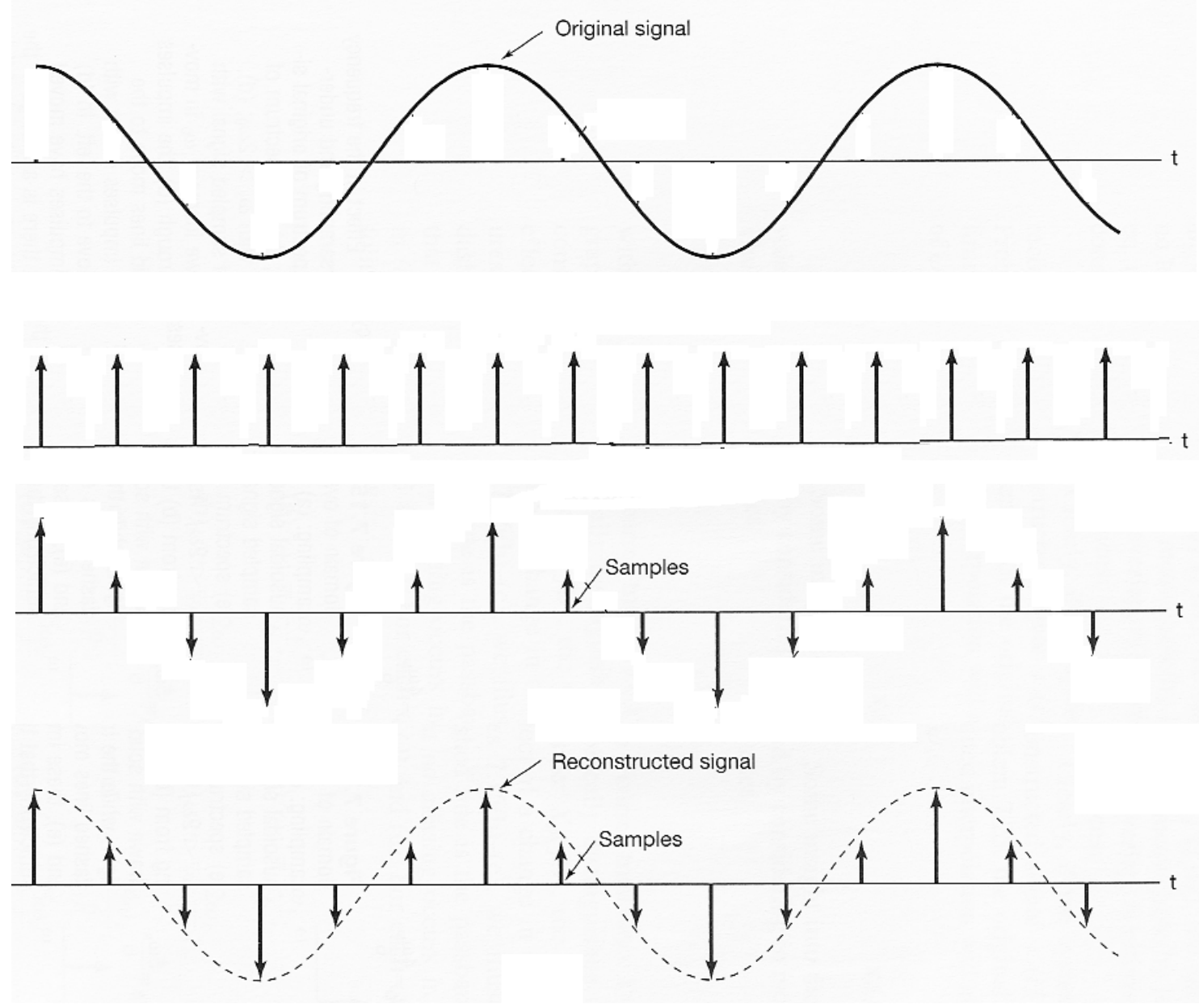
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$

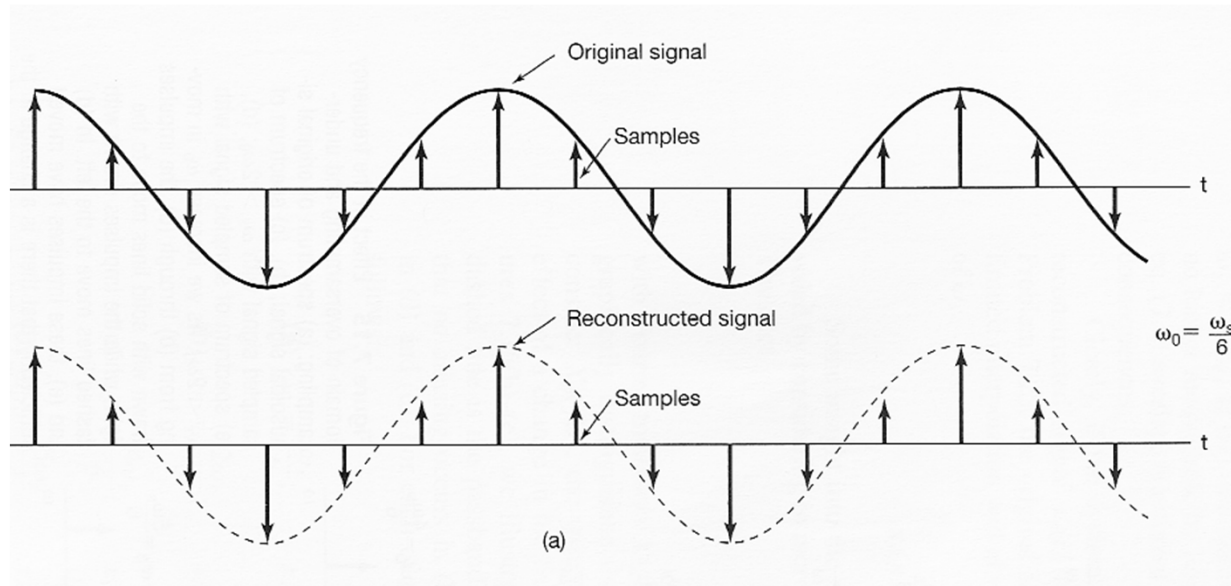


Overlapping in Frequency-Domain: Aliasing

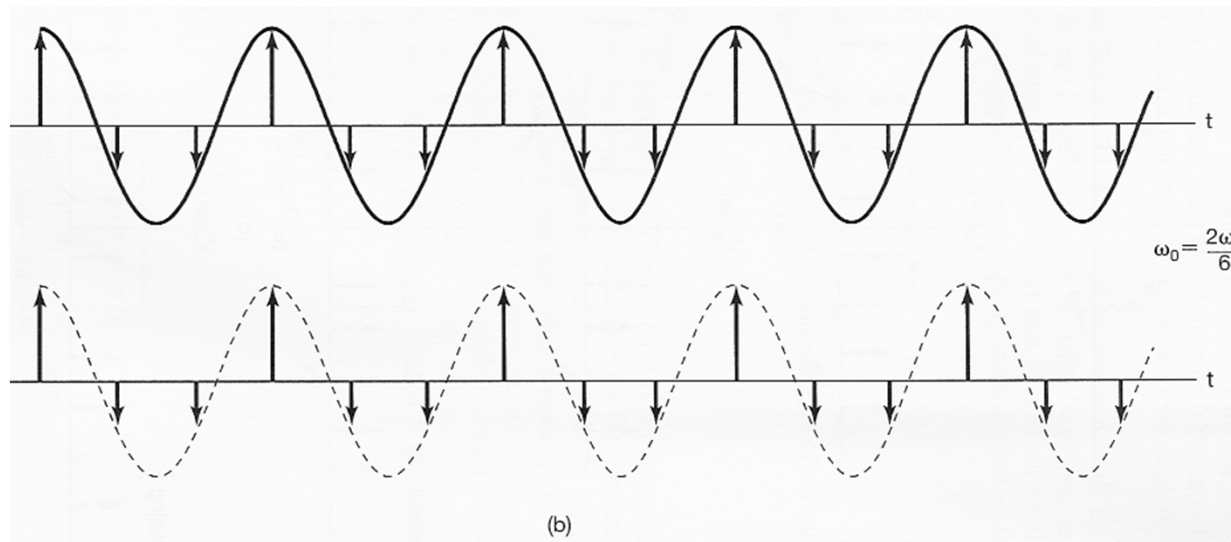
$$\omega_0 = \frac{\omega_s}{6}$$



Overlapping in Frequency-Domain: Aliasing

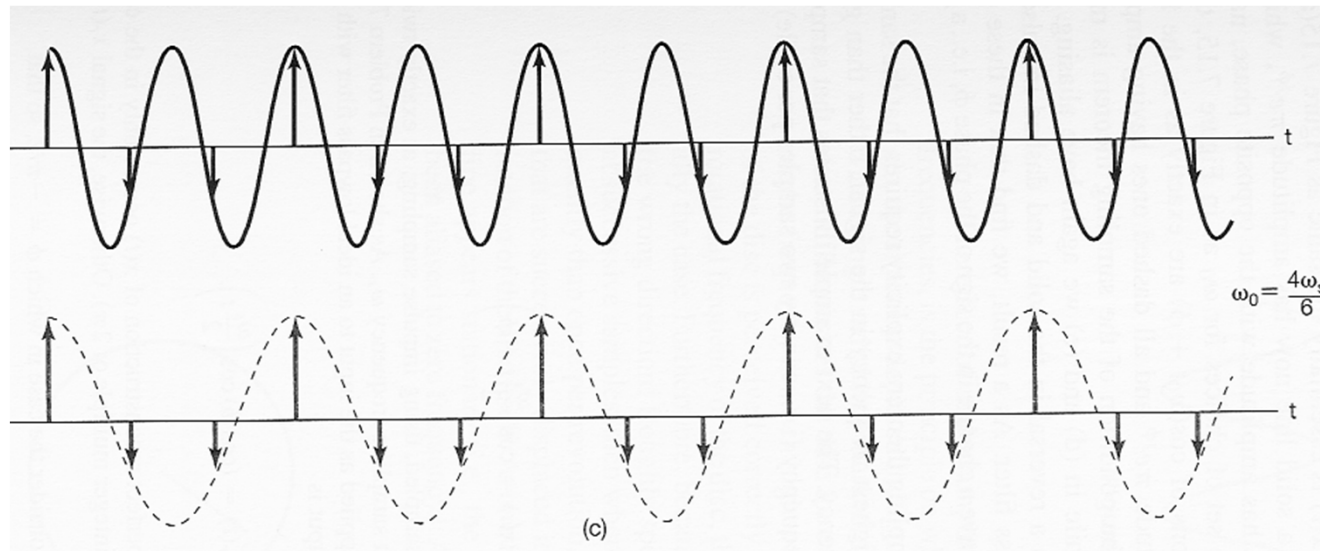


$$\omega_0 = \frac{\omega_s}{6}$$



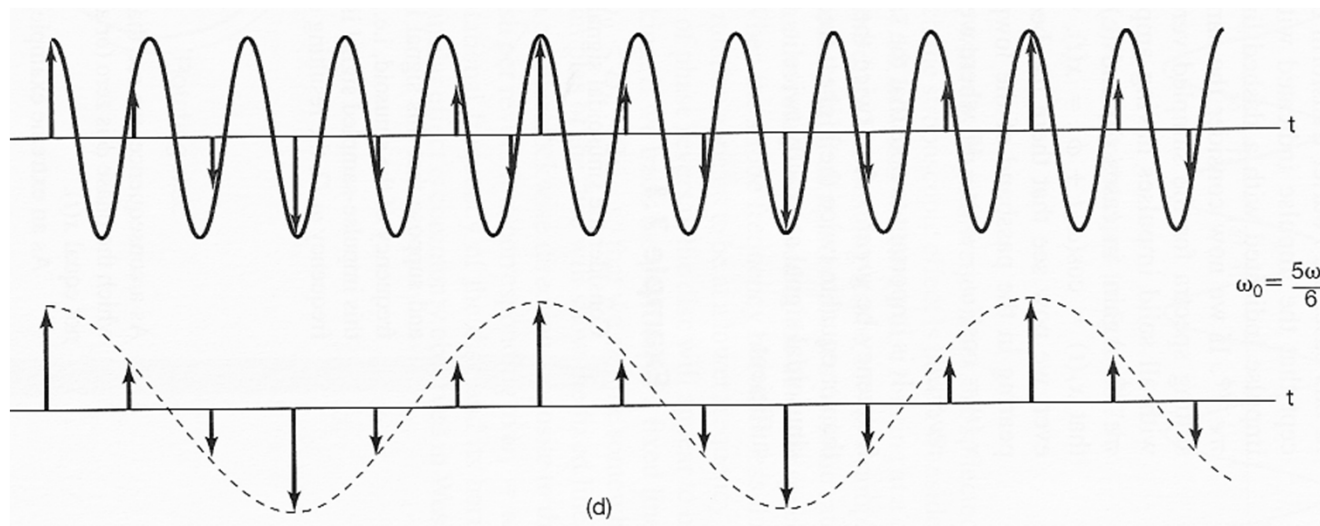
$$\omega_0 = \frac{2\omega_s}{6}$$

Overlapping in Frequency-Domain: Aliasing



$$\omega_0 = \frac{4\omega_s}{6}$$

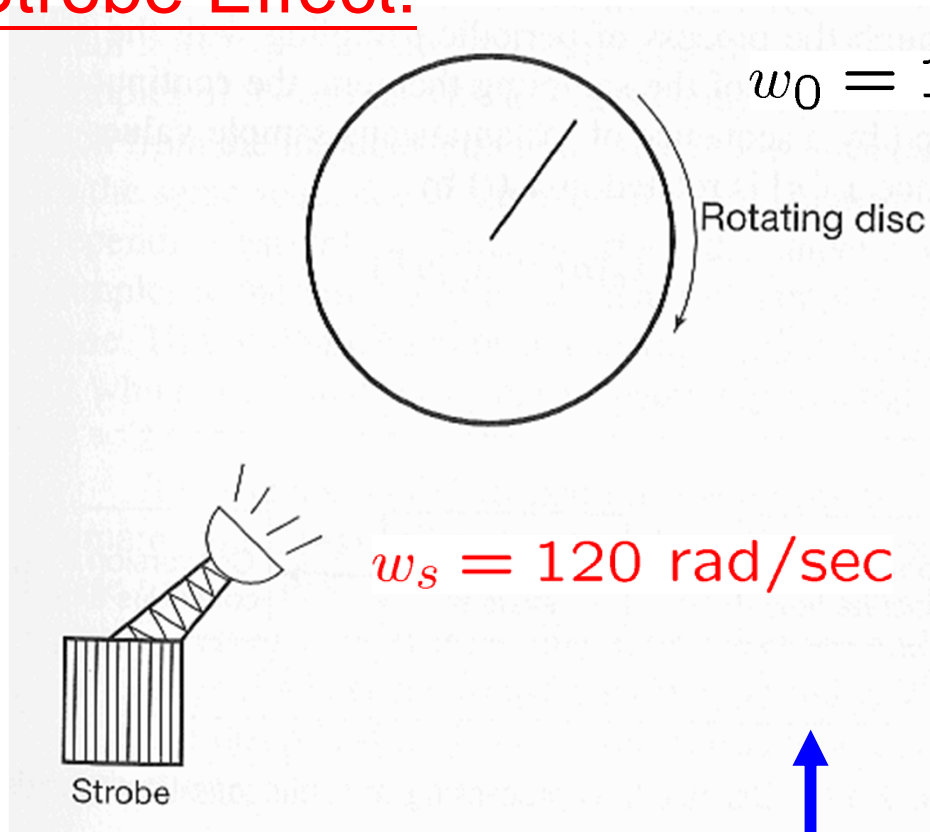
$$\begin{aligned} \omega_s - \omega_0 &= \omega_s - \frac{4\omega_s}{6} \\ &= \frac{2}{6}\omega_s \end{aligned}$$



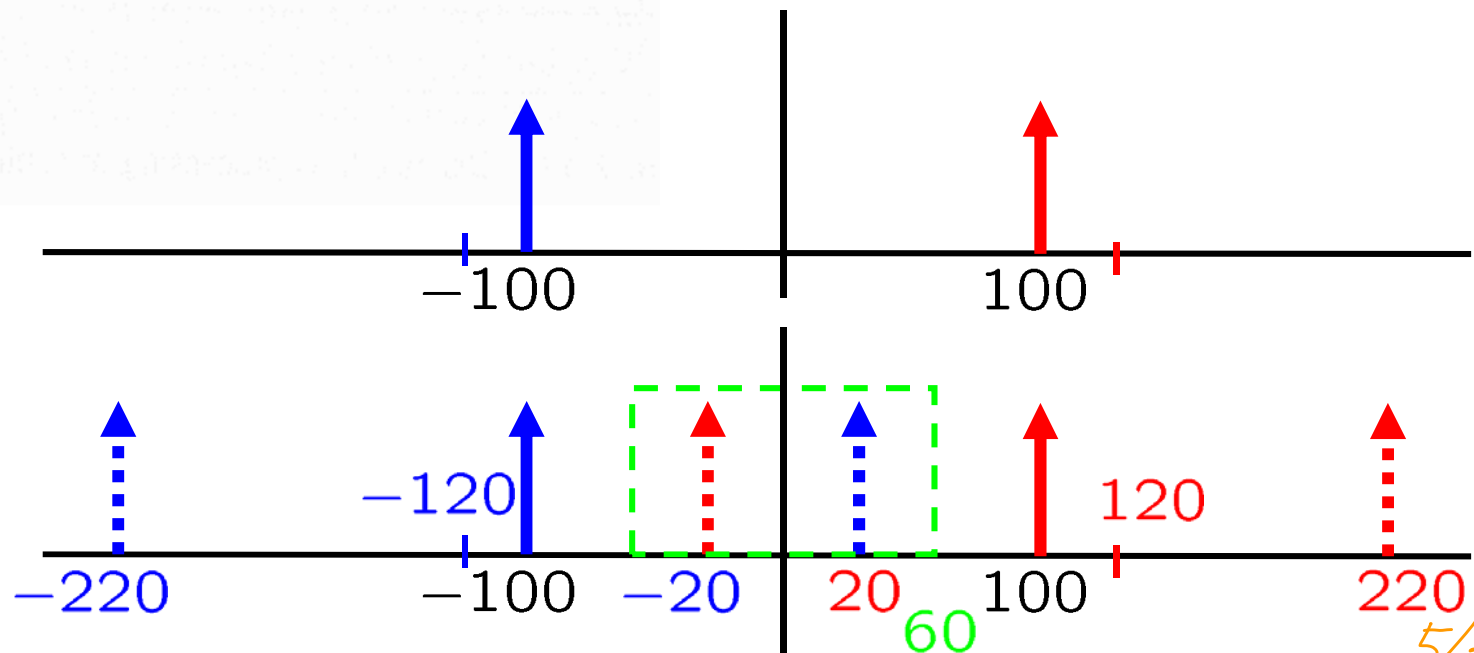
$$\omega_0 = \frac{5\omega_s}{6}$$

$$\begin{aligned} \omega_s - \omega_0 &= \omega_s - \frac{5\omega_s}{6} \\ &= \frac{1}{6}\omega_s \end{aligned}$$

Strobe Effect:



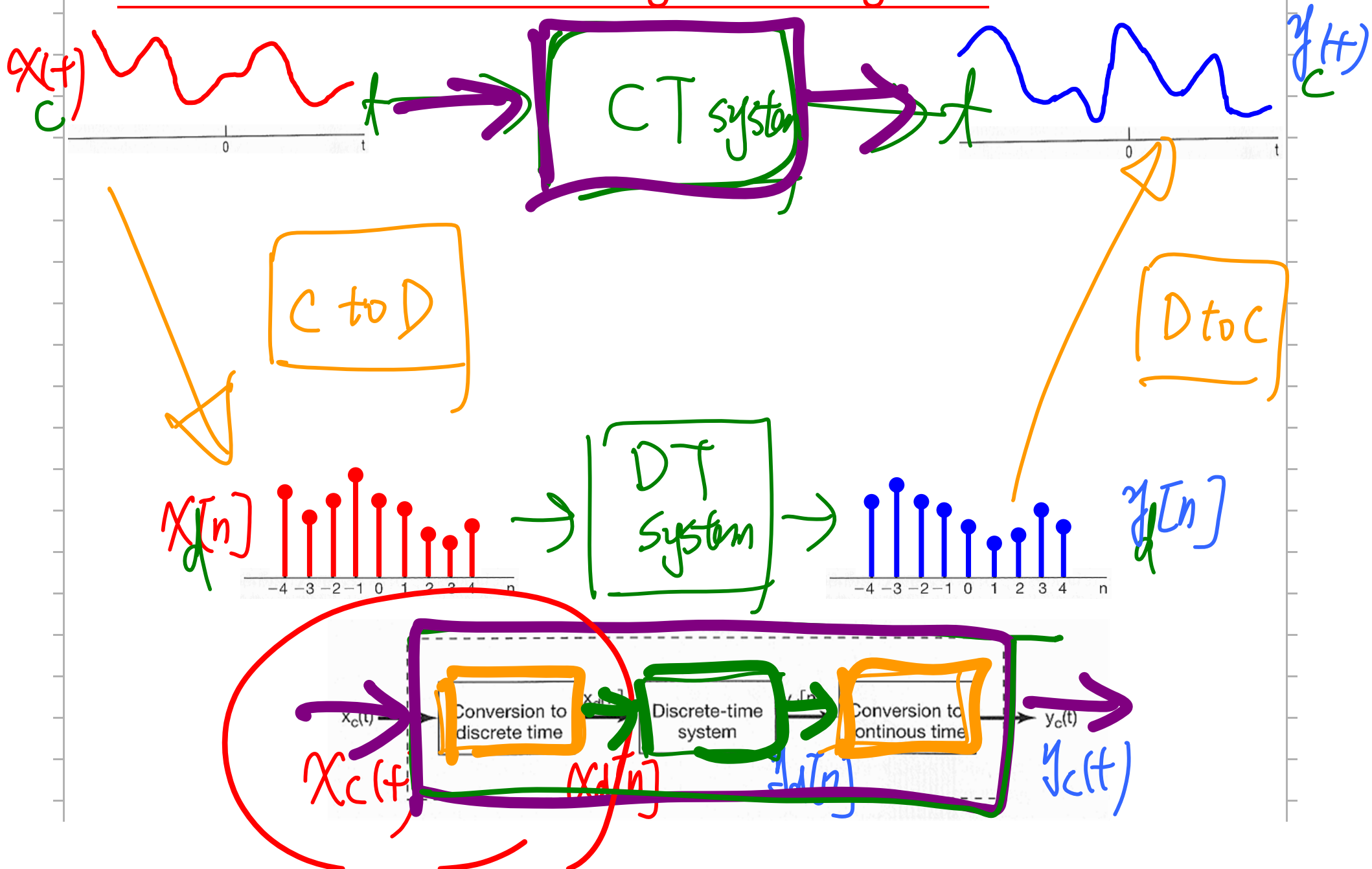
$$\Rightarrow \omega = \pm \omega_s \pm \omega_0$$
$$= +20, -20$$



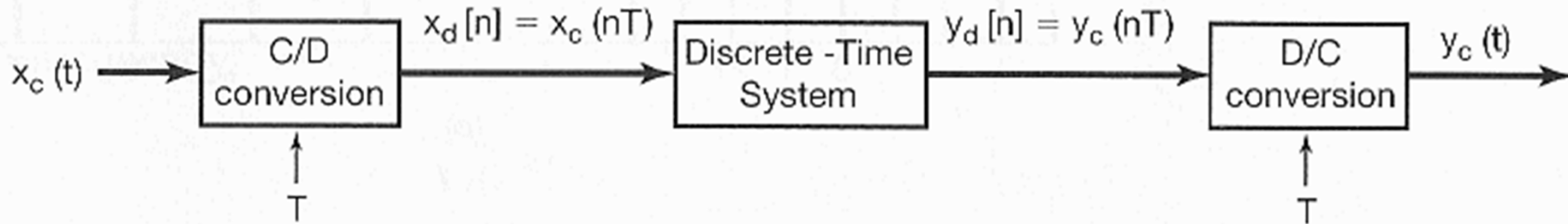
5/2/13
3:10 pm

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

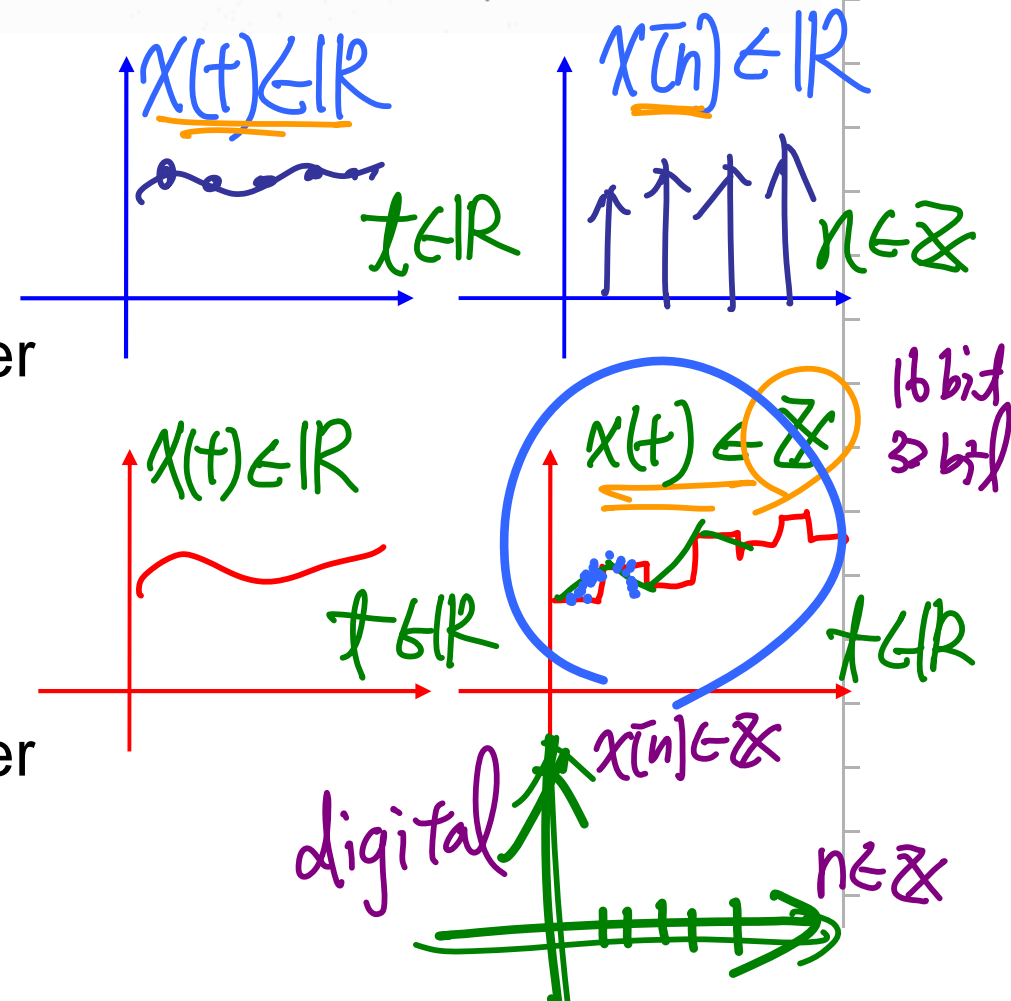
Discrete-Time Processing of CT Signals:



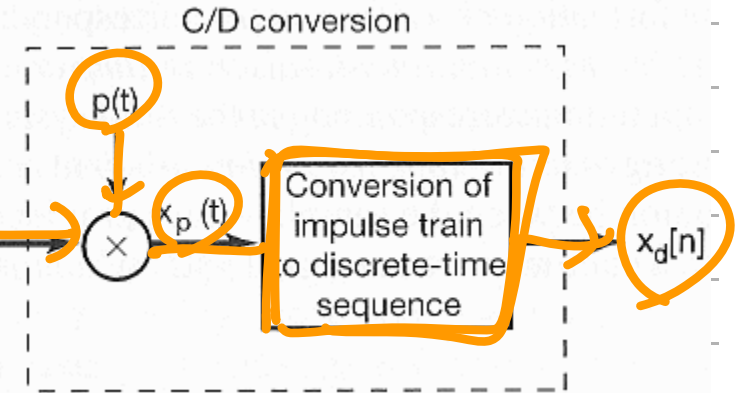
- C/D or A-to-D (ADC) and D/C or D-to-A (DAC):



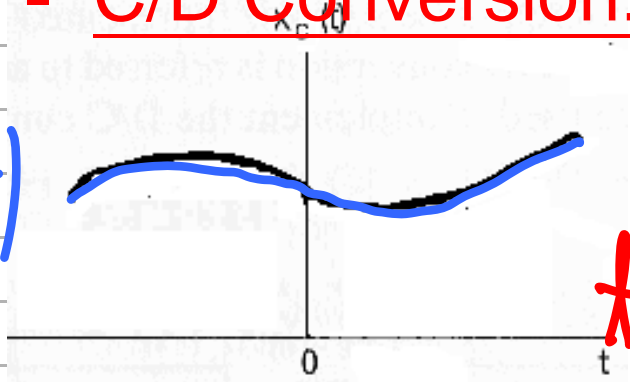
- C/D: continuous-to-discrete-time conversion
- A-to-D: analog-to-digital converter
- D/C: discrete-to-continuous-time conversion
- D-to-A: digital-to-analog converter



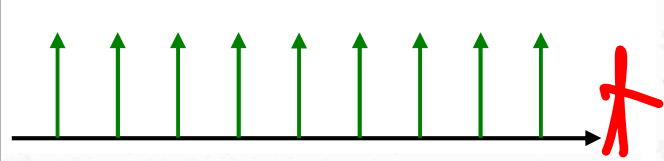
C/D Conversion:



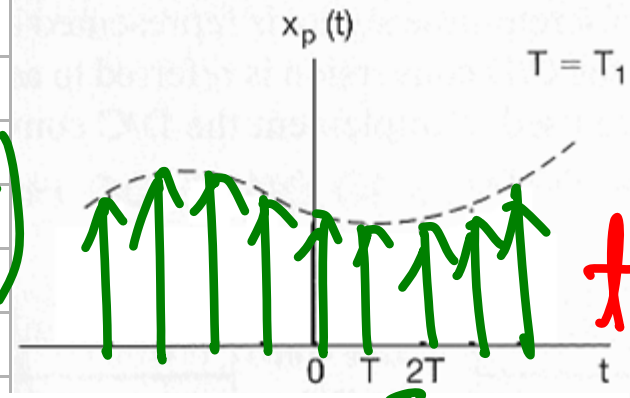
$x_c(t)$



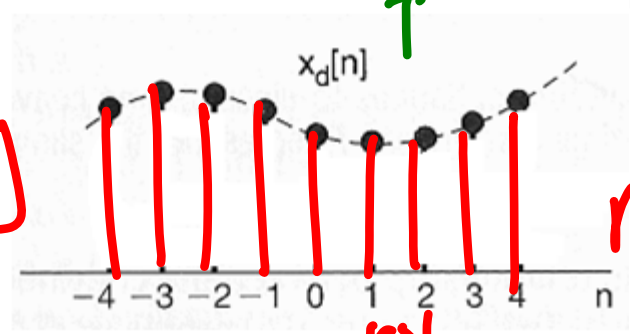
$p(t)$



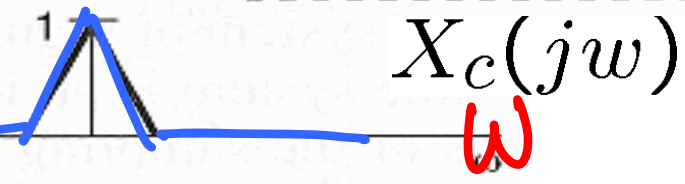
$x_p(t)$



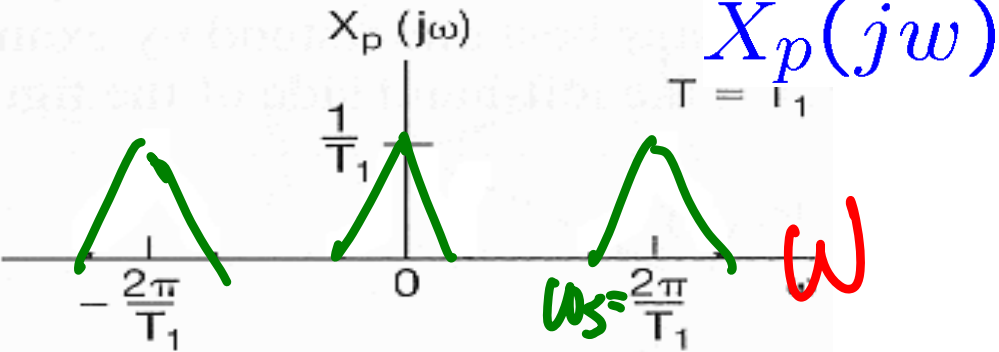
$x_d[n]$



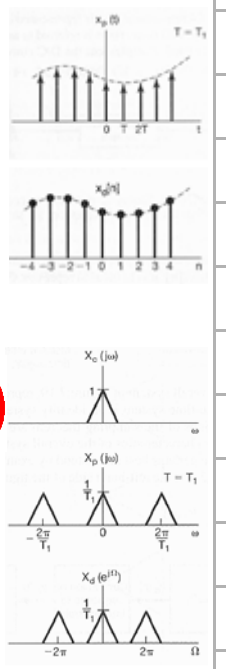
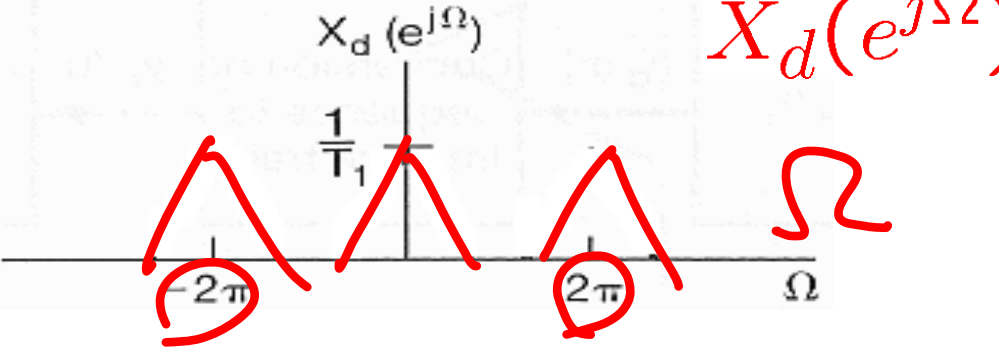
$x_c(t)$



$x_p(t)$



$x_d[n]$



C/D Conversion:

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT) \delta(t - nT)$$

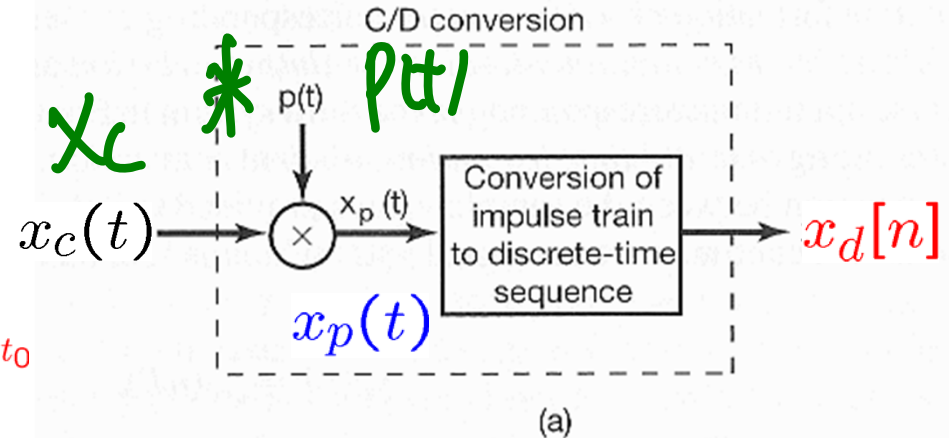


Table 4.2, p. 329

$$\delta(t - t_0) \xleftrightarrow{\mathcal{F}} e^{-j\omega t_0}$$

$$X_p(j\omega) = \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-j\omega nT}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s))$$

Eq 7.3, 7.6, p. 517

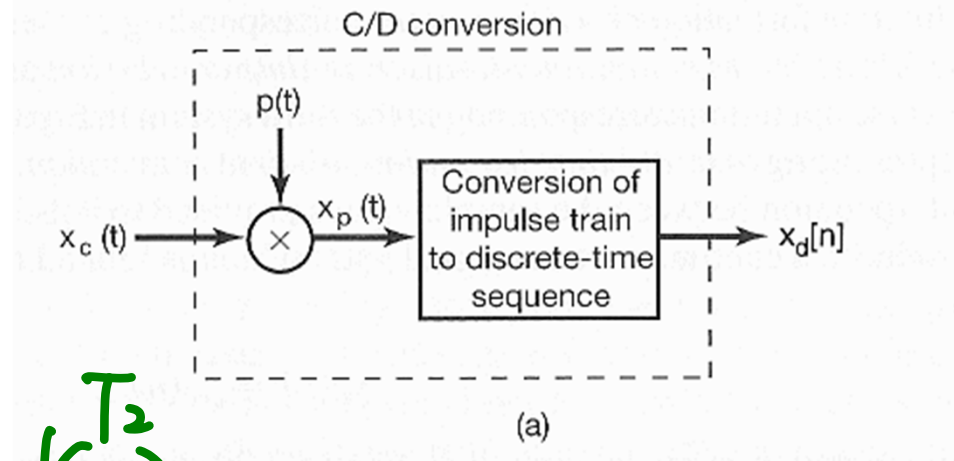
$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n] e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-j\Omega n}$$

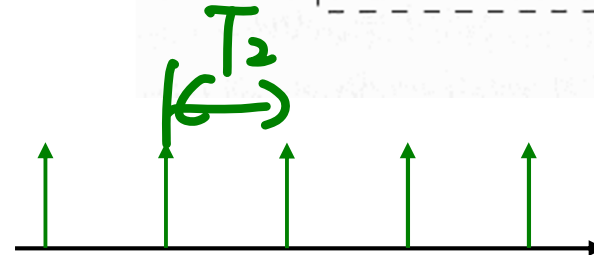
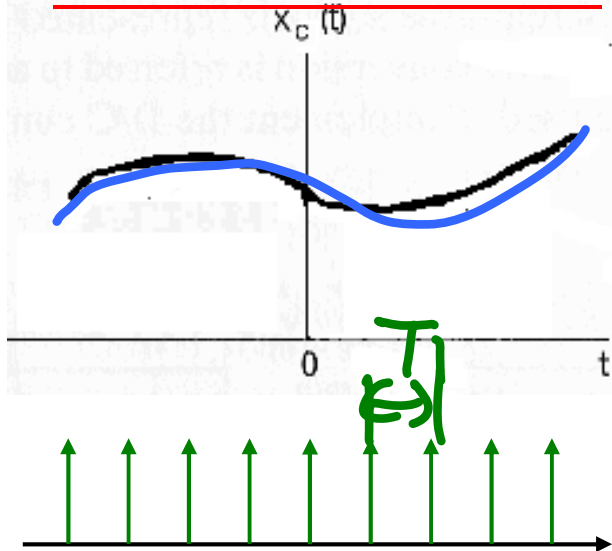
$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c \left(j \left(\frac{\Omega}{T} - k \frac{2\pi}{T} \right) \right)$$

$$\Rightarrow X_d(e^{j\Omega}) = X_p \left(j \frac{\Omega}{T} \right)$$

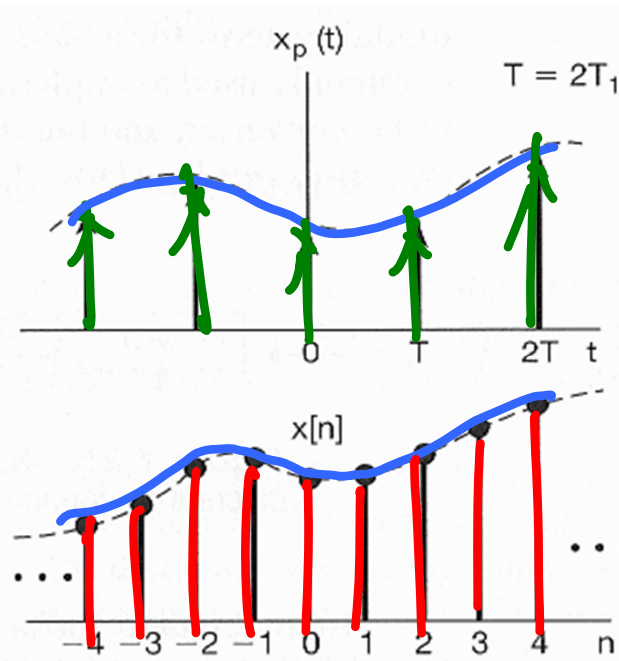
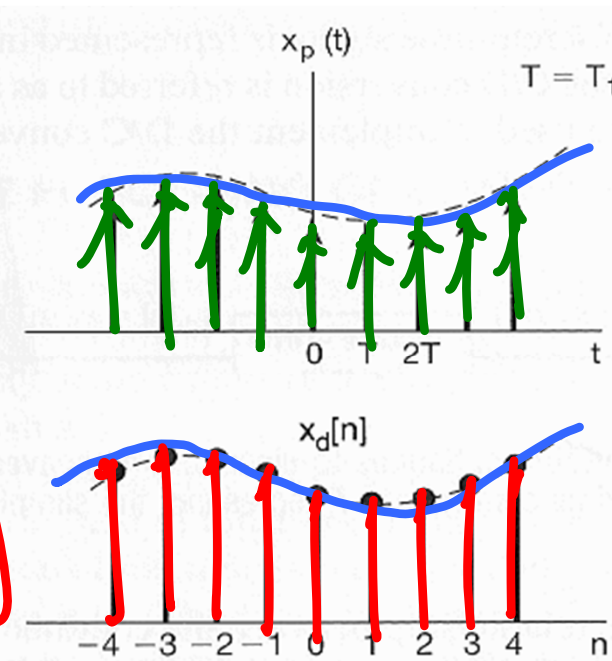
C/D Conversion:



$p(t)$

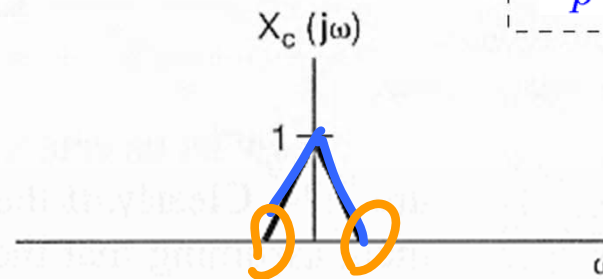
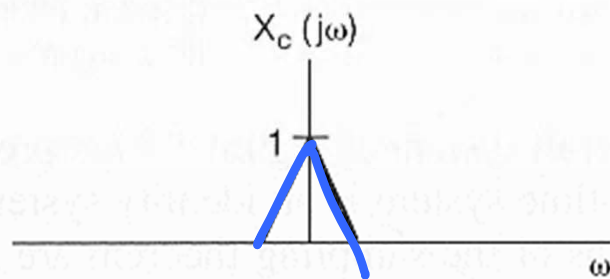
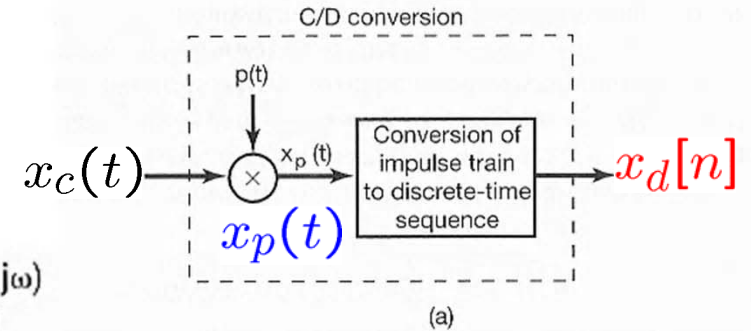


$x_d[n]$

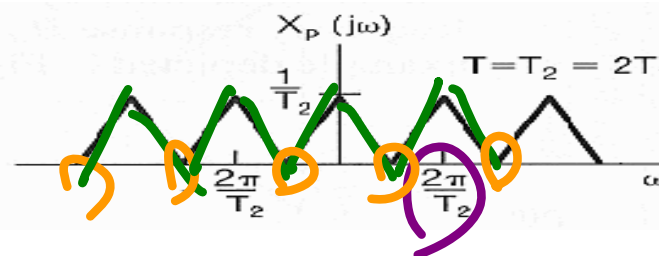
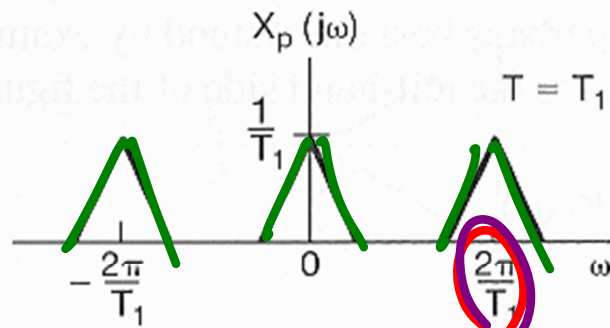


(c)

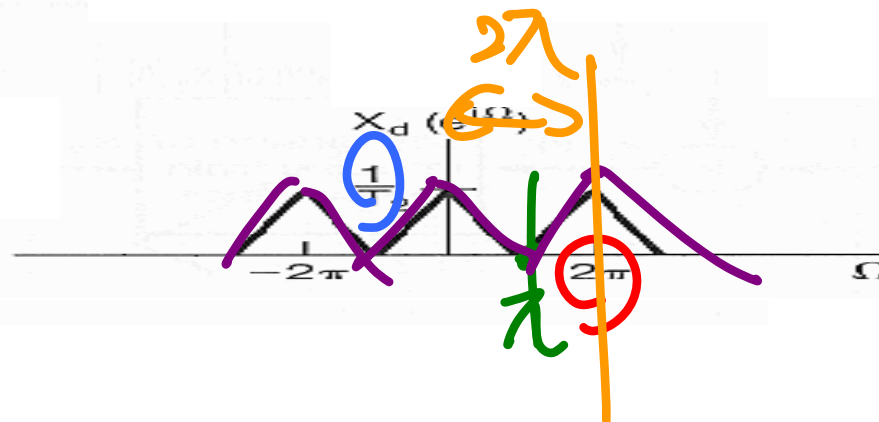
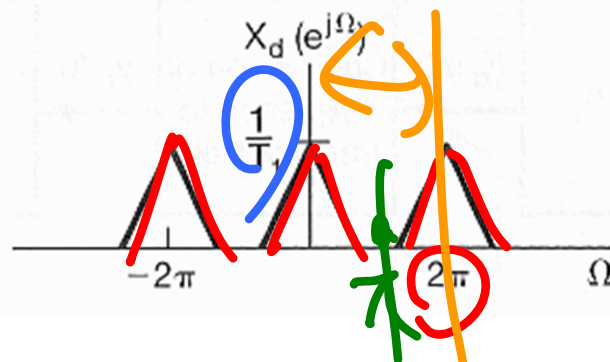
C/D Conversion:



$$X_c(j\omega)$$

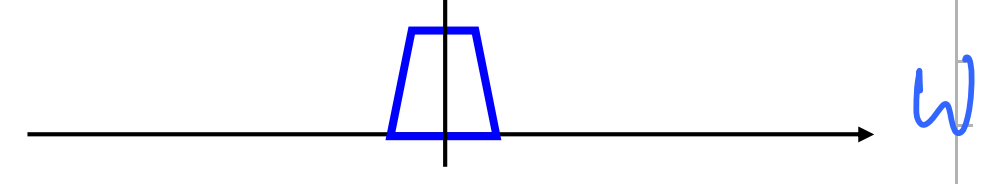
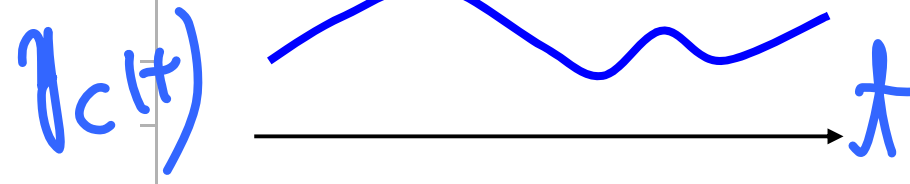
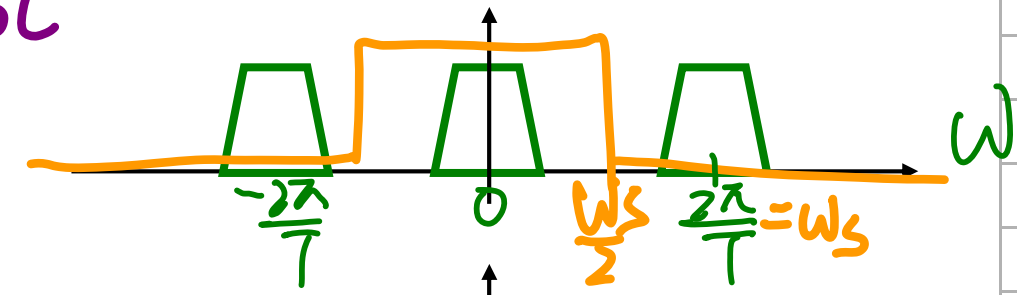
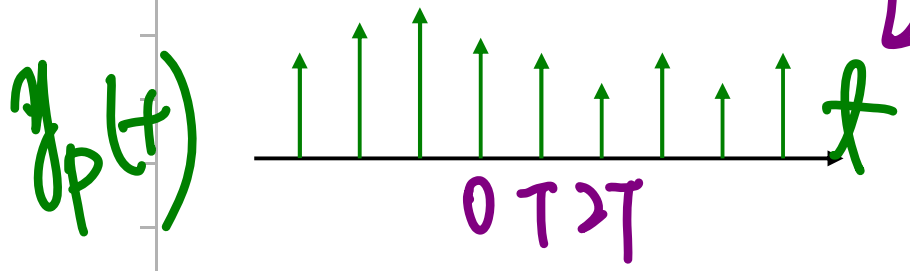
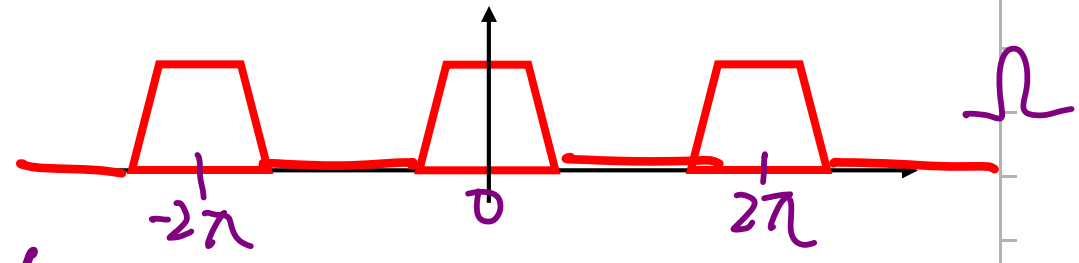
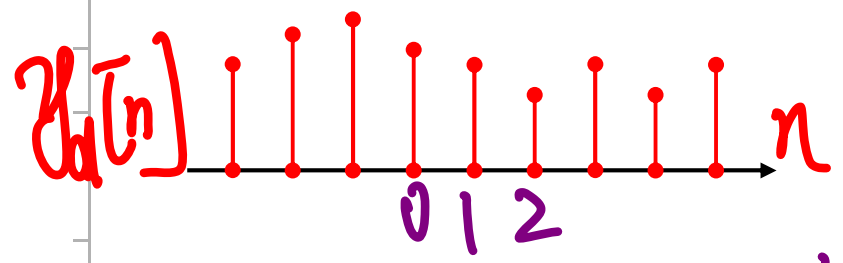
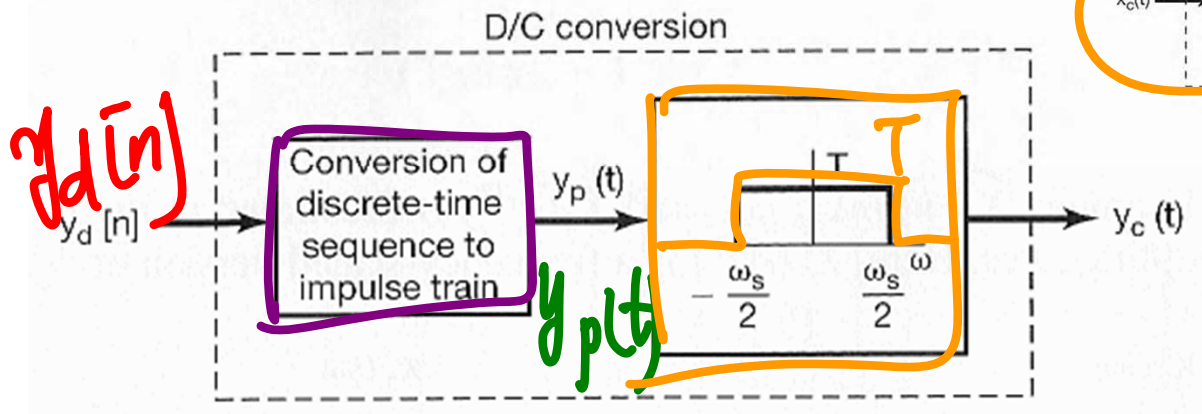
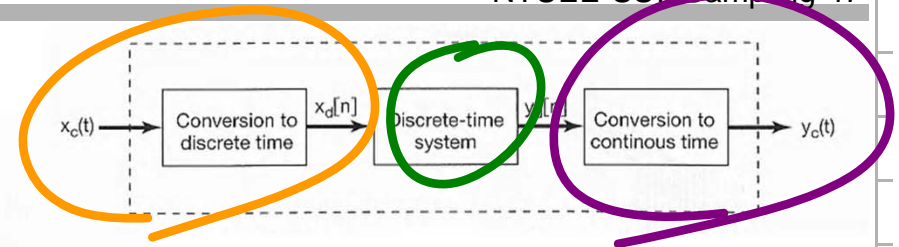


$$X_p(j\omega)$$

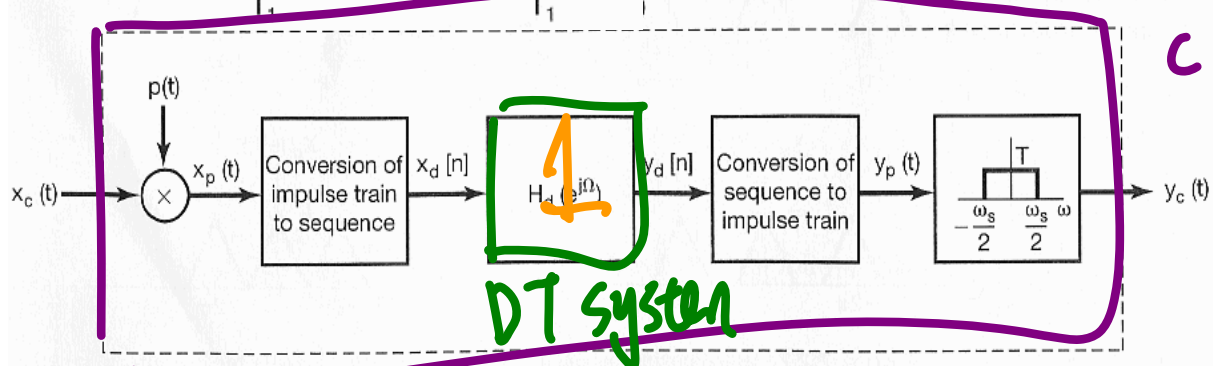
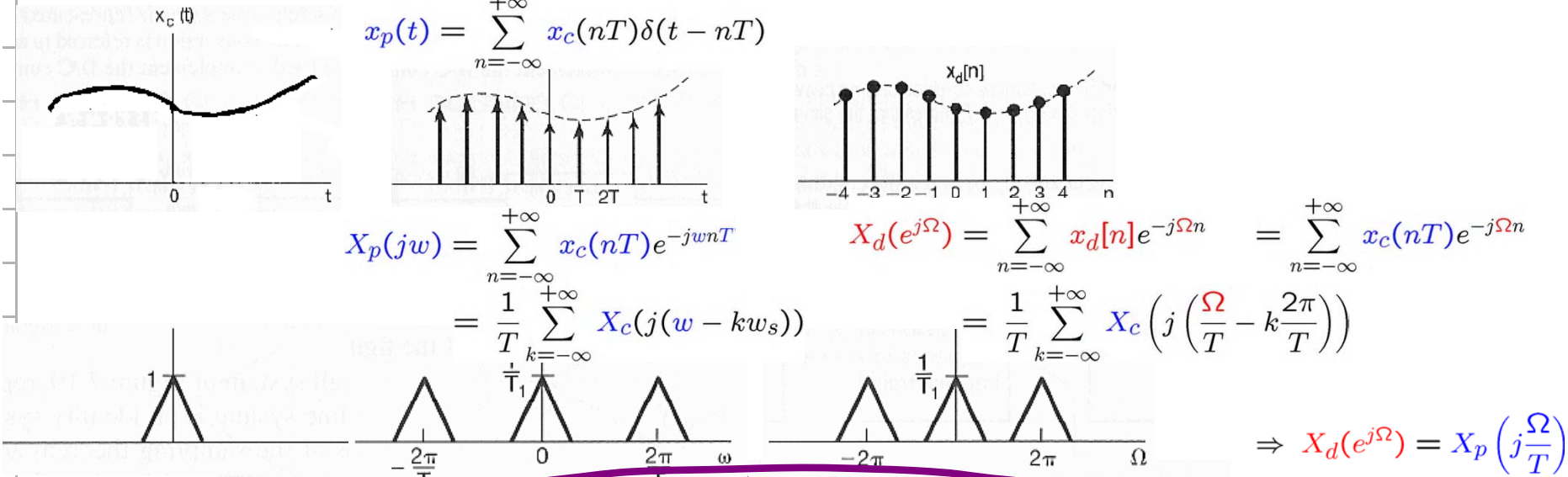


$$X_d(e^{j\Omega})$$

D/C Conversion:

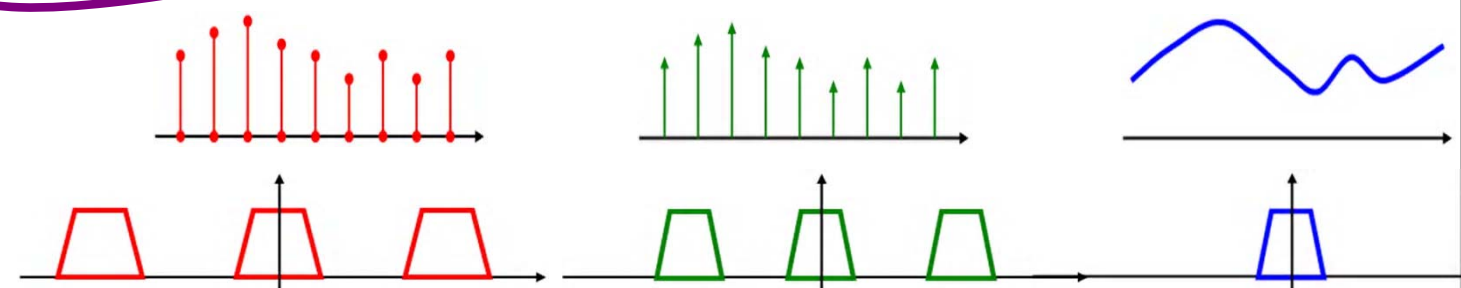


Overall System:



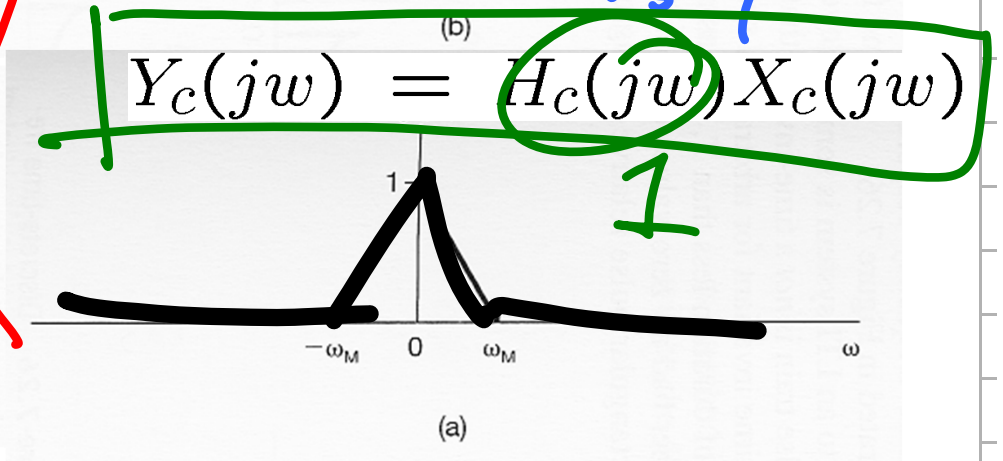
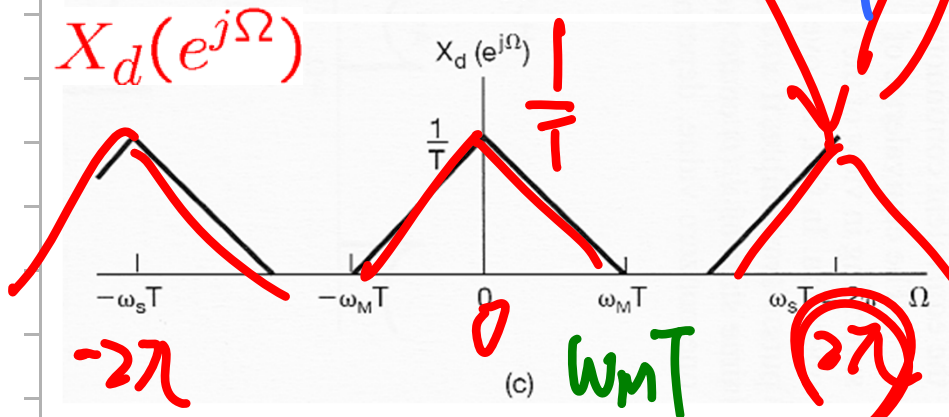
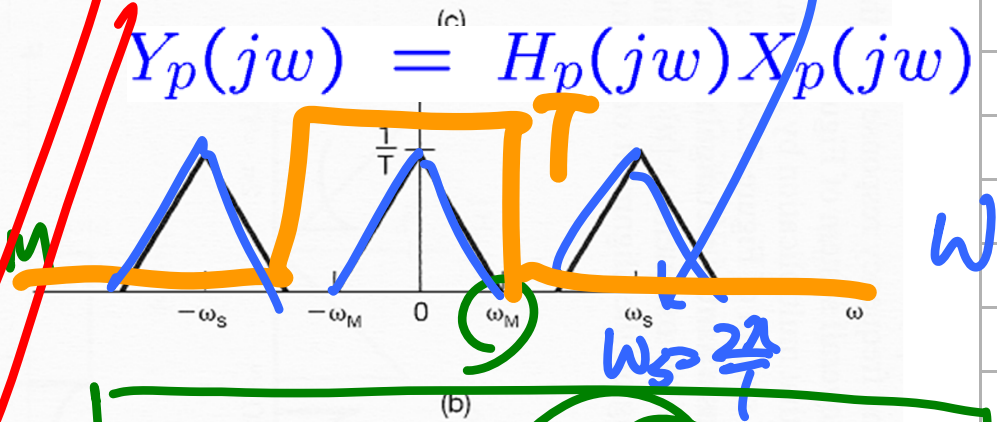
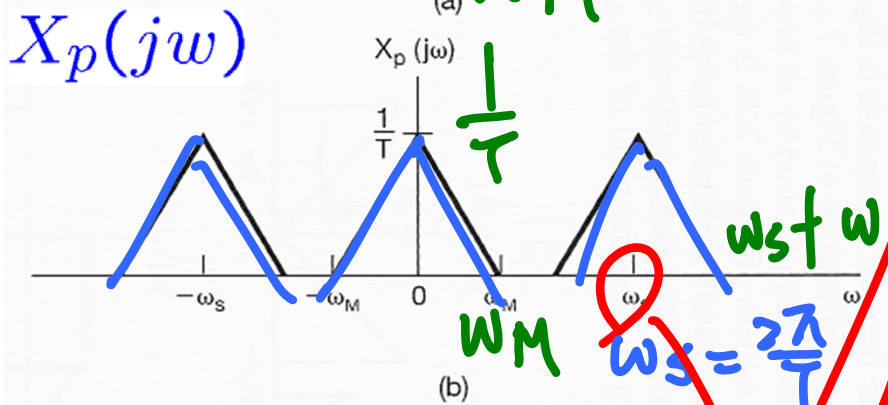
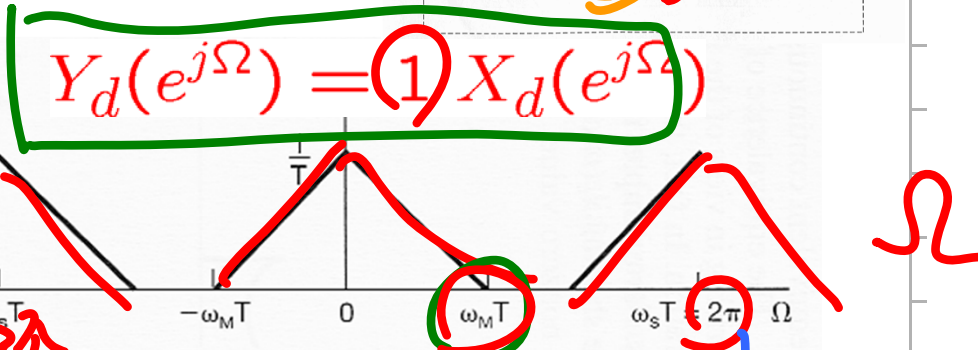
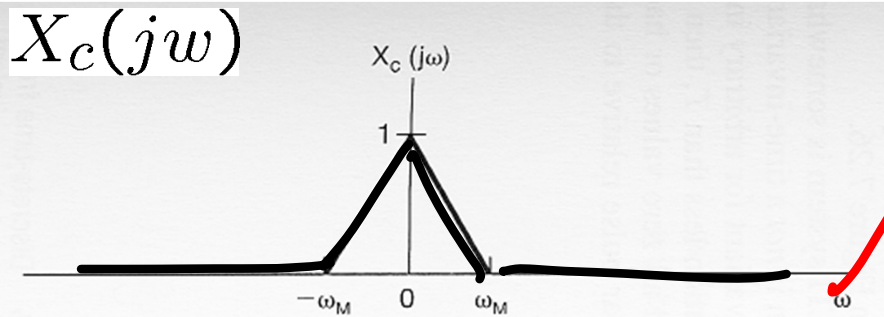
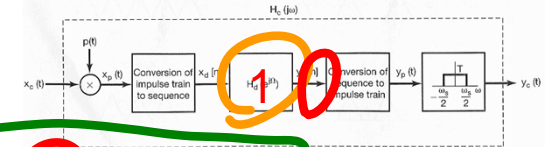
CT system

DT system

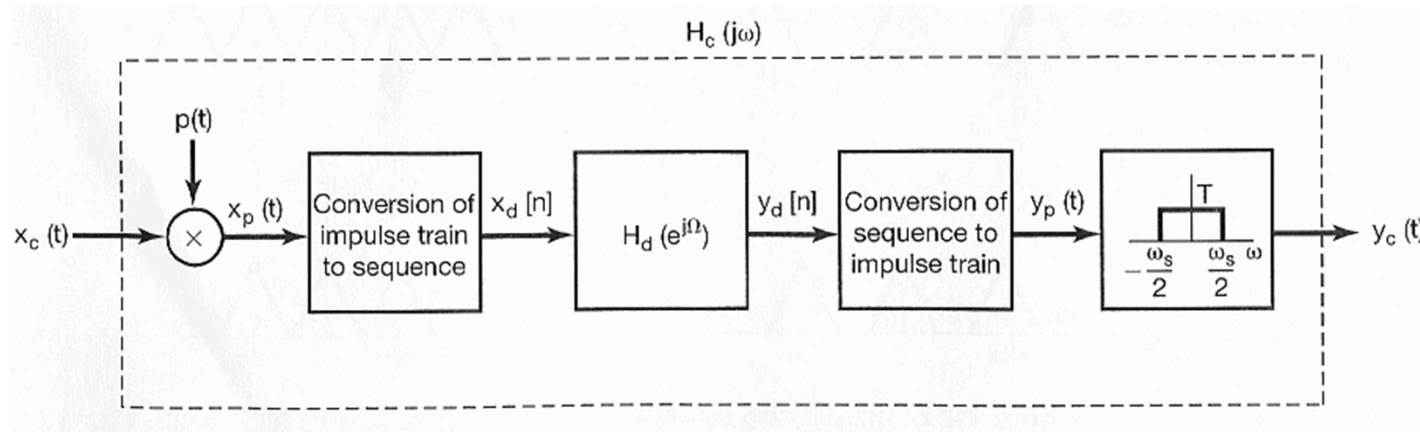


5/6/13
10:14 am

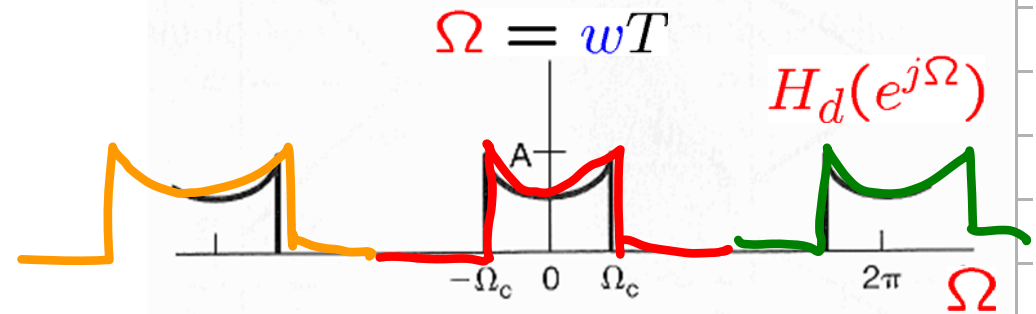
Frequency-Domain Illustration:



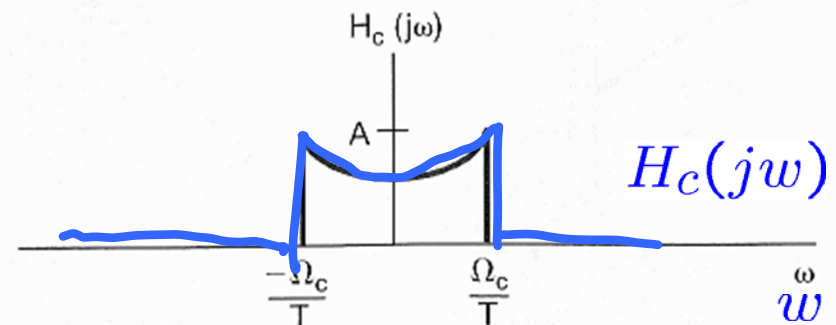
CT & DT Frequency Responses:



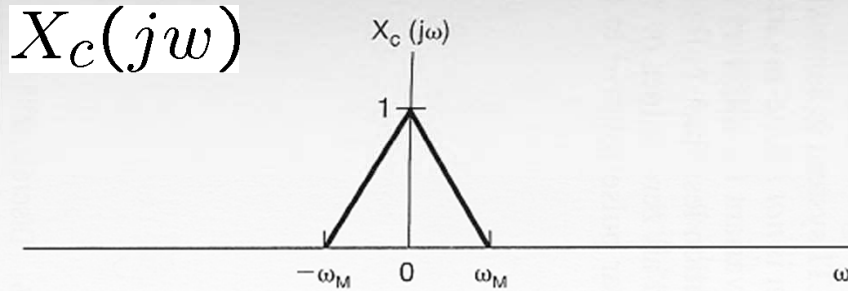
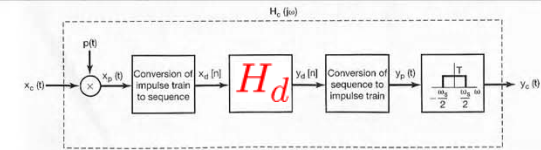
$$Y_c(j\omega) = X_c(j\omega)H_c(j\omega)$$



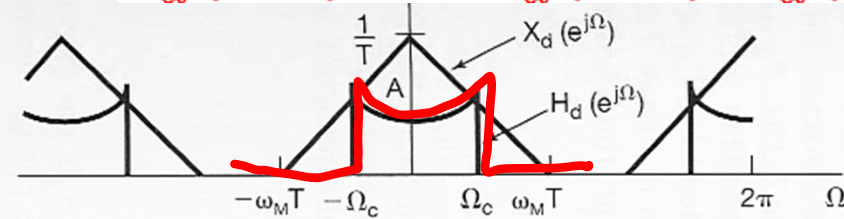
$$H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}), & |\omega| < \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$



Frequency-Domain Illustration:

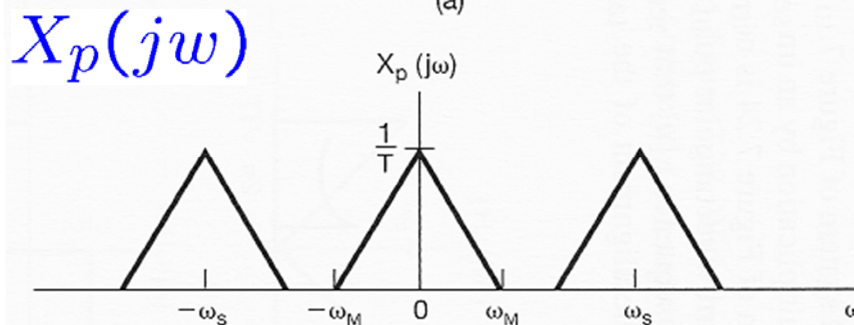


$$Y_d(e^{j\Omega}) = H_d(e^{j\Omega})X_d(e^{j\Omega})$$

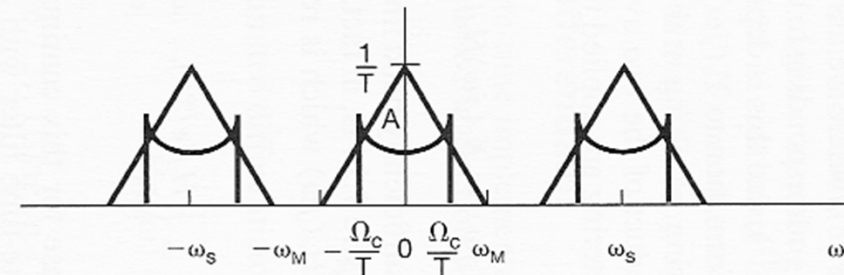


(a)

(d)

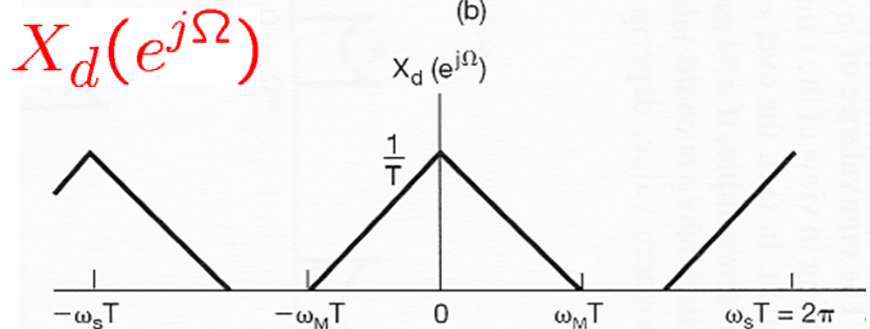


$$Y_p(j\omega) = H_p(j\omega)X_p(j\omega)$$

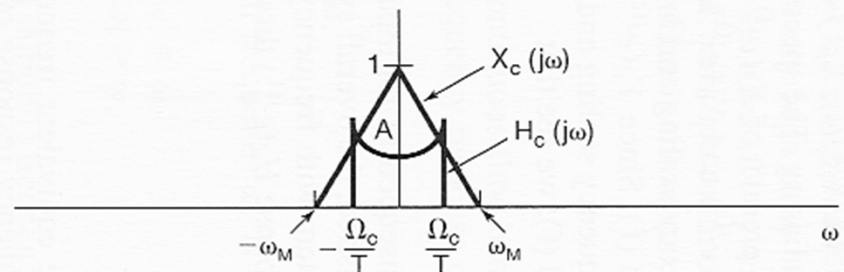


(b)

(e)



$$Y_c(j\omega) = H_c(j\omega)X_c(j\omega)$$

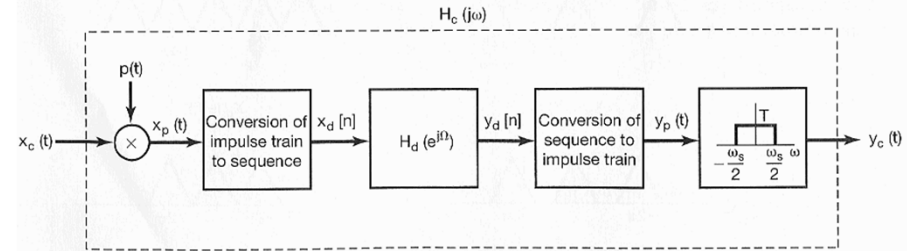


(c)

(f)

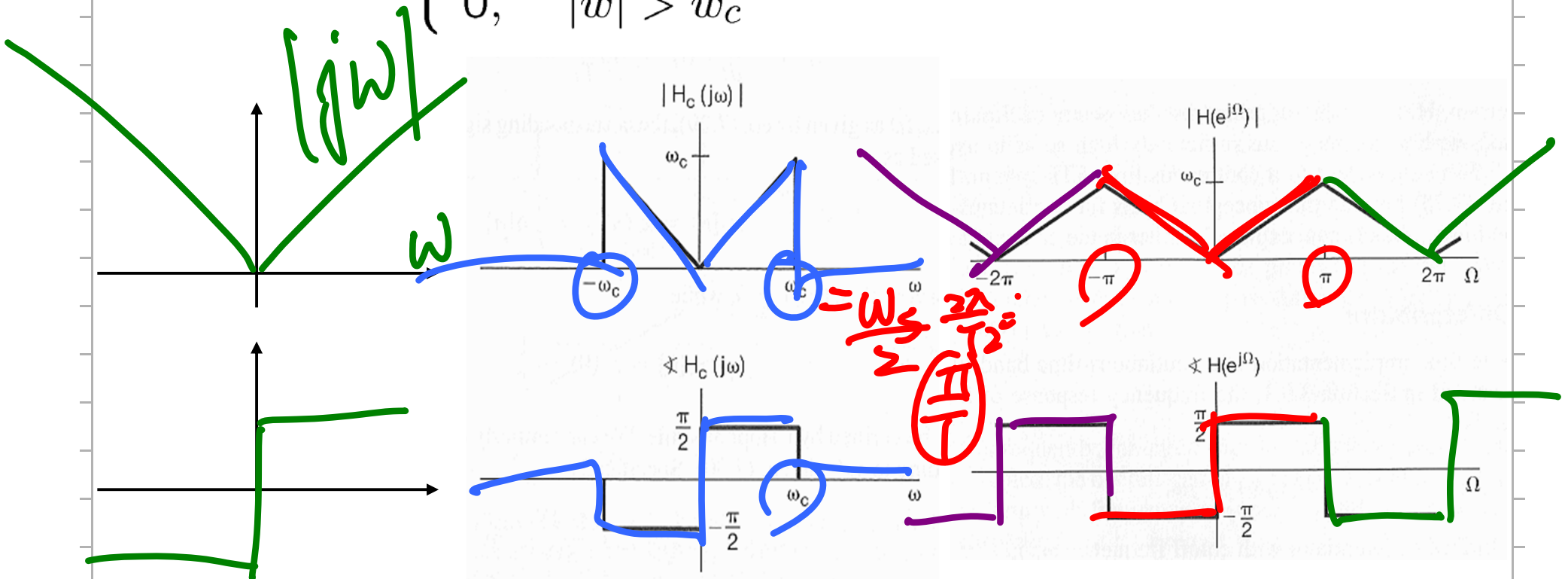
Digital Differentiator: (band-limited)

Ex 4.16, p. 317



$$H_c(j\omega) = \begin{cases} j\omega, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

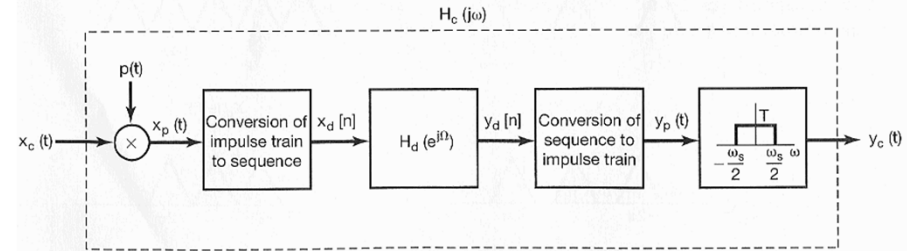
$$H_d(e^{j\Omega}) = j \left(\frac{\Omega}{T} \right), \quad |\Omega| < \pi$$



$$\Omega = \omega T, \quad \omega_s = 2\omega_c$$

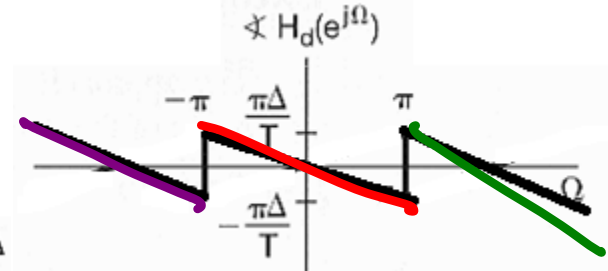
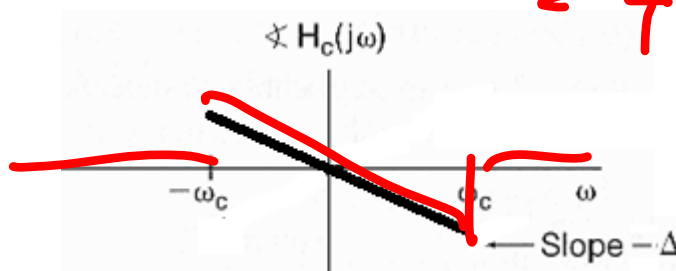
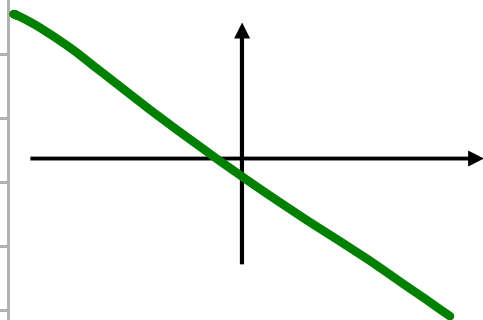
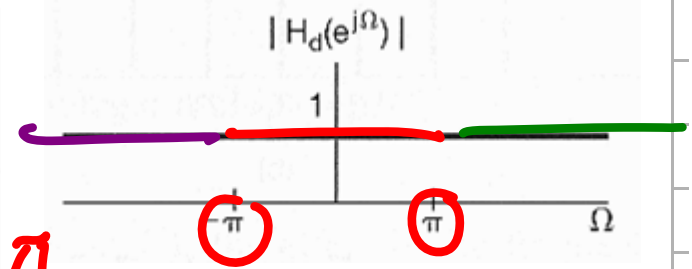
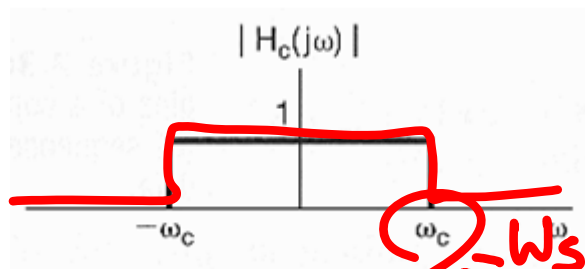
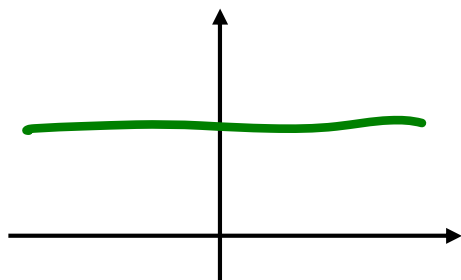
Delay: (band-limited)

Ex 4.15, p. 317



$$H_c(j\omega) = \begin{cases} e^{-j\omega\Delta}, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

$$H_d(e^{j\Omega}) = e^{-j\Omega\Delta/T}, \quad |\Omega| < \pi$$



$$\Omega = \omega T, \quad \omega_s = 2\omega_c$$

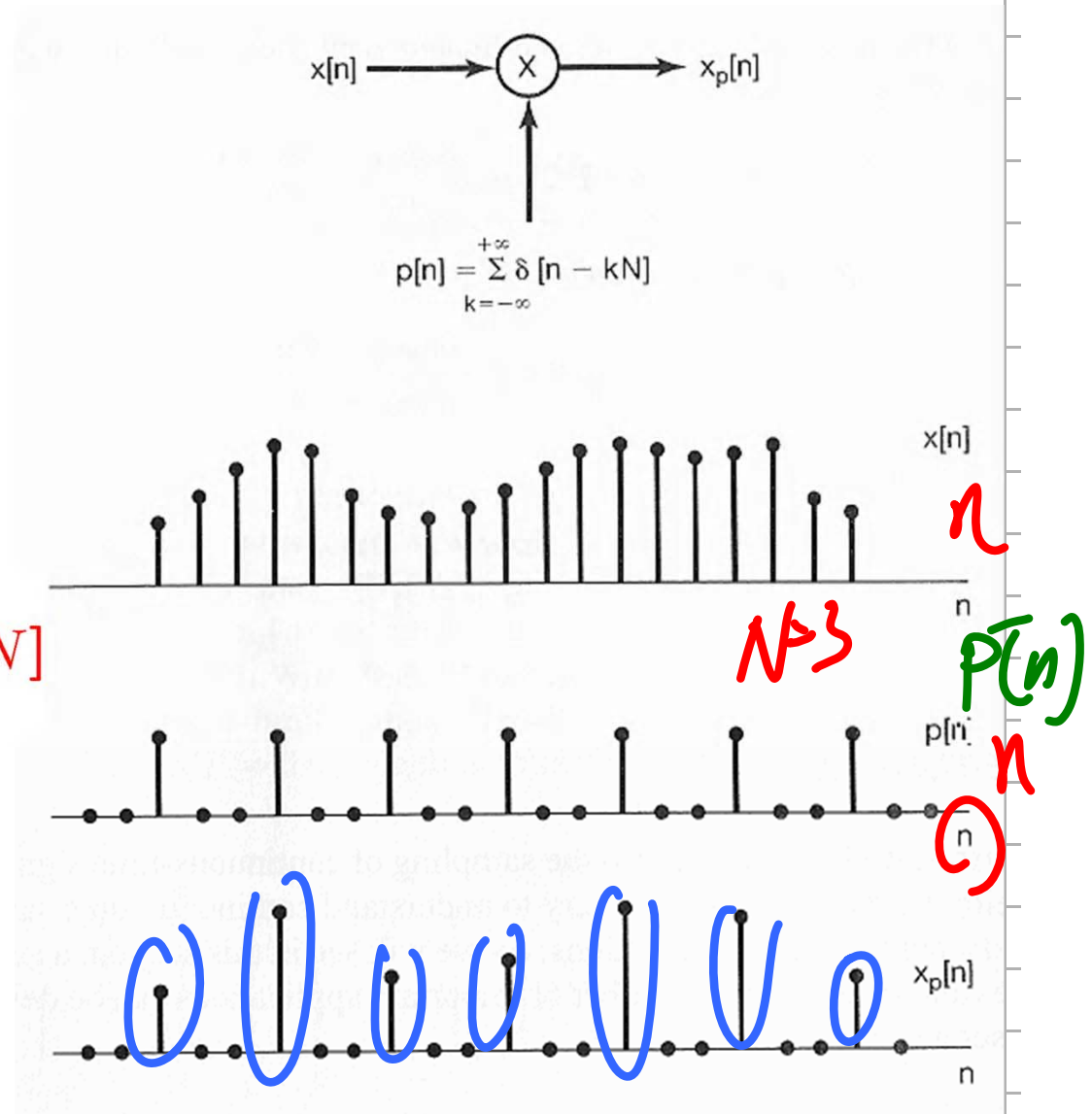
- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- **Sampling** of Discrete-Time Signals

Impulse-Train Sampling:

$$x_p[n] = \begin{cases} x[n], & \text{if } n = kN \\ 0, & \text{otherwise} \end{cases}$$

$$x_p[n] = x[n] p[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[kN] \delta[n - kN]$$

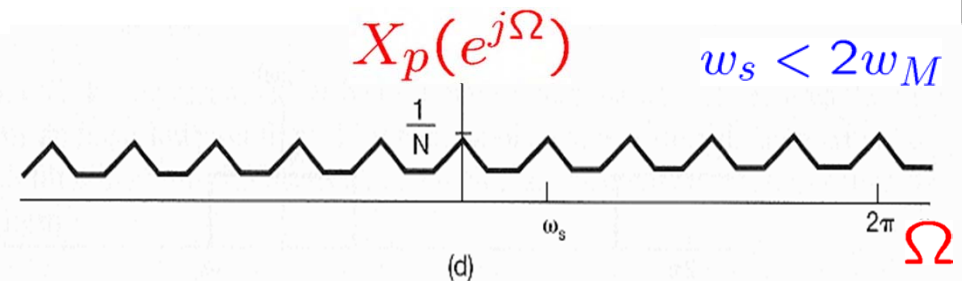
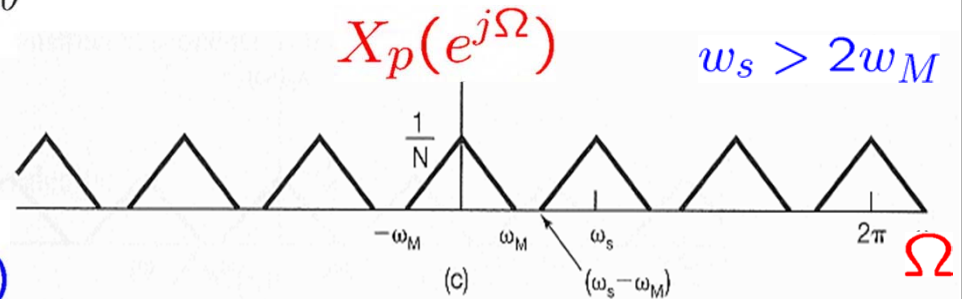
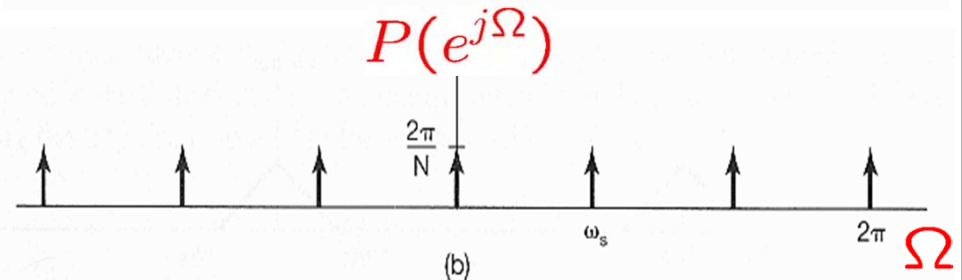
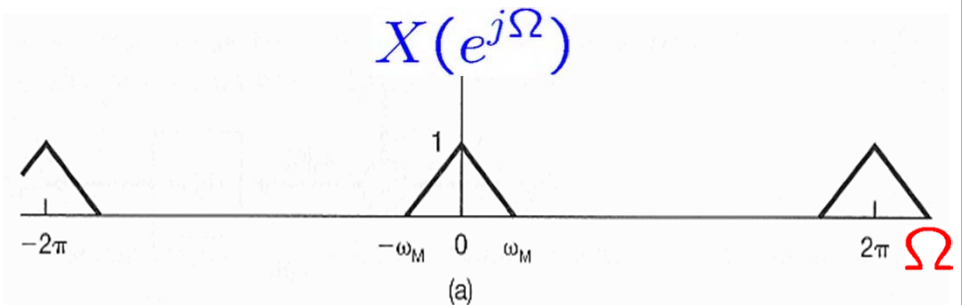


Impulse-Train Sampling:

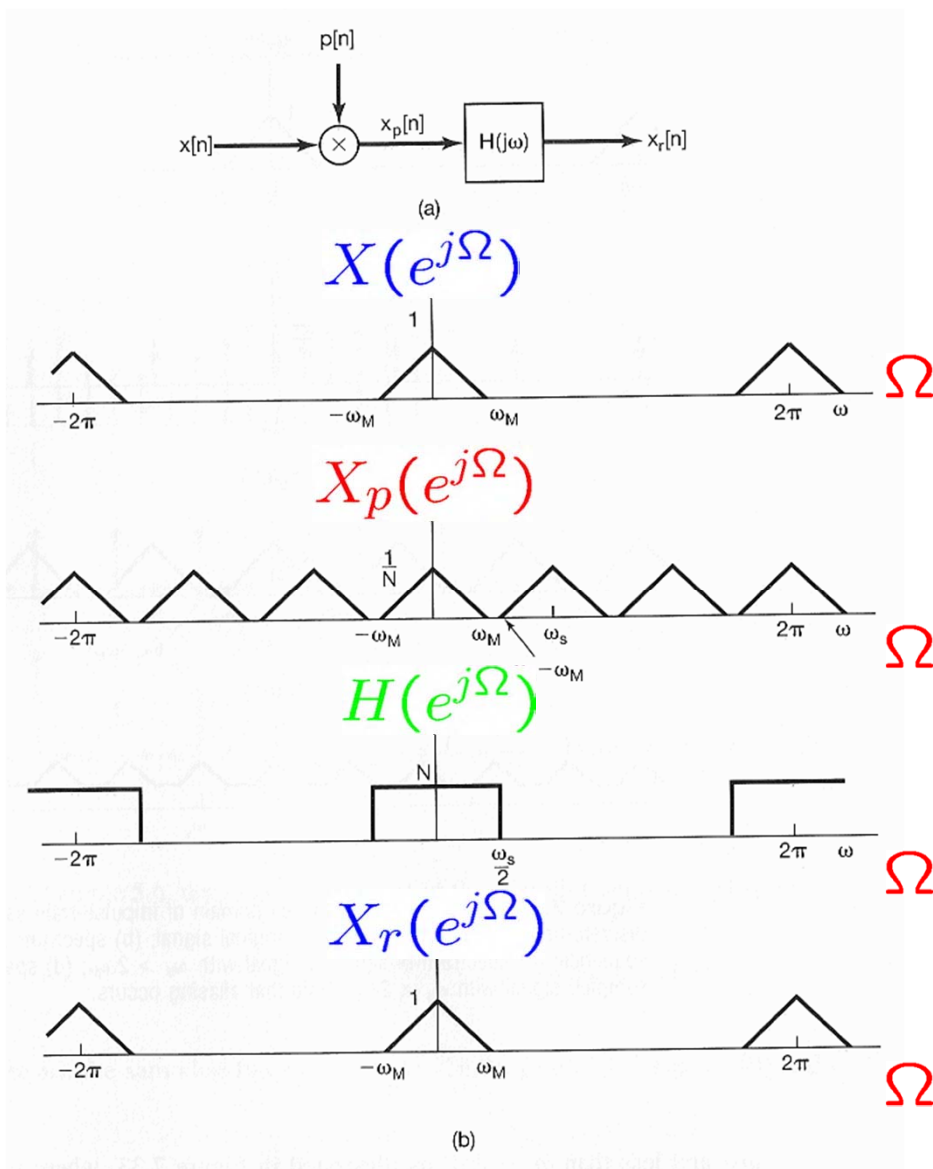
$$P(e^{j\Omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\Omega - k\omega_s)$$

$$X_p(e^{j\Omega}) = \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) X(e^{j(\Omega-\theta)}) d\theta$$

$$\Rightarrow X_p(e^{j\Omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\Omega - k\omega_s)})$$

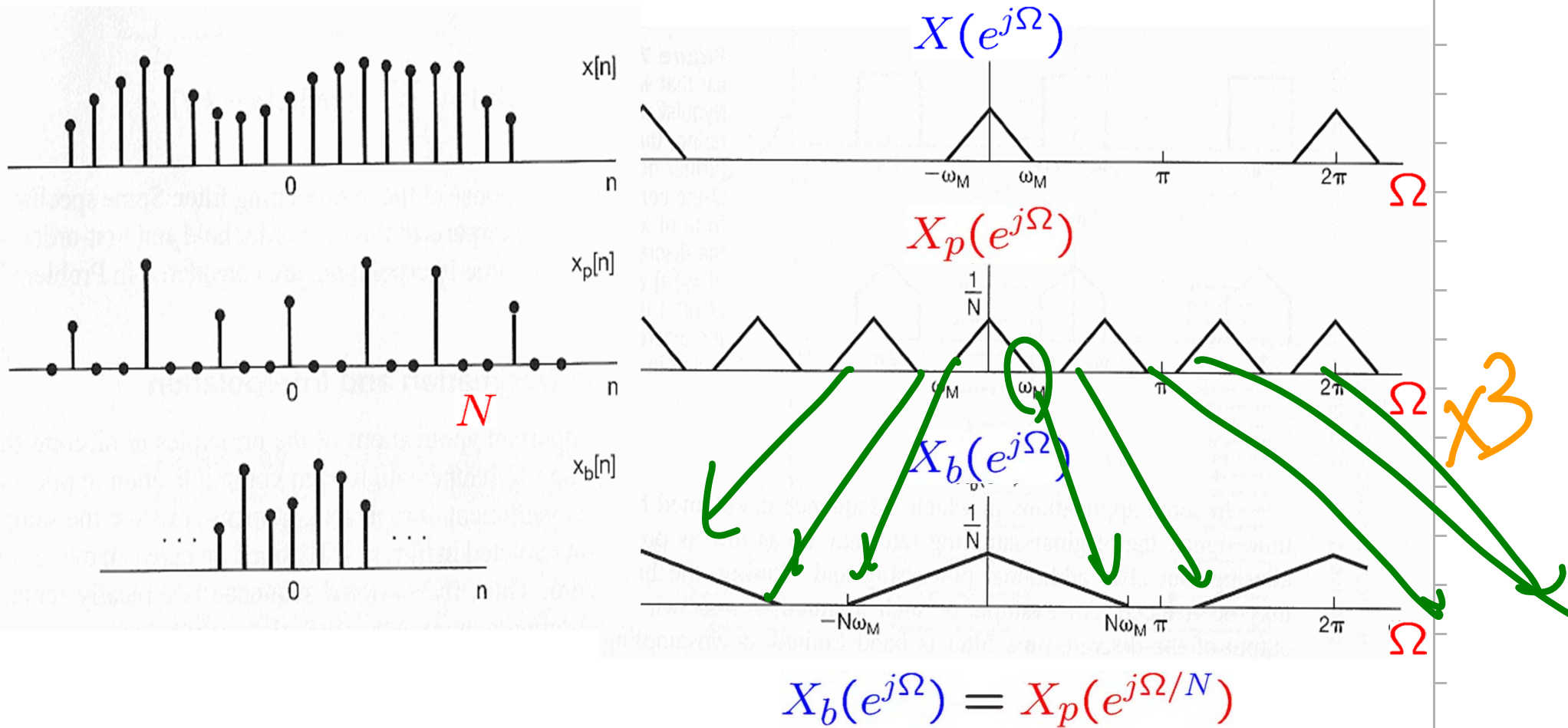


Exact Recovery Using Ideal Lowpass Filter:

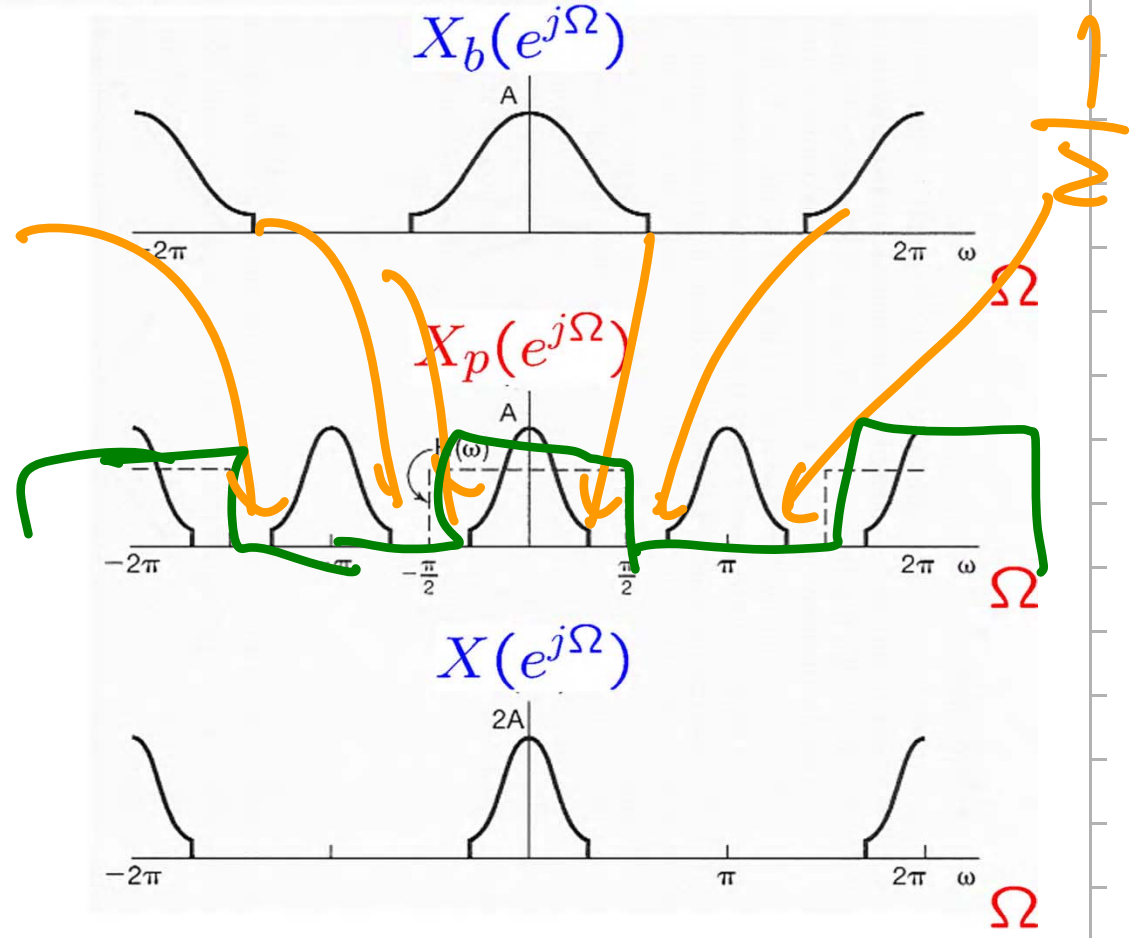
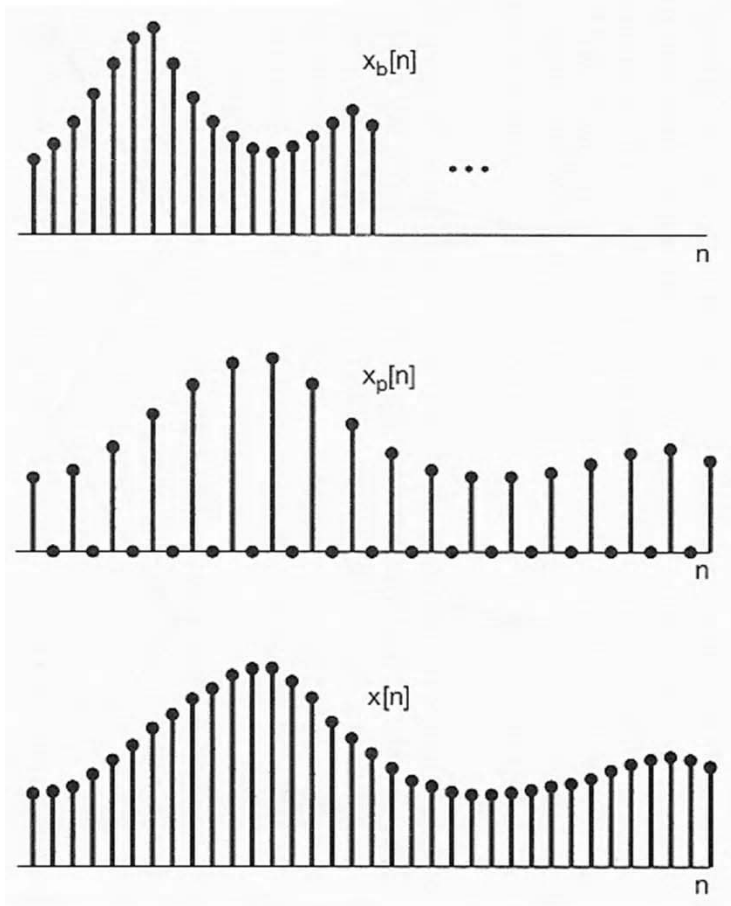
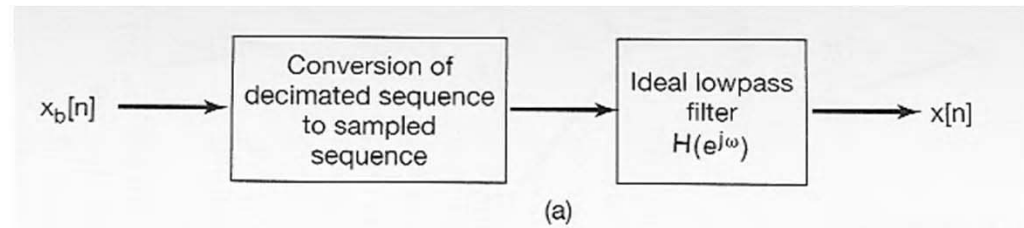


DT Decimation & Interpolation: Down-sampling

Eq 5.45, p. 378: Time expansion



Higher Equivalent Sampling Rate: Up-sampling



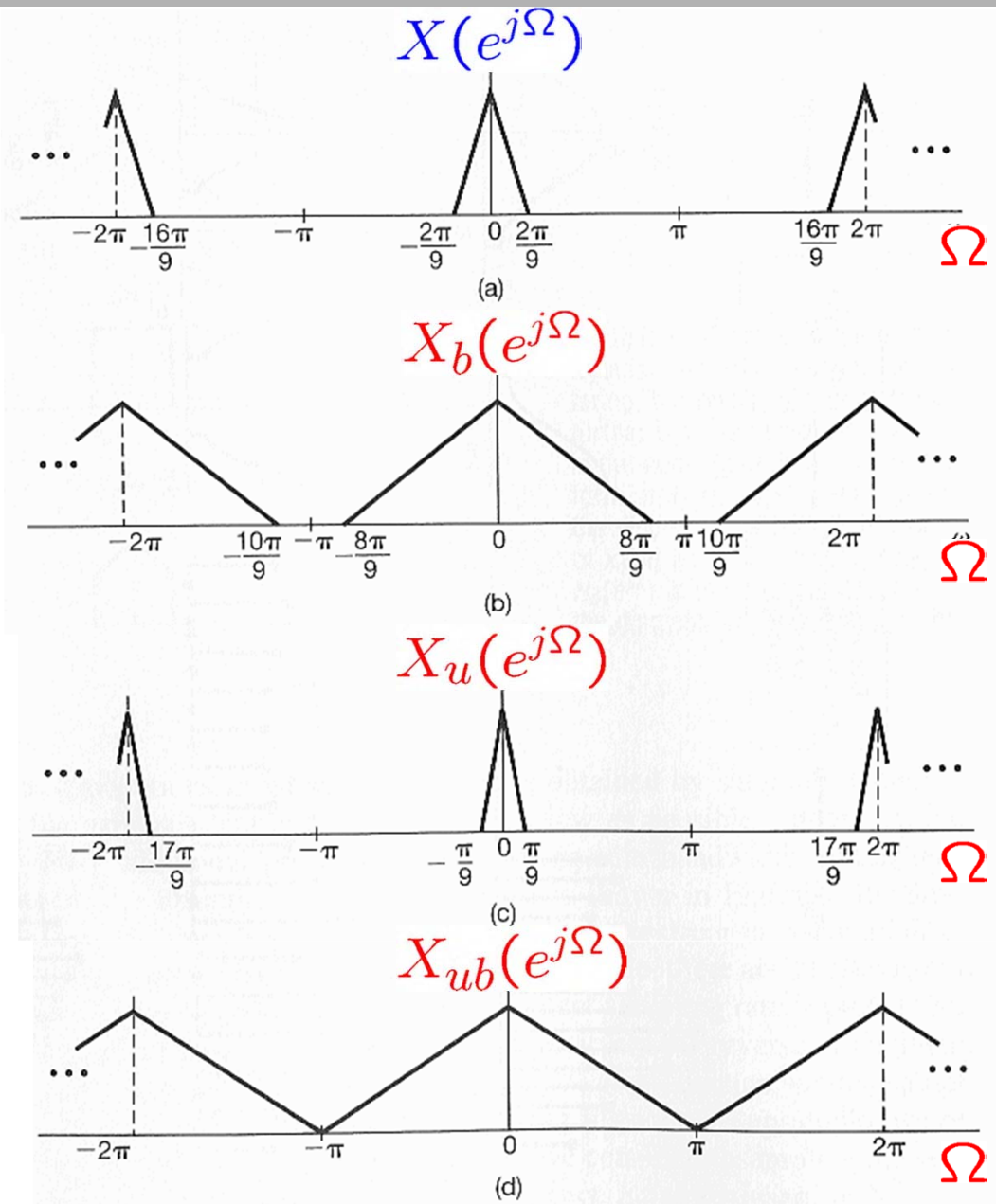
Down-sampling + Up-sampling:

$$\frac{2\pi}{9} \times 4 < \pi$$

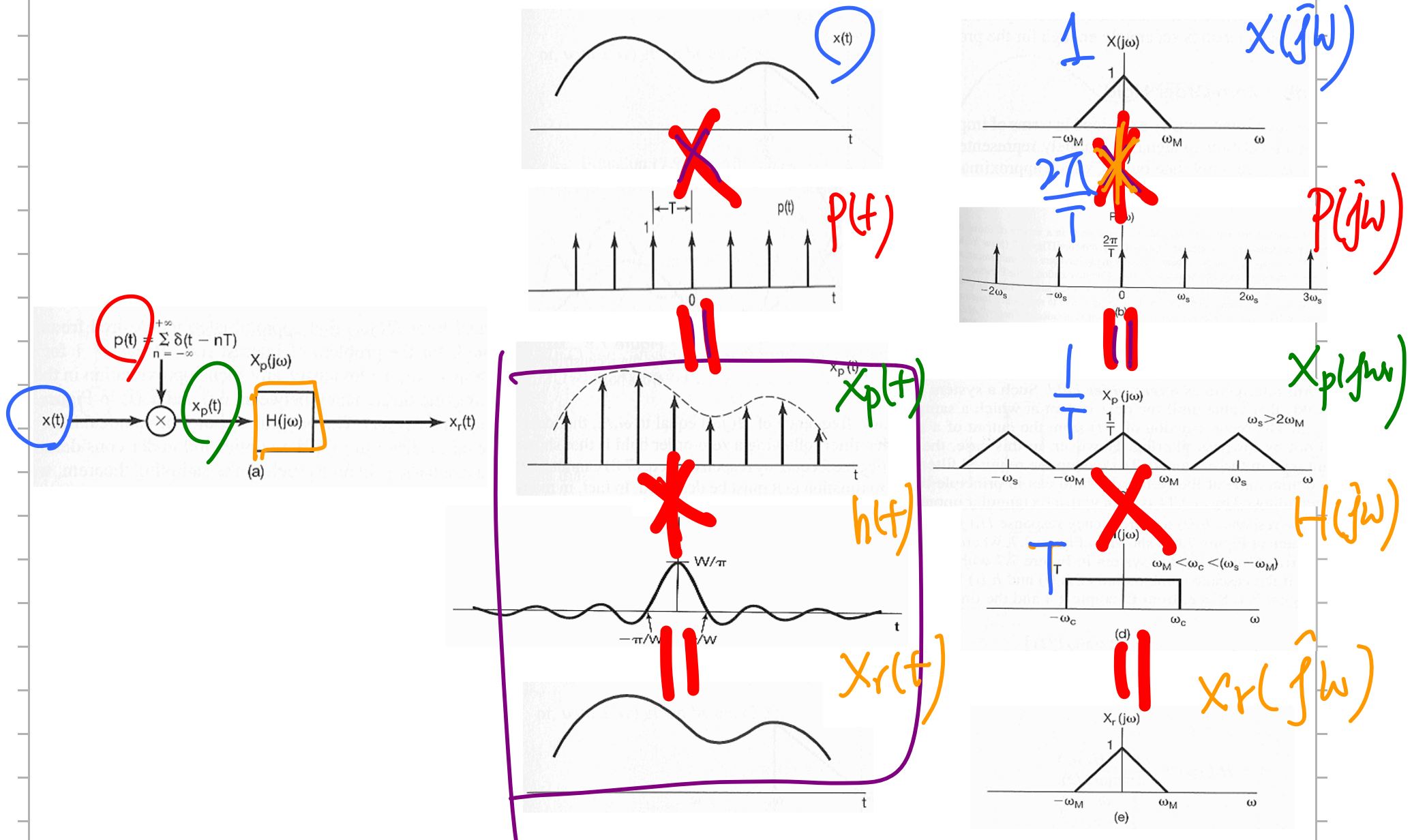
$$\frac{2\pi}{9} \times \frac{9}{2} = \pi$$

$$\frac{2\pi}{9} \times \frac{1}{2} = \frac{\pi}{9}$$

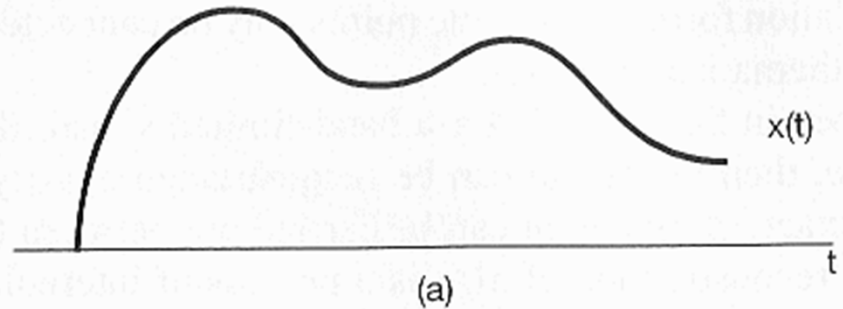
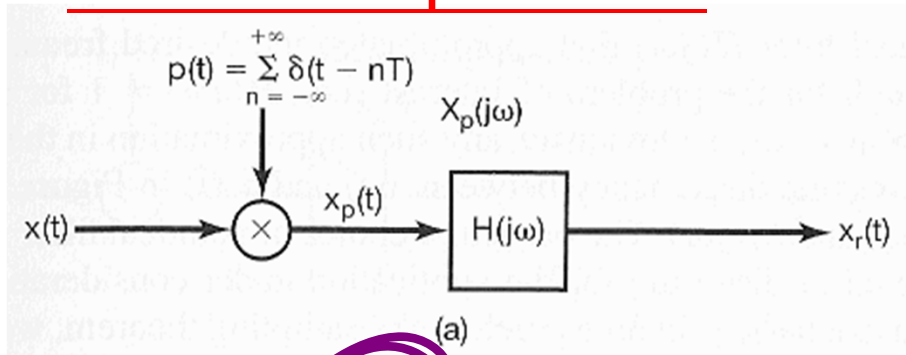
$$\frac{\pi}{9} \times 9 = \pi$$



Exact Recovery by an Ideal Lowpass Filter:



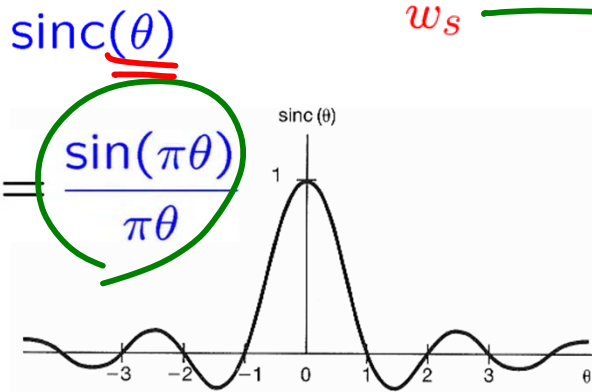
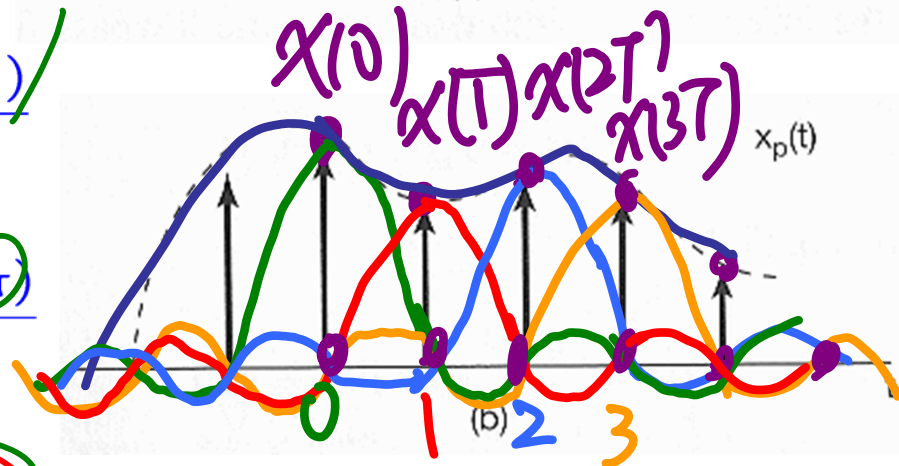
Exact Interpolation:



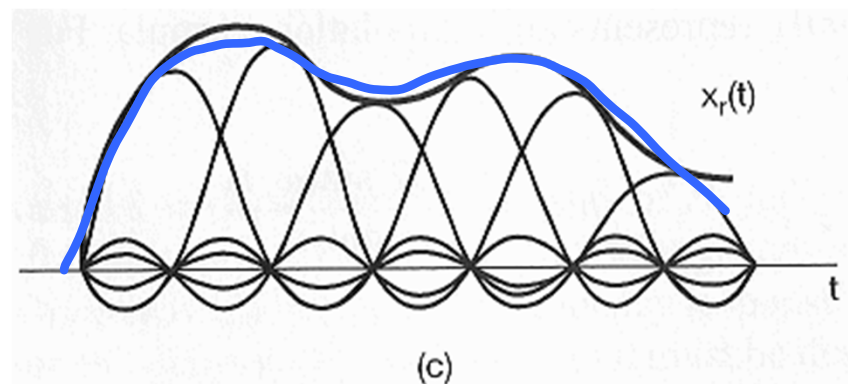
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{\omega_c T \sin(\omega_c(t - nT))}{\pi \omega_c(t - nT)}$$

$$\frac{\omega_c}{\pi} \frac{2\pi}{\omega_s} \frac{\sin \pi(\omega_c(t - nT)/\pi)}{\pi \omega_c(t - nT)/\pi}$$

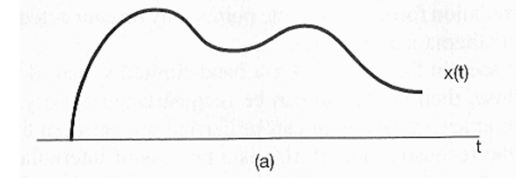
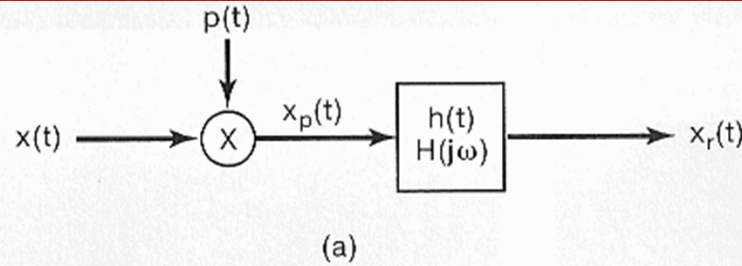
$$\frac{2\omega_c}{\omega_s} \text{sinc}(\omega_c(t - nT)/\pi)$$



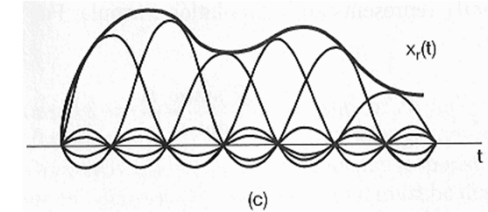
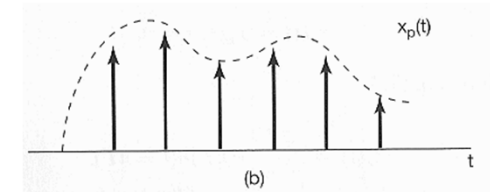
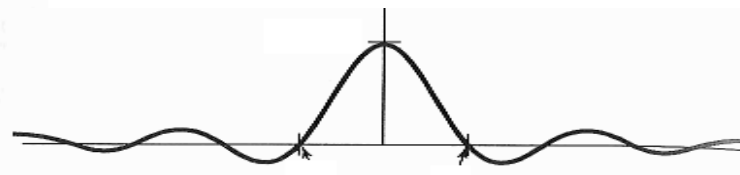
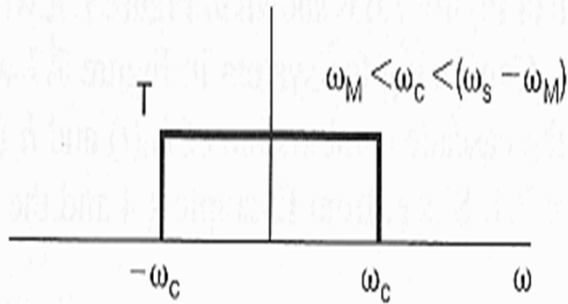
$n=0$
 $n=1$
 $n=2$
 $n=3$



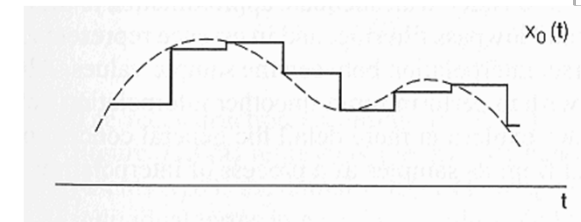
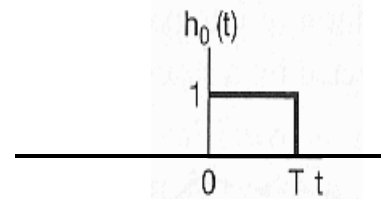
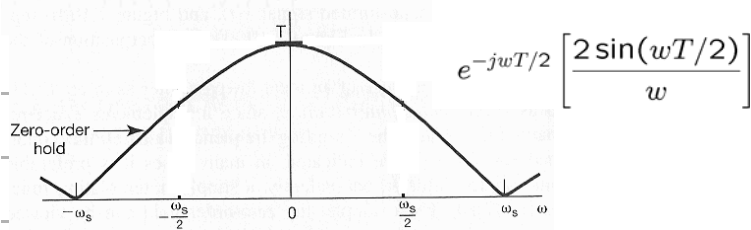
Three Filters: Ideal Lowpass, Zero-Order, First-Order



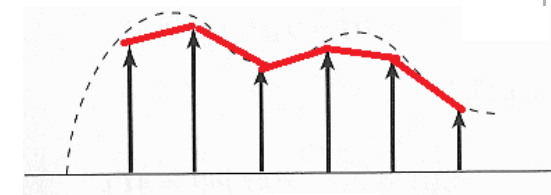
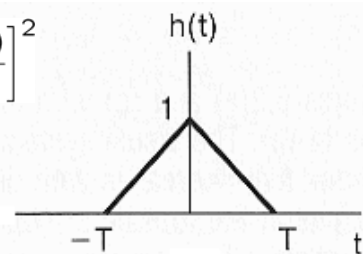
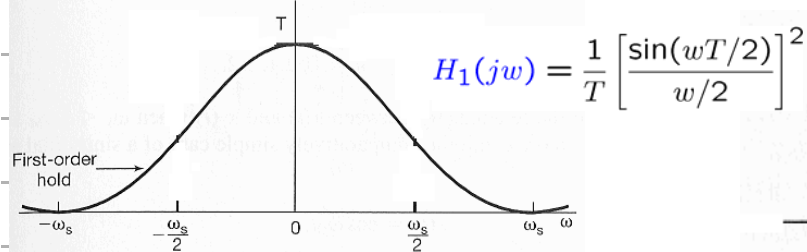
ideal lowpass



zero-order hold $H_0(j\omega) =$

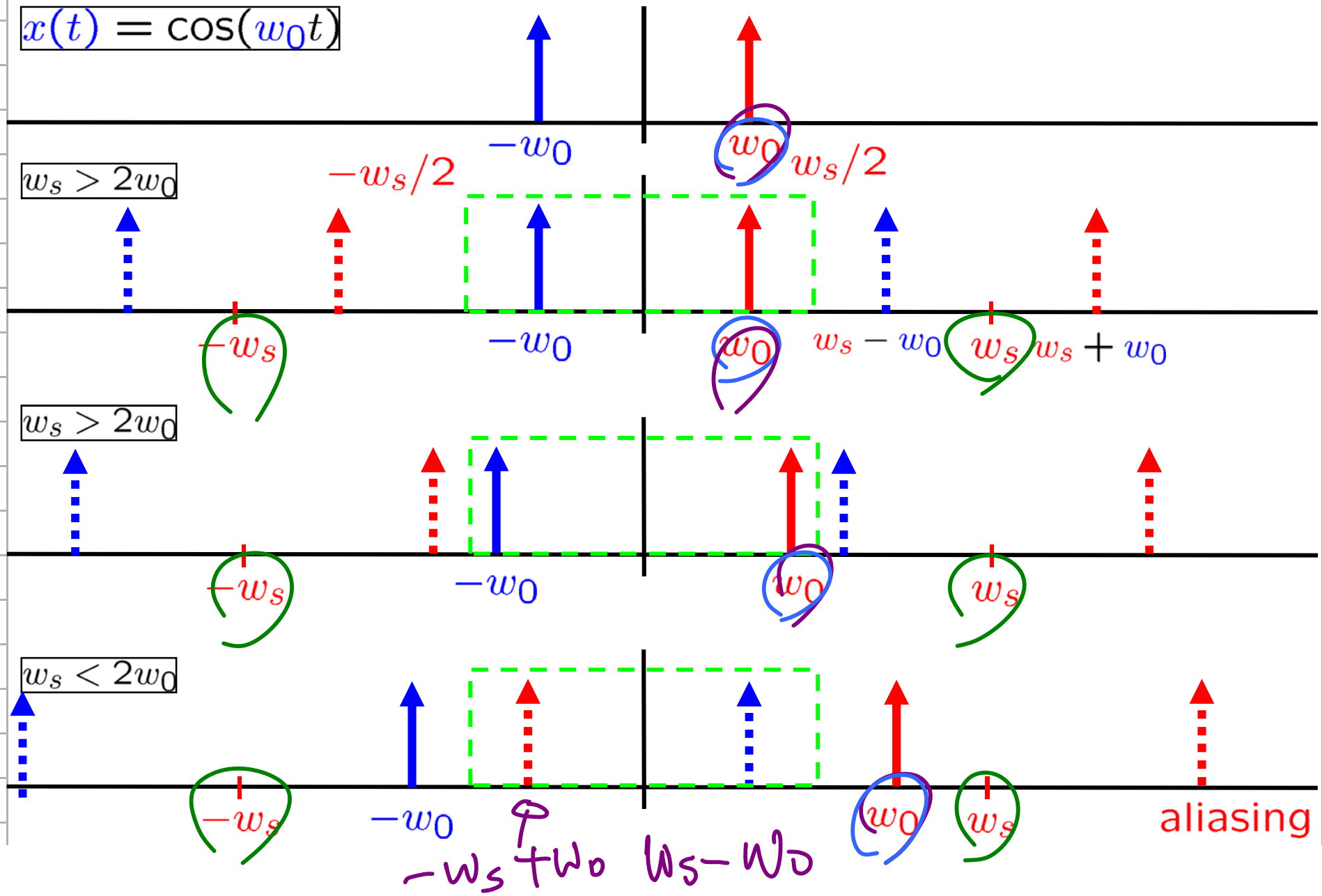


first-order hold



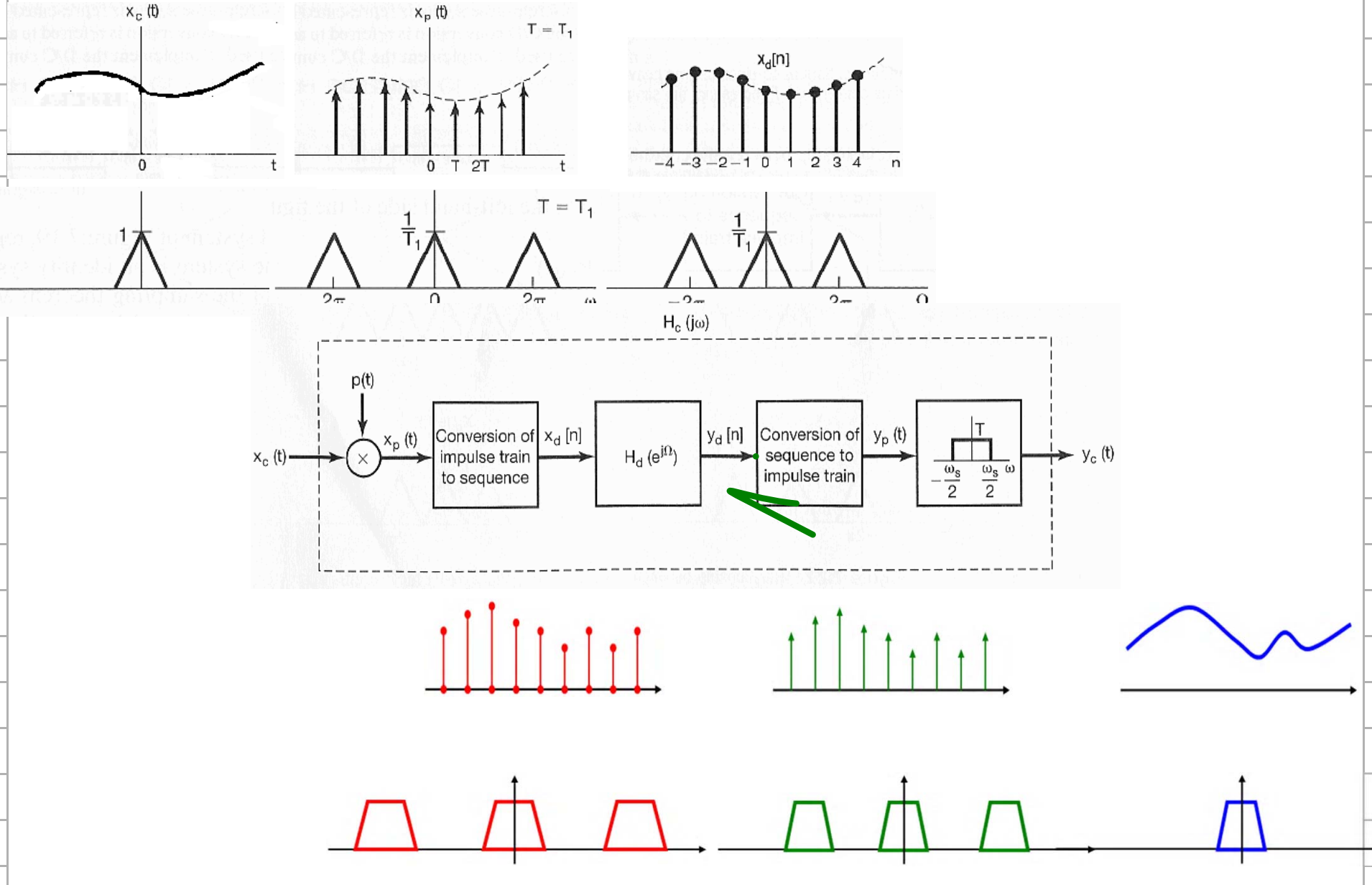
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$

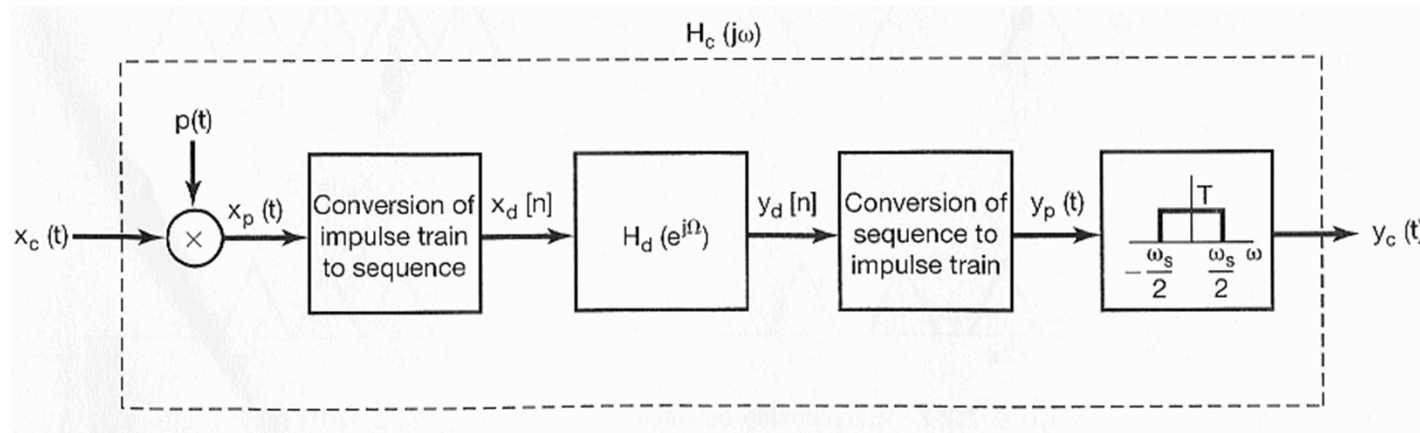


In Summary

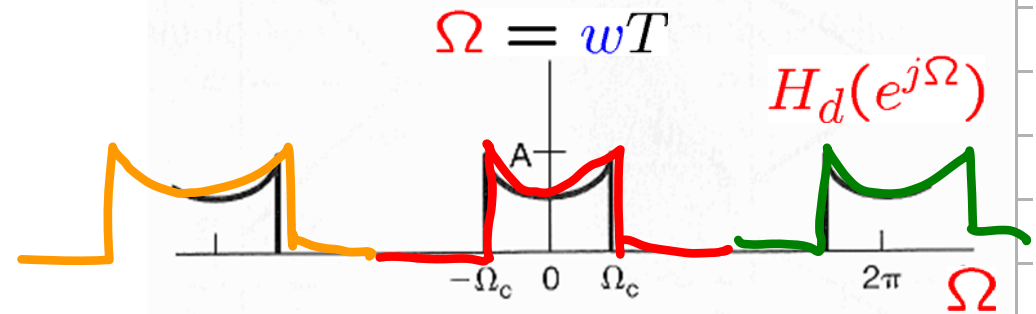
Discrete-Time Processing of CT Signals



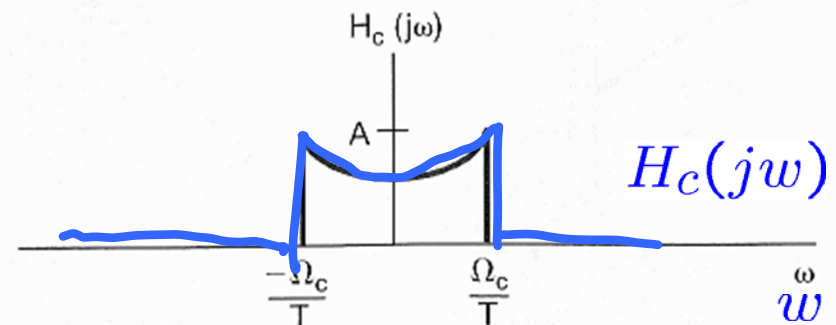
CT & DT Frequency Responses:



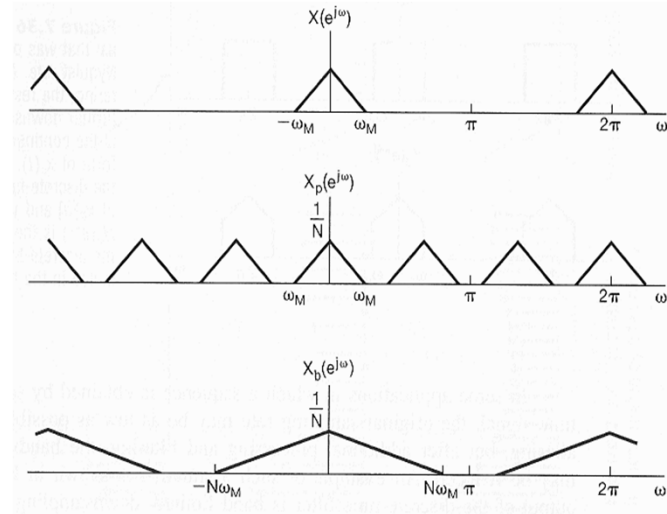
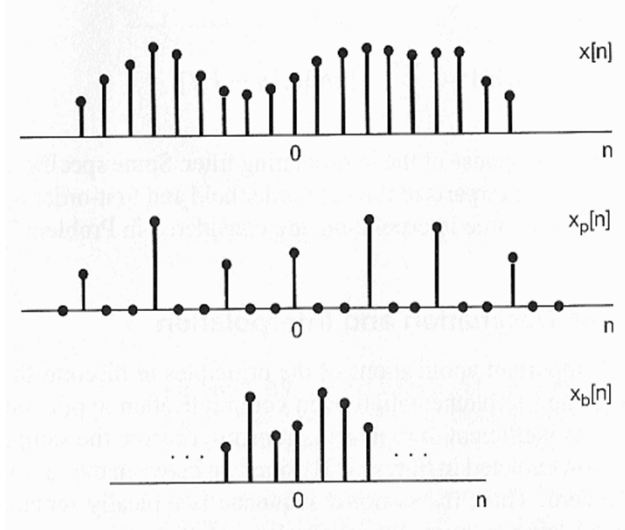
$$Y_c(j\omega) = X_c(j\omega)H_c(j\omega)$$



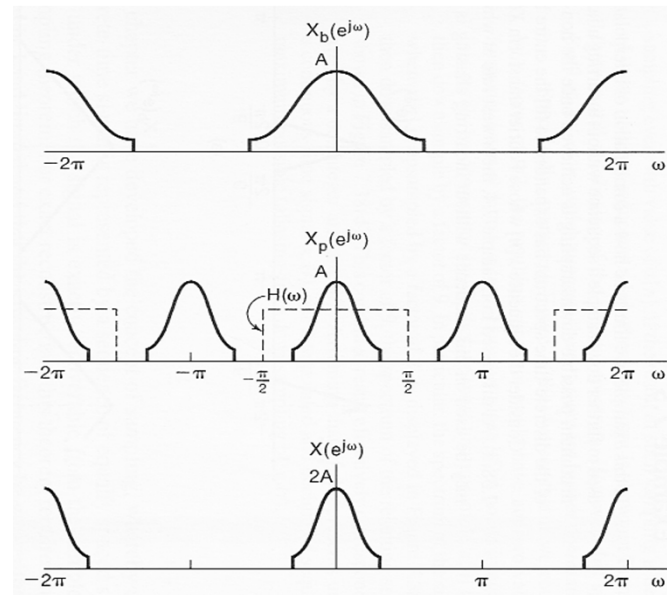
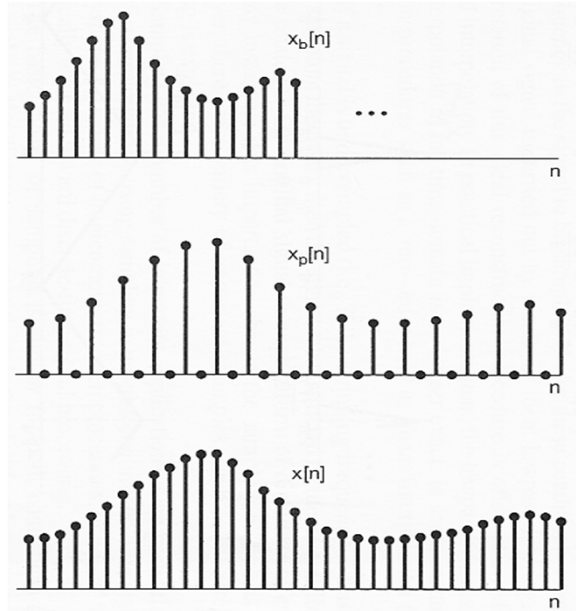
$$H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}), & |\omega| < \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$



Decimation (Down-sampling):

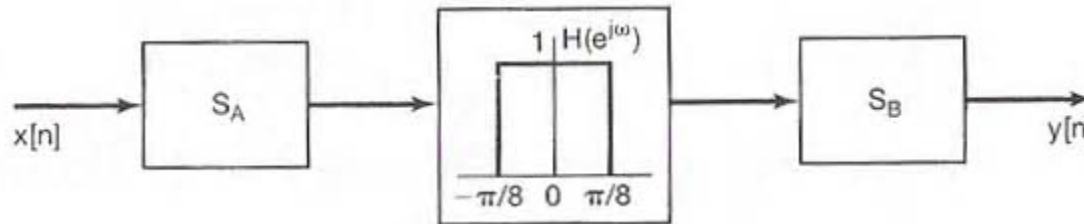


Interpolation (Up-sampling):

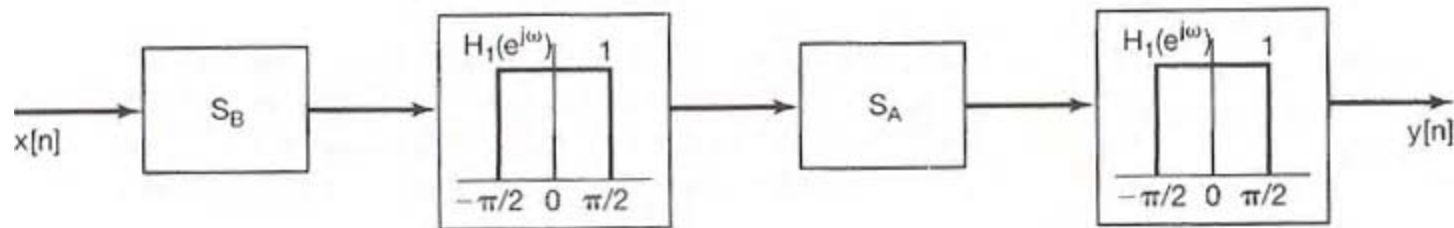


■ **Problem 7.20 (p.560):**

■ (a)



■ (b)



■ S_A : Inserting one zero after each sample

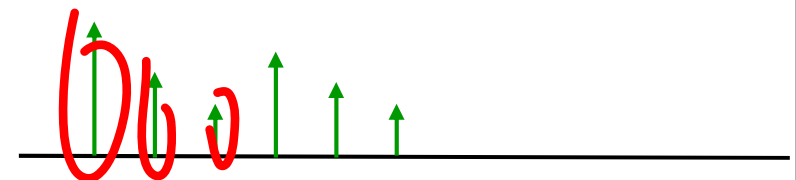
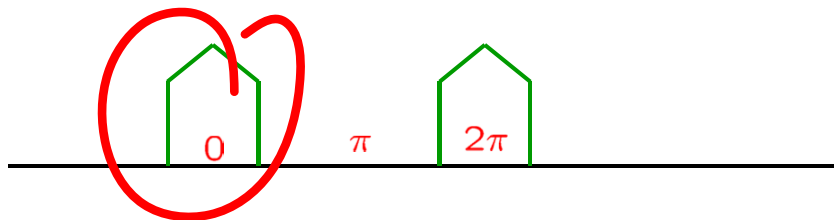
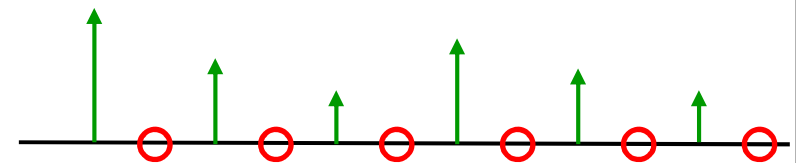
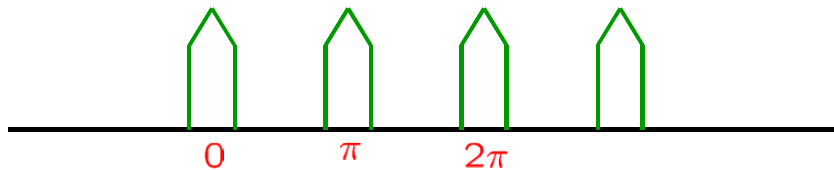
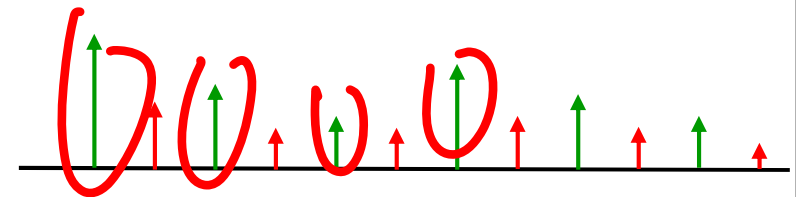
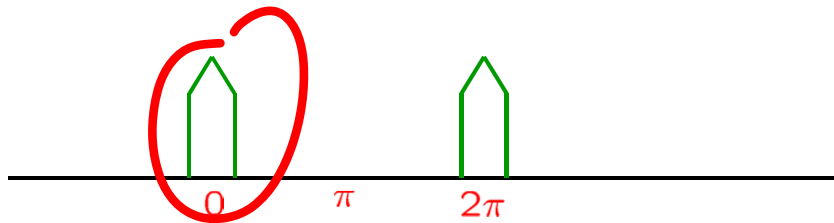
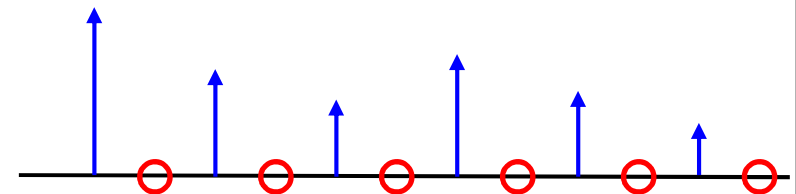
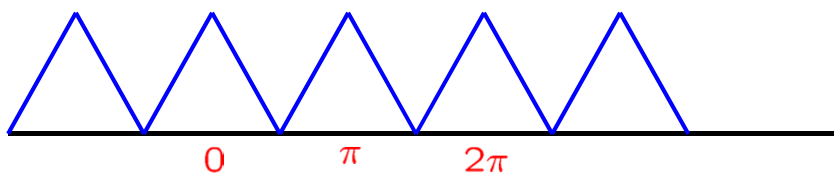
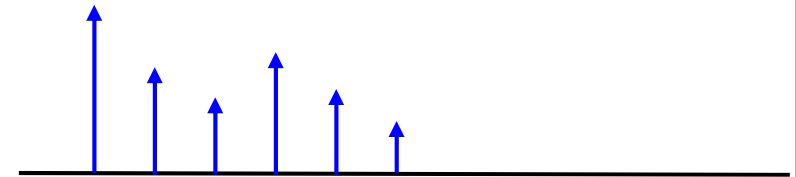
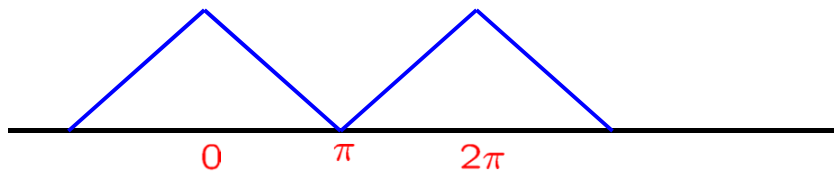
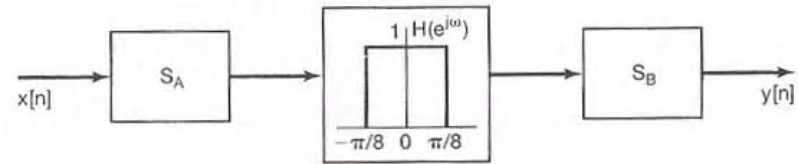
■ S_B : Decimation 2:1, extracting every second sample

■ Which corresponds to **low-pass filtering** with $\omega_c = \pi/4$?

Sampling of Discrete-Time Signals

■ Problem 7.20 (p.560):

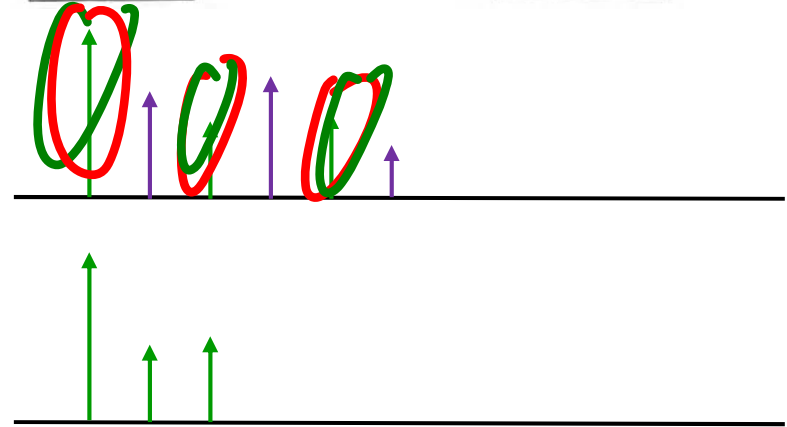
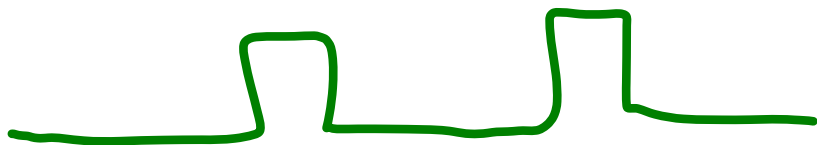
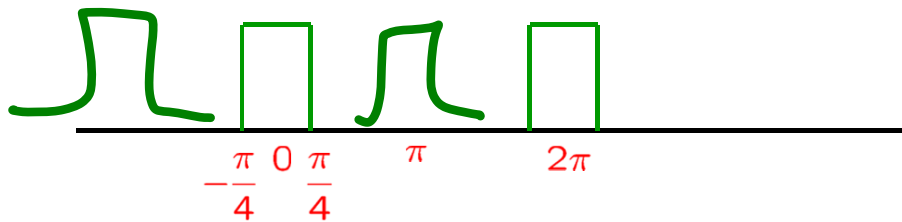
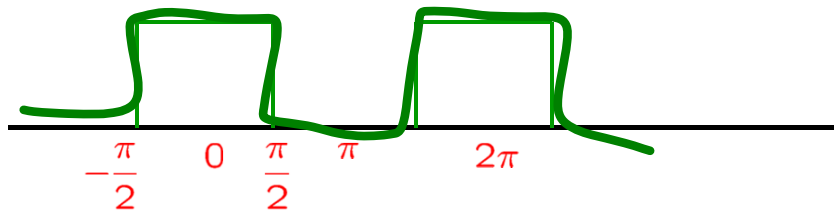
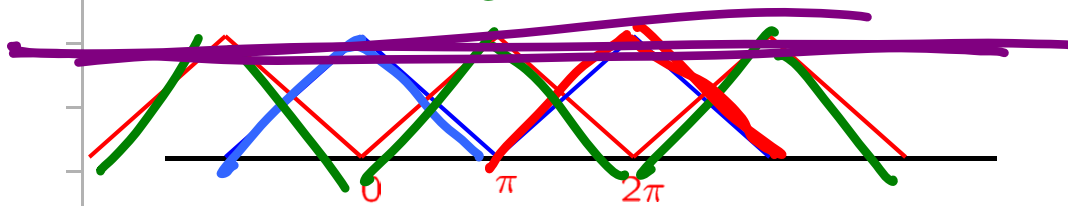
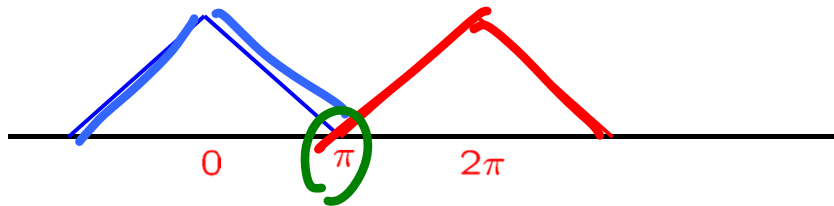
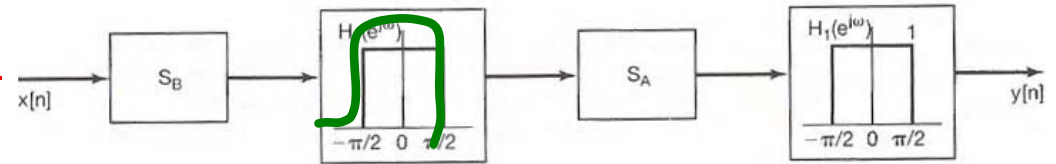
■ (a)



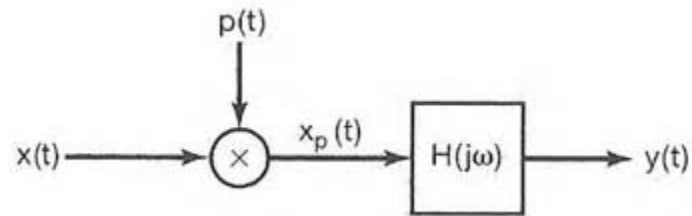
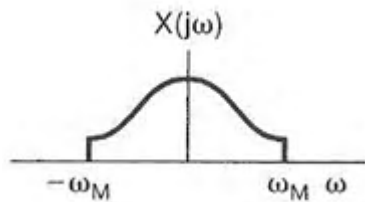
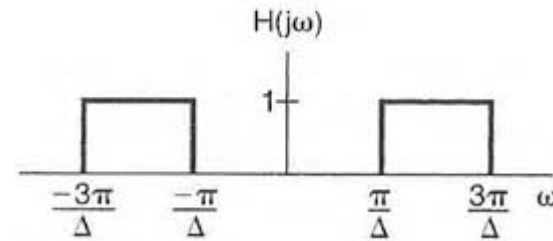
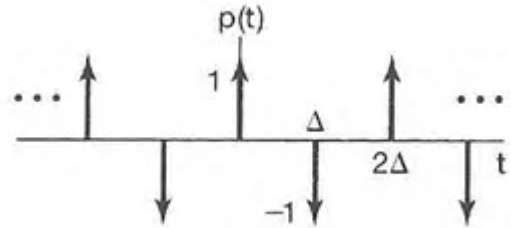
Sampling of Discrete-Time Signals

■ Problem 7.20 (p.560):

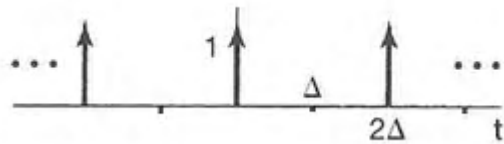
■ (b)



■ Problem 7.23 (p.562):

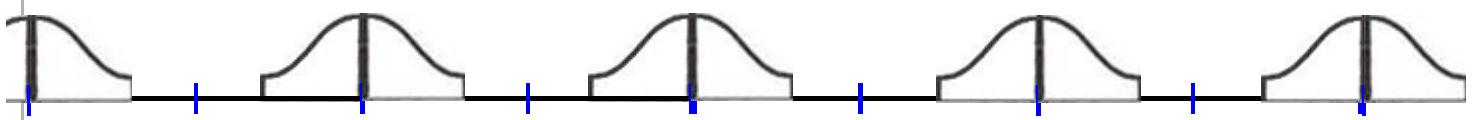
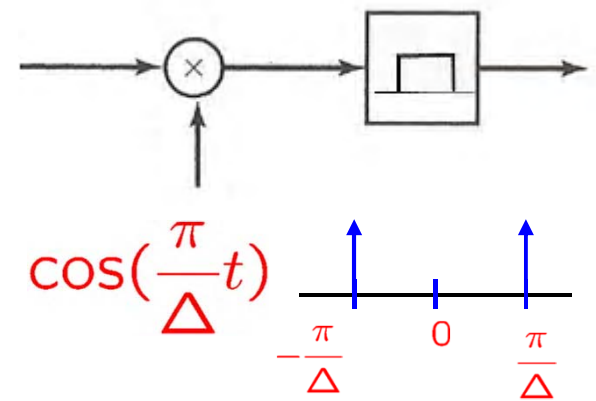
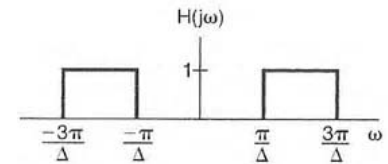
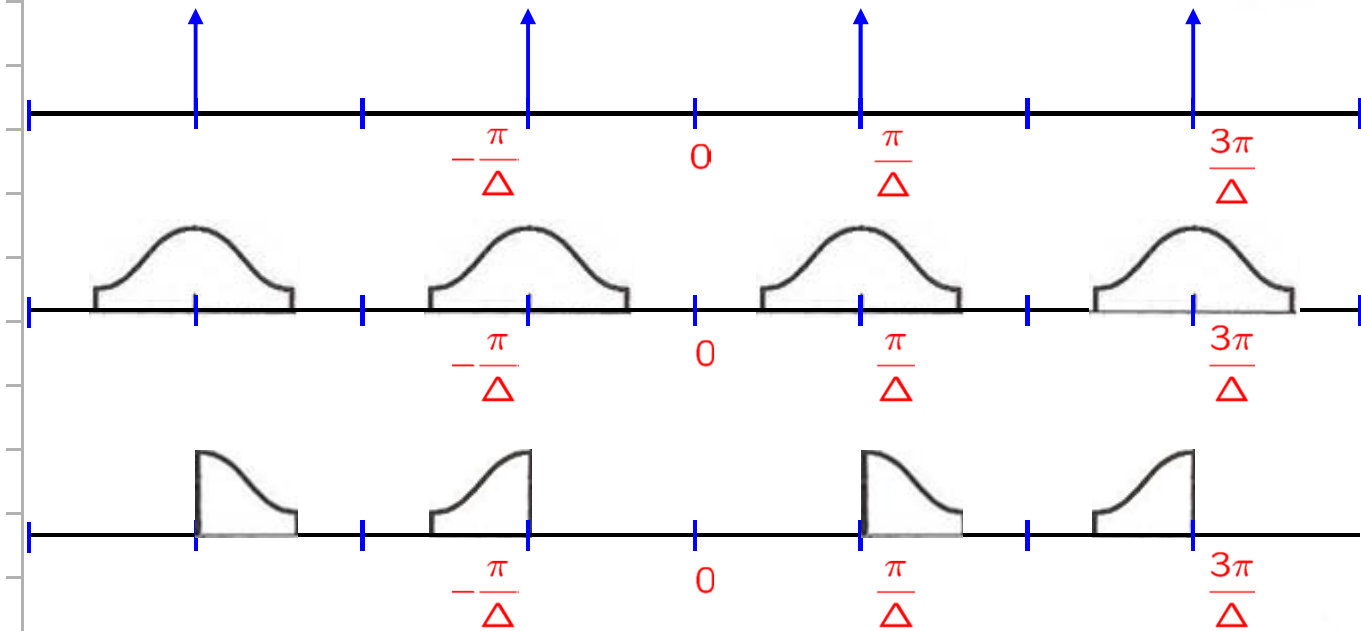
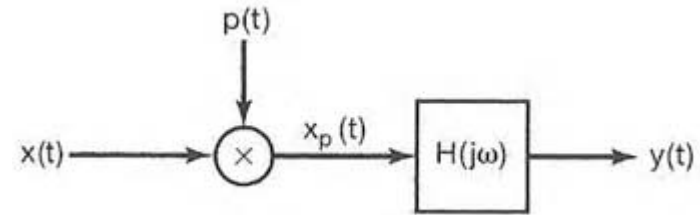


$$p_1(t)$$

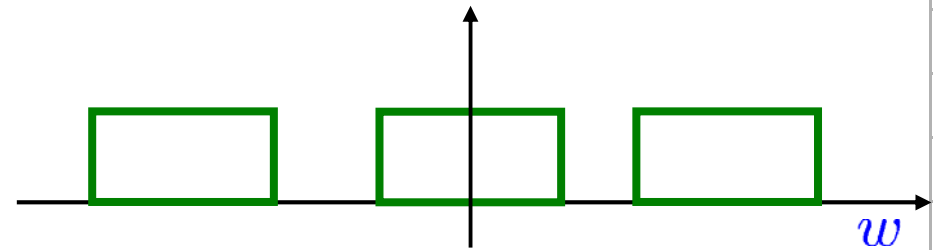
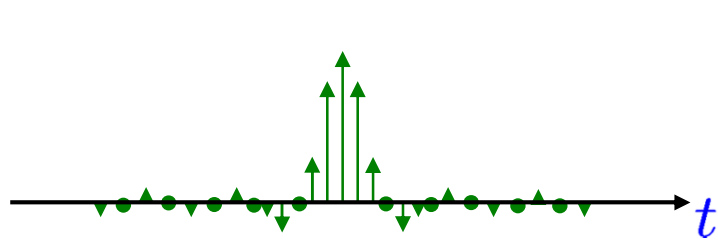
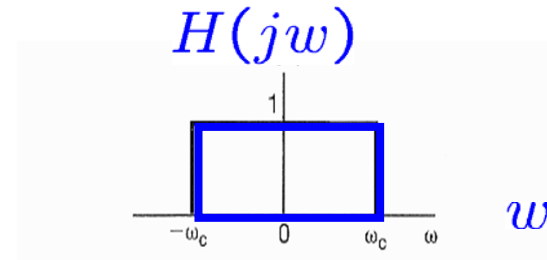
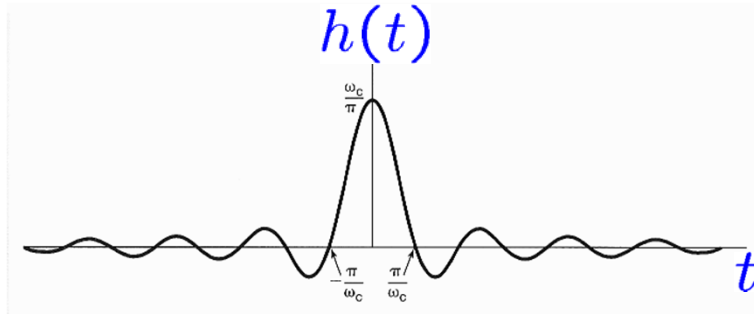


$$p(t) = p_1(t) - p_1(t - \Delta)$$

■ Problem 7.23 (p.562):



In Summary

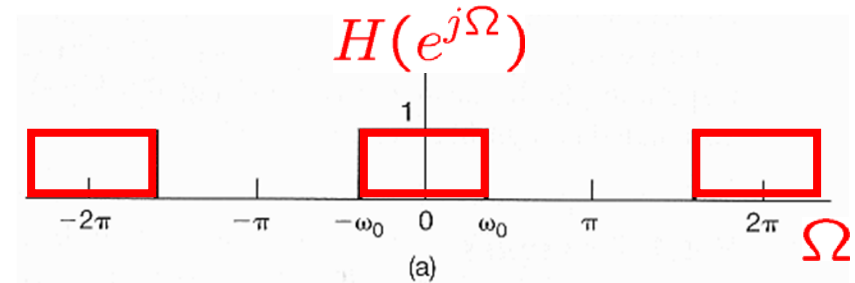
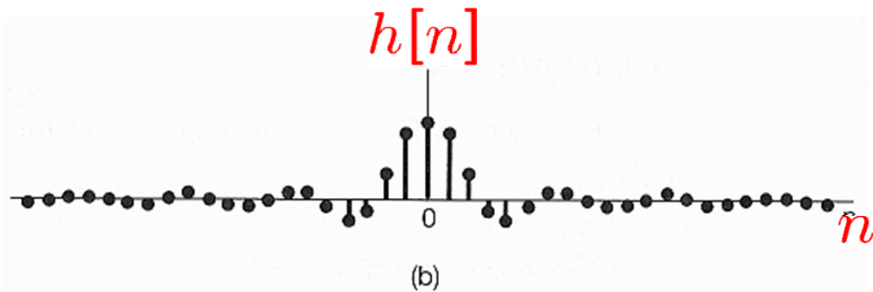


$$h(t) \xleftrightarrow{\text{C.T.F.T.}} H(j\omega)$$

$$\omega_s = \frac{2\pi}{T}$$

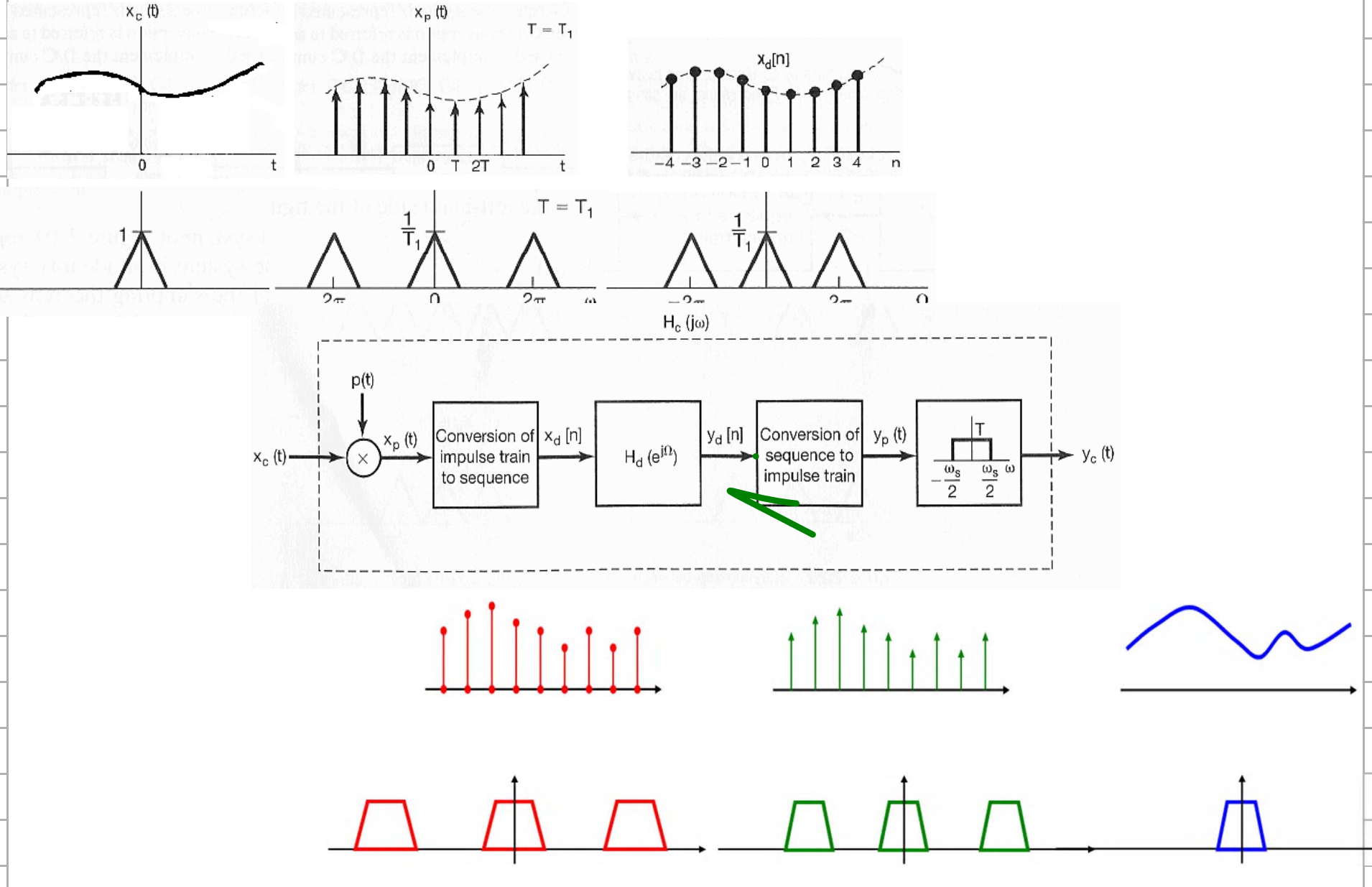
$$\Omega = \omega T$$

$$h[n] \xleftrightarrow{\text{D.T.F.T.}} H(e^{j\Omega})$$



In Summary

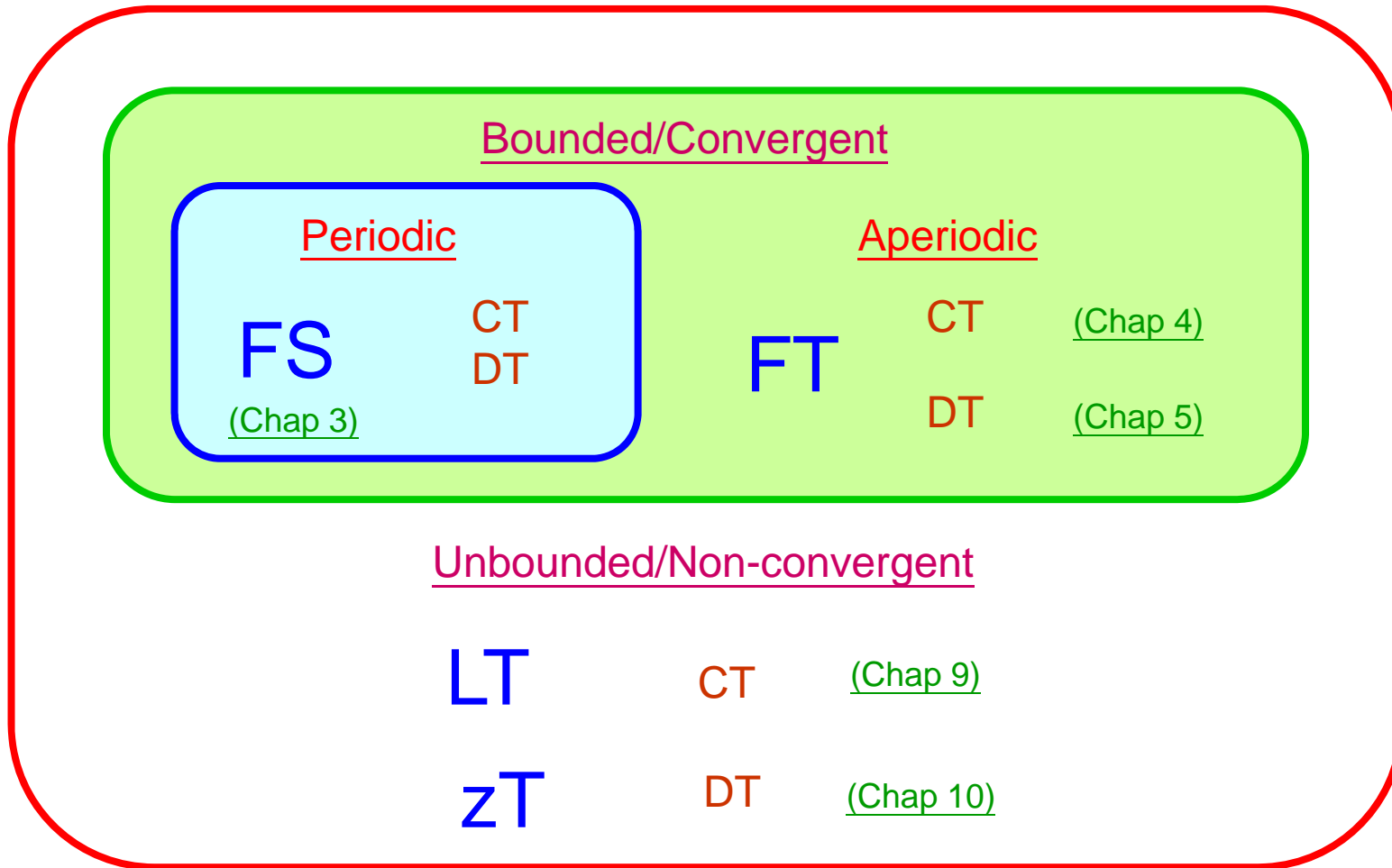
Discrete-Time Processing of CT Signals



- Representation of of a CT Signal by Its Samples:
 - The Sampling Theorem
- Reconstruction of of a Signal from Its Samples
 - Using exact interpolation
 - Using zero-order hold
 - Using higher-order hold
- The Effect of Under-sampling
 - Overlapping in Frequency-Domain
 - Aliasing
- DT Processing of CT Signals
- Sampling of Discrete-Time Signals
 - Down-sampling
 - Up-sampling

Introduction [\(Chap 1\)](#)

LTI & Convolution [\(Chap 2\)](#)



Time-Frequency [\(Chap 6\)](#)

Communication [\(Chap 8\)](#)

Digital
Signal
Processing [\(dsp-8\)](#)

CT-DT [\(Chap 7\)](#)

Control [\(Chap 11\)](#)