Exams

Final in 2014 – 1: Lowpass Filter, Linear Phase

1. (6%) An engineer is designing a lowpass filter. He starts with

$$h_0[n] = \frac{\sin w_c n}{\pi n}, \quad H_0(e^{jw}) = \begin{cases} 1, & |w| \le w_c, \\ 0, & w_c < |w| \le \pi. \end{cases}$$

Because $h_0[n]$ is not causal, he first truncates $h_0[n]$ at $n = \pm N$ to have

$$h_1[n] = \begin{cases} h_0[n], & |n| \le N, \\ 0, & \text{else}, \end{cases}$$

and then shifts $h_1[n]$ to have $h_2[n] = h_1[n - N]$.

(a) (4%) Prove that $H_1(e^{jw})$, the discrete-time Fourier transform of $h_1[n]$, is real and even.

(b) (2%) Show that $H_2(e^{jw})$ has linear phase.

Final in 2014 – 2: Group Delay

2. (2%) For a system function $H(jw)$, what is the group delay for it? Explain what that means.

Final in 2013 – 3: Lowpass, Highpass, Cutoff Freq, s -> 1/s

3. (10%) Consider a continuous-time system with the transfer function

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}.$$

(a) (2%) Find the poles and zeros and sketch the magnitude response of this system. Determine whether the system is low pass or high pass, and find the cutoff frequency (the value of ω for which $|H(j\omega)| = 1/\sqrt{2}$).

(b) (2%) Sketch a block diagram of the system in the direct form.

(c) (3%) Perform the transformation of variables in which s is replaced by $1/s$ in $H(s)$. Repeat Part (a) for the transformed system.

(d) (3%) Find the transformation that converts $H(s)$ to a high-pass system with cutoff frequency $\omega = 100$.

4. (13%) Consider a discrete-time right sided LTI system with transfer function

$$H(z)=\frac{1-a^{*}z}{z-a},\quad |a|<1.$$

(a) (3%) Sketch a pole-zero plot for this system in the z-plane.

(b) (3%) Use the graphical method to show that the magnitude response of the system is unity for all frequencies.

(c) (3%) Use the graphical method to sketch the phase response of the system for $a=1/2$.

(d) (4%) Use the result from (b) to prove that any system with a transfer function of the form

$$H(z)=\prod_{k=1}^{K}\frac{1-a_k^{*}z}{z-a_k},\quad |a_k|<1,$$

corresponds to a stable and causal all-pass system.

Final in 2012 – 1: Lowpass Filter, Cutoff Freq

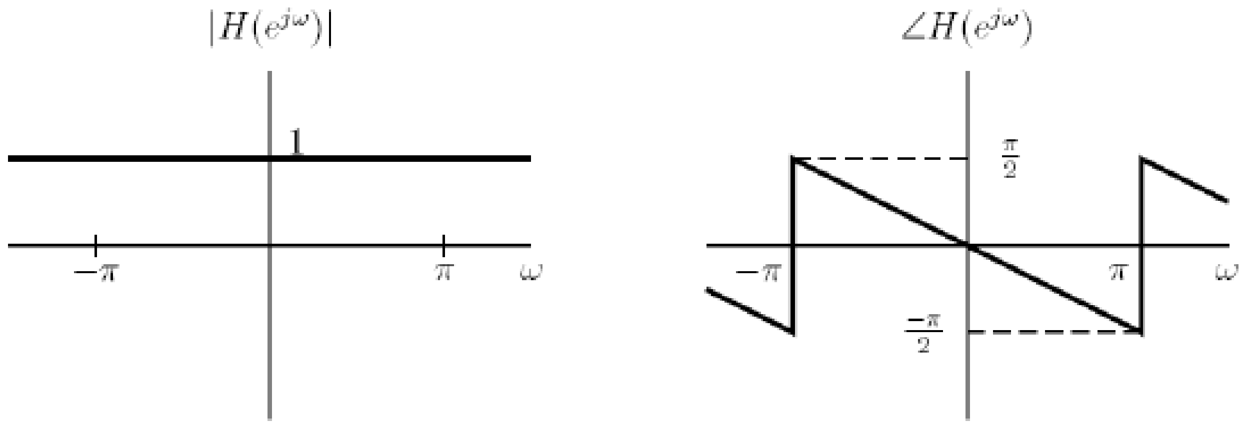
1. (10%) Consider the continuous-time signal $x(t)$ as follows:

$$x(t) = \sum_{k=0}^{\infty} 2^{-k/2} \cos(40\pi k t) .$$

To design a low-pass filter that removes no more than 10% of the signal power, what should the cutoff frequency of the filter be?

Final in 2011 – 1: x[n] -> H -> y[n]

1. [6] The magnitude and phase of the frequency response of a DT LTI system are given below:



Suppose the input sequence to the system is $x[n] = \cos(\frac{5\pi}{2}n - \frac{\pi}{4})$. Determine the output.