Spring 2015

信號與系統
Signals and Systems

Chapter SS-6
Time & Frequency Characterization of Signals and Systems

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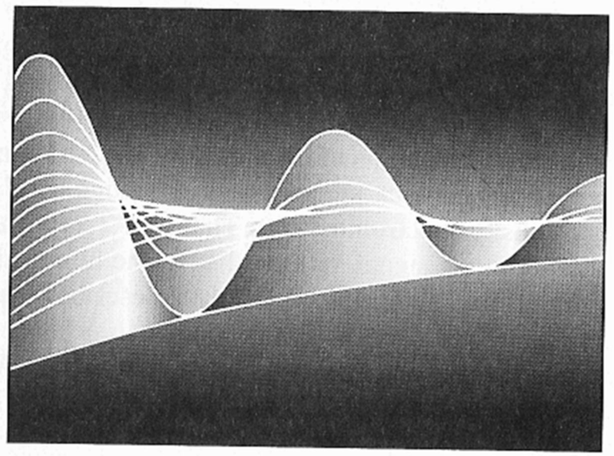
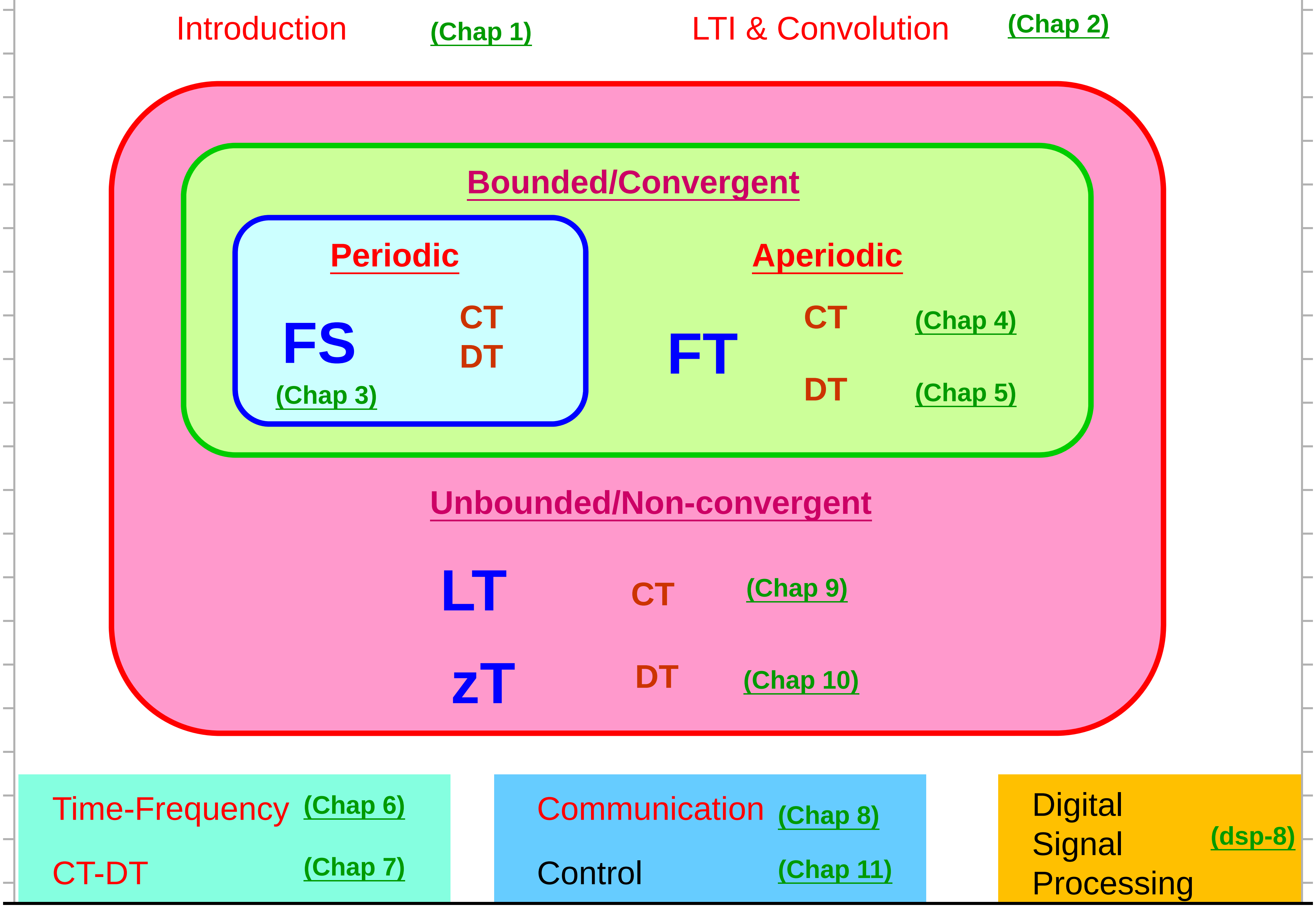
NTU-EE

Feb15 – Jun15



Figures and images used in these lecture notes are adopted from
"Signals & Systems" by Alan V. Oppenheim and Alan S. Willsky, 1997


Introduction
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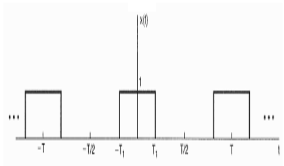
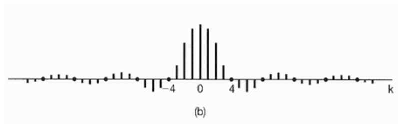
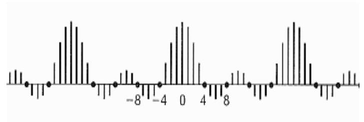
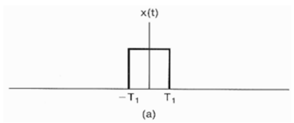
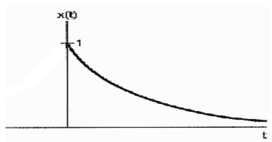
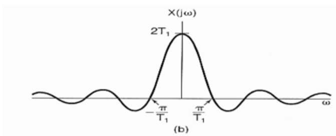
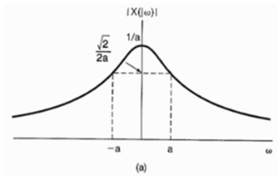
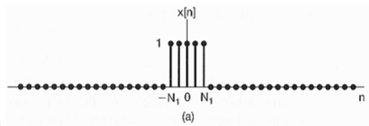
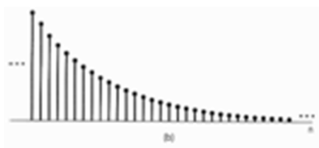
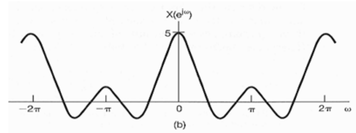
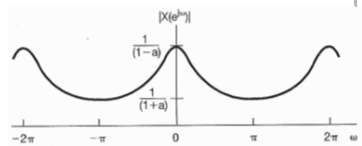
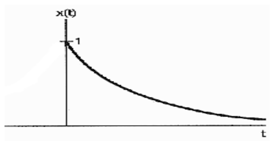
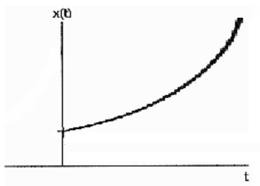
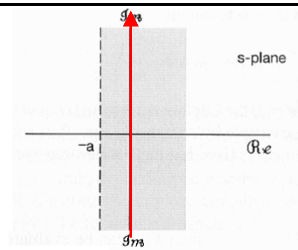
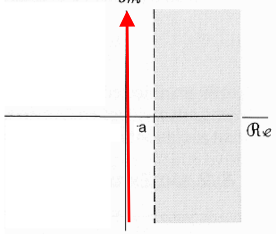
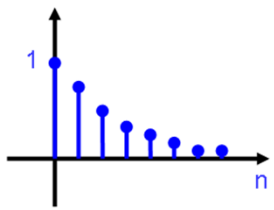
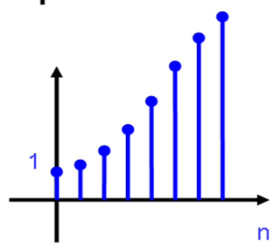
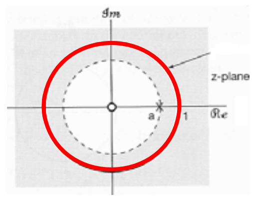
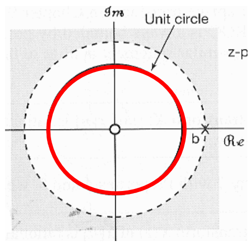
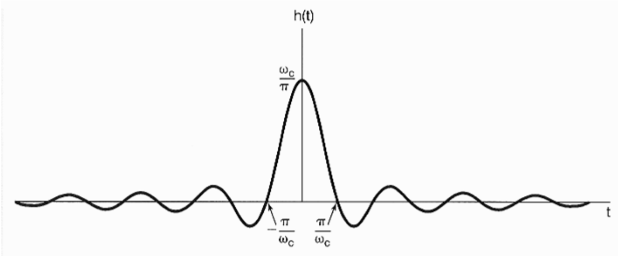
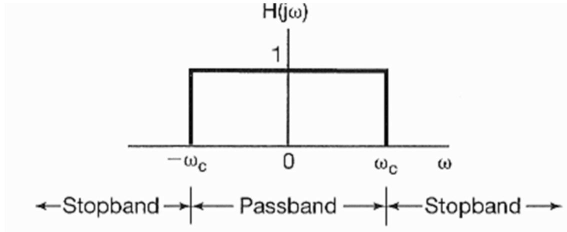
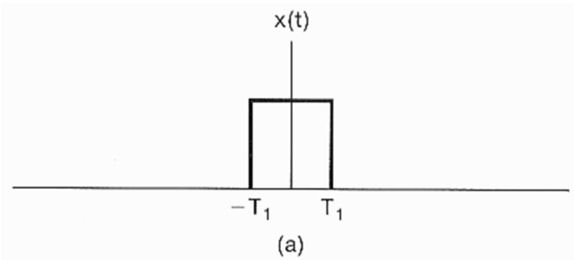
Fourier Series, Fourier Transform, Laplace Transform, z-Transform

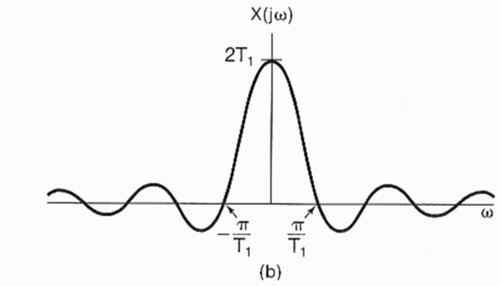
	CT		DT	
	time	frequency	time	frequency
FS				
FT	 	 	 	 
LT/zT	 	 	 	 

- Time-Domain & Frequency-Domain Characterization:



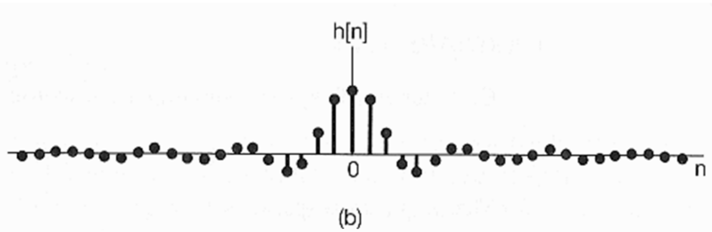


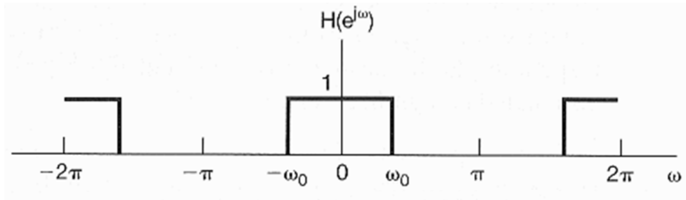


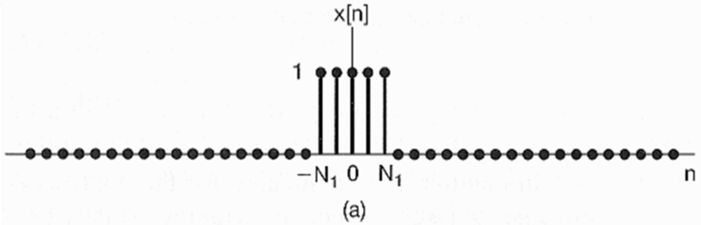


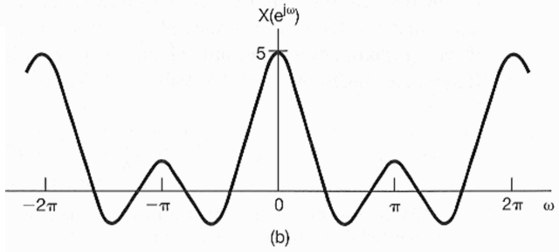
$$h(t) \stackrel{\mathcal{F}}{\longleftrightarrow} H(jw)$$

$$h[n] \stackrel{\mathcal{F}}{\longleftrightarrow} H(e^{jw})$$









▪ Time-Domain & Frequency-Domain Characterization:

$$H(jw) = \int_{-\infty}^{\infty} h(t) e^{-jwt} dt \qquad H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-jwn}$$

$\delta(t)/\delta[n]$ $h(t)/h[n]$ $h(t)/h[n]$

$x(t)/x[n]$ → LTI System → $y(t)/y[n]$

$X(jw)/X(e^{jw})$ $H(jw)/H(e^{jw})$ $Y(jw)/Y(e^{jw})$

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(jw) \qquad h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(jw) \qquad y(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(jw)$$

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{jw}) \qquad h[n] \overset{\mathcal{F}}{\longleftrightarrow} H(e^{jw}) \qquad y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{jw})$$

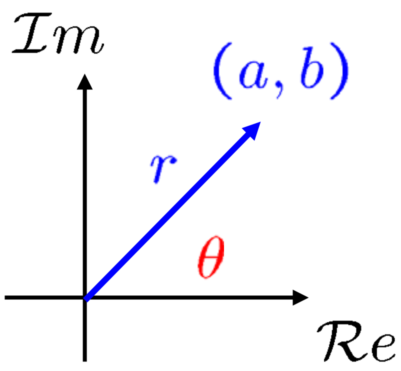
$$y(t) = x(t) * h(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(jw) = X(jw)H(jw)$$

$$y[n] = x[n] * h[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{jw}) = X(e^{jw})H(e^{jw})$$

Time-Domain		Frequency-Domain
	Convolution ↔ Transformation	
Differential Eqns or Difference Eqns	System Model & Operations	Algebraic Equations
Convolution Multiplication	Techniques	Multiplication Convolution
Time-Domain Considerations	System Design	Frequency-Domain Considerations

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems (p. 427)
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems
- 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

- Magnitude & Phase Representation:



$$a + jb \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = re^{j\theta}$$

$$|a + jb| = \sqrt{a^2 + b^2} \qquad \left|\frac{1}{a+jb}\right| = \frac{1}{\sqrt{a^2+b^2}}$$

$$\measuredangle\, a + jb = \tan^{-1}(\frac{b}{a}) \qquad \measuredangle\, \frac{1}{a+jb} = -\tan^{-1}(\frac{b}{a})$$

$$X(jw) = \mathcal{R}e\,\{X(jw)\} + j\ \mathcal{I}m\,\{X(jw)\} \qquad = \left|X(jw)\right| e^{j\measuredangle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} \qquad = \left|X(e^{jw})\right| e^{j\measuredangle X(e^{jw})}$$

$\left|X(jw)\right|$ or $\left|X(e^{jw})\right|$: magnitude

$\measuredangle\, X(jw)$ or $\measuredangle\, X(e^{jw})$: phase angle

Magnitude Distortion & Phase Distortion: (p. 428)

$X \rightarrow$ H LTI System $\rightarrow Y$

$$Y(jw) = X(jw)\ H(jw)$$

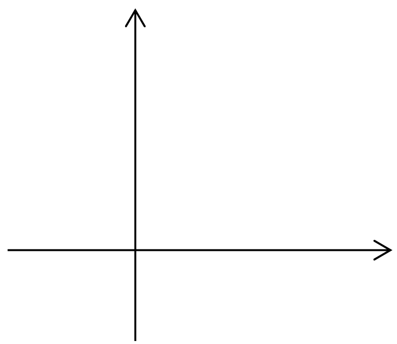
$$Y(e^{jw}) = X(e^{jw})\ H(e^{jw})$$

$$\Rightarrow \ \big|Y(jw)\big| e^{j\measuredangle Y(jw)} = \big|X(jw)\big| e^{j\measuredangle X(jw)} \big|H(jw)\big| e^{j\measuredangle H(jw)}$$

$$= \big|X(jw)\big| \big|H(jw)\big| e^{j(\measuredangle X(jw) + \measuredangle H(jw))}$$

$$\Rightarrow \begin{cases} |Y(jw)| = |X(jw)|\ |H(jw)| & \text{magnitude distortion} \\ \measuredangle Y(jw) = \measuredangle X(jw) + \measuredangle H(jw) & \text{phase distortion} \end{cases}$$

$$\Rightarrow \begin{cases} |Y(e^{jw})| = |X(e^{jw})|\ |H(e^{jw})| \\ \measuredangle Y(e^{jw}) = \measuredangle X(e^{jw}) + \measuredangle H(e^{jw}) \end{cases}$$



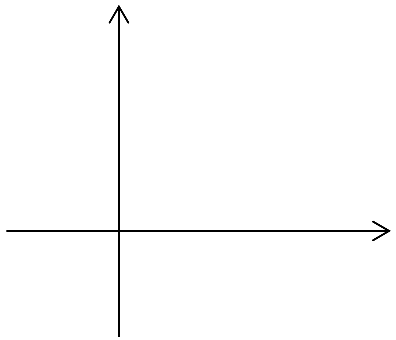
$|H(jw)|$ or $|H(e^{jw})|$: gain of the system

$\measuredangle H(jw)$ or $\measuredangle H(e^{jw})$: phase shift of the system

▪ Log-Magnitude & Phase Plots: (p. 436)

$$Y(jw) = X(jw)\ H(jw)$$

$$\Rightarrow \begin{cases} |Y(jw)| & = & |X(jw)|\ |H(jw)| \\ \measuredangle\, Y(jw) & = & \measuredangle\, X(jw) + \measuredangle\, H(jw) \end{cases}$$



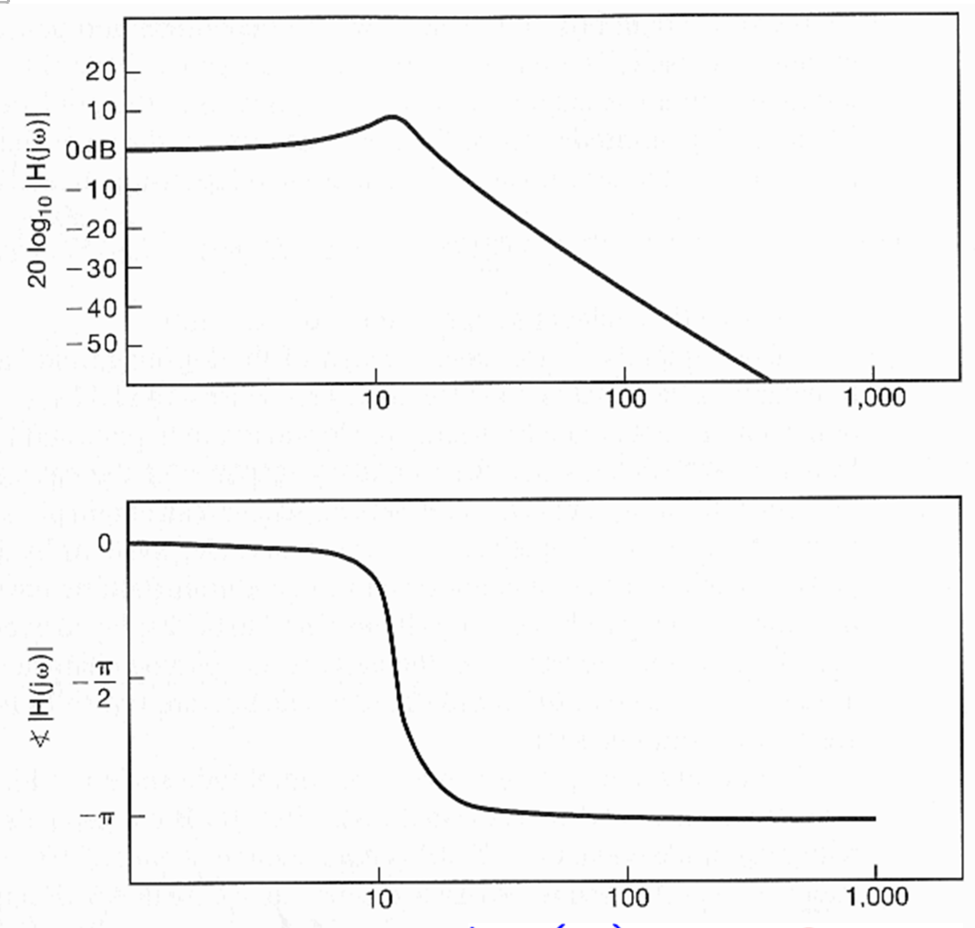
$$\Rightarrow \ \log |Y(jw)| = \log |X(jw)| + \log |H(jw)|$$

$$\Rightarrow \ 20\ \log_{10} |Y(jw)| = 20\ \log_{10} |X(jw)| + 20\ \log_{10} |H(jw)|$$

$$\Rightarrow \begin{cases} 20\ \log_{10}(1) & = & 0\ \text{dB} \\ 20\ \log_{10}(10) & = & 20\ \text{dB} \\ 20\ \log_{10}(0.1) & = & -20\ \text{dB} \end{cases}$$

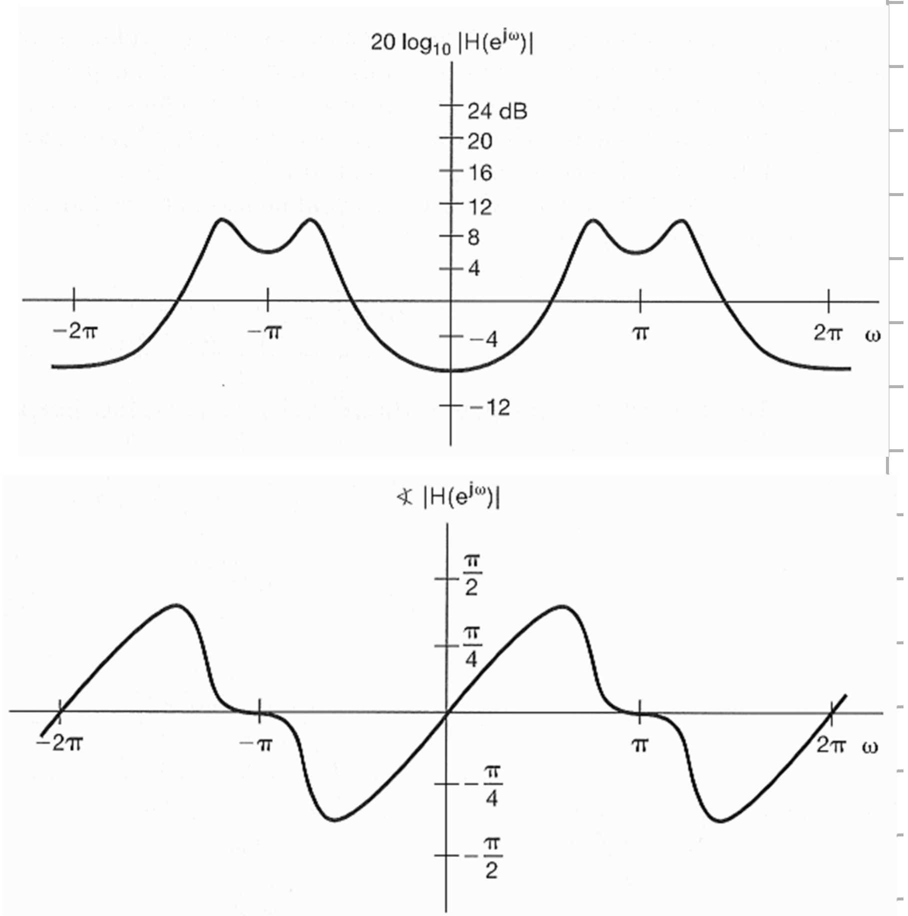
- Log-Magnitude & Bode Plots: (p. 436)

Continuous-Time Bode plot



$\log(w)$, $w : 0 \leftrightarrow \infty$

Discrete-Time Bode plot



(w), $w : -\pi \leftrightarrow \pi$

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems (p. 448)
- 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

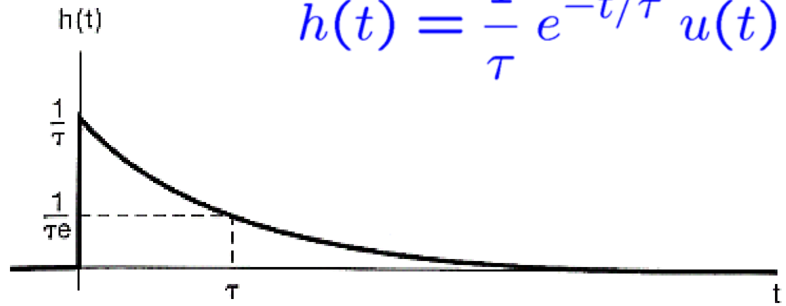
First-Order CT Systems: (p. 448)

$x(t) \rightarrow$ LTI System $\rightarrow y(t)$

$$Y(jw) = X(jw)H(jw)$$

$$\tau \frac{d}{dt} y(t) + y(t) = x(t)$$

$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$



(a)

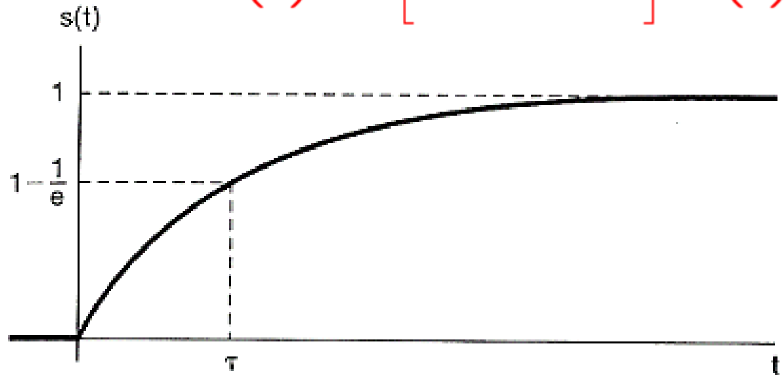
$$\Rightarrow \quad H(jw) = \frac{1}{jw\tau + 1}$$

$$\Rightarrow \quad h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

$$\Rightarrow \quad s(t) = h(t) * u(t)$$

$$= \left[1 - e^{-t/\tau}\right] u(t)$$

$$s(t) = \left[1 - e^{-t/\tau}\right] u(t)$$



(b)

Time Constant

- **First-Order CT Systems:**

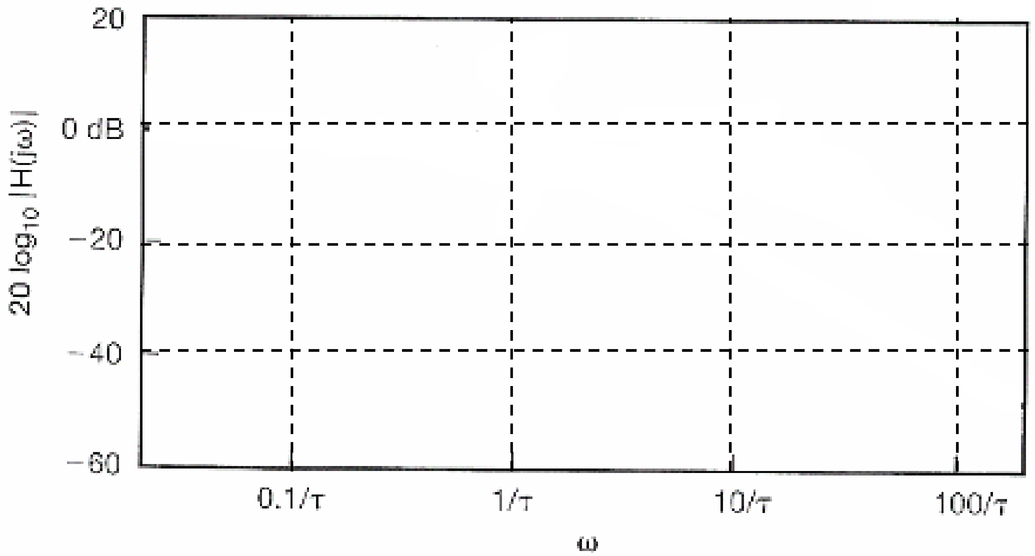
$$H(jw) = \frac{1}{jw\tau + 1}$$

$$|H(jw)| = \frac{1}{\sqrt{(w\tau)^2 + 1}}$$

$$20\log_{10}|H(jw)| = -10\log_{10}\left[(w\tau)^2 + 1\right]$$

$$\approx \begin{cases} -10\log_{10}\left[(w\tau)^2 + 1\right] = 0 & w << \frac{1}{\tau} \\ -10\log_{10}\left[(w\tau)^2 + 1\right] = -10\log_{10}(2) \approx -3dB & w = \frac{1}{\tau} \\ -10\log_{10}\left[(w\tau)^2 + 1\right] = -20\log_{10}(w\tau) & w >> \frac{1}{\tau} \end{cases}$$

$$= -20\log_{10}(w) - 20\log_{10}(\tau)$$

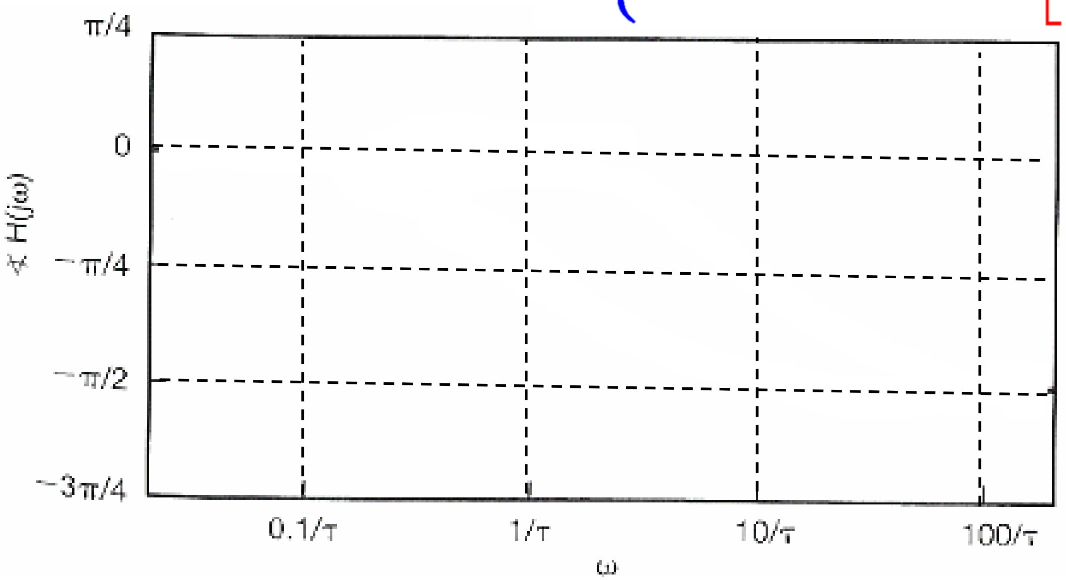


▪ First-Order CT Systems:

$$H(jw) = \frac{1}{jw\tau + 1}$$

$$\sphericalangle H(jw) = -\tan^{-1}(w\tau)$$

$$\approx \begin{cases} 0 & w \le \frac{0.1}{\tau} \\ -\frac{\pi}{4} & w = \frac{1}{\tau} \\ -\frac{\pi}{2} & w \ge \frac{10}{\tau} \\ -(\frac{\pi}{4})[\log_{10}(w\tau) + 1] & \frac{0.1}{\tau} \le w \le \frac{10}{\tau} \\ \quad = -(\frac{\pi}{4})\left[\log_{10}(w) + \log_{10}(\tau) + 1\right] & \end{cases}$$



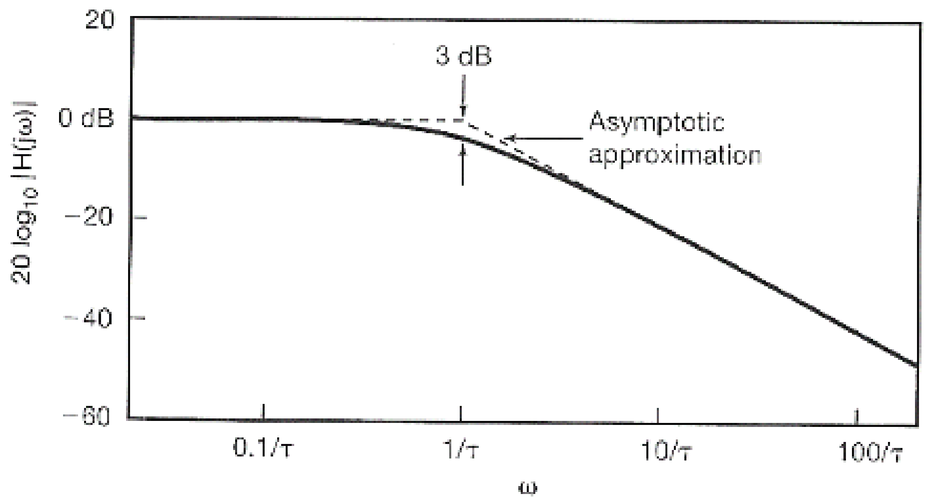
$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

- First-Order CT Systems:

$$H(jw) = \frac{1}{jw\tau + 1}$$

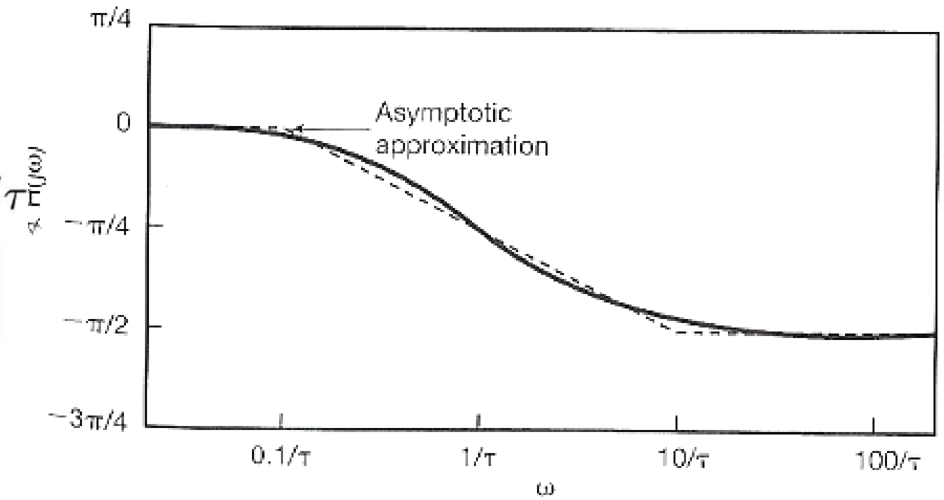
$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << 1/\tau \\ -10\log_{10}(2) \approx -3dB & w = 1/\tau \\ -20\log_{10}(w\tau) & w >> 1/\tau \\ \quad = -20\log_{10}(w) - 20\log_{10}(\tau) & \end{cases}$$



$$\sphericalangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1/\tau \\ -(\pi/4)[\log_{10}(w\tau) + 1] & 0.1/\tau \le w \le 10/\tau \\ \quad = -(\pi/4)\left[\log_{10}(w) + \log_{10}(\tau) + 1\right] & \\ -\pi/4 & w = 1/\tau \\ -\pi/2 & w \ge 10/\tau \end{cases}$$



- Second-Order CT Systems: (p. 451)

$$\frac{d^2}{dt^2}y(t) + 2\ \zeta\ w_n\ \frac{d}{dt}y(t) + w_n^2\ y(t) = w_n^2\ x(t)$$

$$\Rightarrow\ H(jw) = \frac{{w_n}^2}{(jw)^2 + 2\zeta w_n (jw) + {w_n}^2}$$

$$= \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

$$\Rightarrow\ H(jw) = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

$$\begin{cases} c_1 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1} \\ c_2 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1} \end{cases}$$

▪ Second-Order CT Systems:

$$\Rightarrow \quad H(jw) = \frac{{w_n}^2}{(jw)^2 + 2\zeta w_n (jw) + {w_n}^2}$$

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

ζ : damping ratio

w_n : undamped natural frequency

$$\begin{cases} 0 < \zeta < 1: & \text{underdamped} \\ \zeta = 1: & \text{critically damped} \\ \zeta > 1: & \text{overdamped} \end{cases}$$

Second-Order CT Systems:

$$H(jw) = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

- For $\zeta = 1 \;\Rightarrow\; c_1 = c_2 = -w_n$:

$$\Rightarrow \; H(jw) = \frac{w_n^2}{(jw + w_n)^2}$$

$$\Rightarrow \; h(t) = w_n^2 \, t \, e^{-w_n t} u(t)$$

- For $\zeta \neq 1 \;\Rightarrow\; c_1 \neq c_2$:

$$\Rightarrow \; H(jw) = \frac{M}{jw - c_1} - \frac{M}{jw - c_2}$$

$$M = \frac{w_n}{2\sqrt{\zeta^2 - 1}}$$

$$\Rightarrow \; h(t) = M \left[e^{c_1 t} - e^{c_2 t} \right] u(t)$$

Second-Order CT Systems:

$$M = \frac{w_n}{2\sqrt{\zeta^2 - 1}}$$

- For $0 < \zeta < 1$, c_1, c_2: complex:

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1} = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

$$\Rightarrow \ h(t) = M\left[e^{c_1 t} - e^{c_2 t}\right] u(t) \qquad \begin{cases} c_1 = -\zeta w_n + j w_n \sqrt{1-\zeta^2} \\ c_2 = -\zeta w_n - j w_n \sqrt{1-\zeta^2} \end{cases}$$

$$= \frac{w_n e^{-\zeta w_n t}}{2j\sqrt{1-\zeta^2}} \left\{ e^{j(w_n\sqrt{1-\zeta^2})t} - e^{-j(w_n\sqrt{1-\zeta^2})t} \right\} u(t)$$

$$= \frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin\left((w_n\sqrt{1-\zeta^2})t \right) \right] u(t)$$

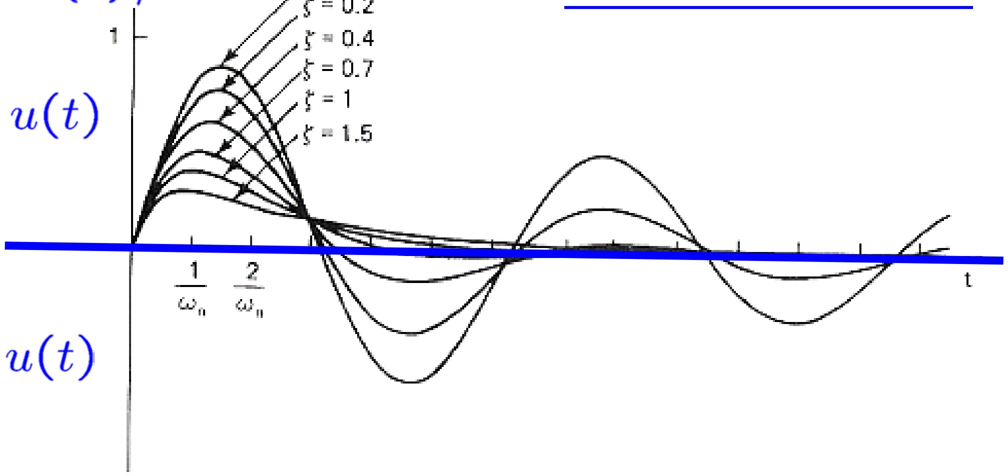
$$\Rightarrow \ s(t) = h(t) * u(t) = \left\{ 1 + M\left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2} \right] \right\} u(t) \qquad \zeta \neq 1$$

Second-Order CT Systems:

$$h(t) = \frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin \left((w_n \sqrt{1-\zeta^2}) t \right) \right] u(t)$$

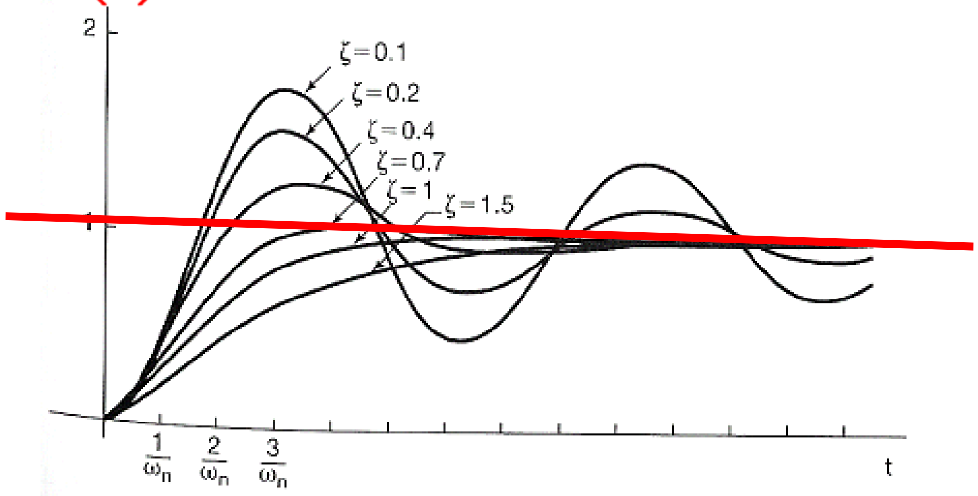
$$\frac{h(t)}{w_n} = \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin \left((w_n \sqrt{1-\zeta^2}) t \right) \right] u(t)$$

$h(t)/w_n$



(a)

$s(t)$



(b)

$$s(t) = \left\{ 1 + M \left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2} \right] \right\} u(t)$$

- **Second-Order CT Systems:** $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

$$|H(jw)| = \frac{1}{\sqrt{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2}}$$

$$20\log_{10}|H(jw)| = -10\log_{10}\left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\}$$

$$\left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\} \quad \left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\}$$

$$Q = \frac{1}{2\zeta}$$

$$\approx \begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

$$|H(jw)| = \frac{1}{\sqrt{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2}}$$

$$\Rightarrow \left(\frac{w}{w_n}\right)^4 - 2\left(\frac{w}{w_n}\right)^2 + 1 + 4\zeta^2\left(\frac{w}{w_n}\right)^2 = \left(\frac{w}{w_n}\right)^4 + (4\zeta^2-2)\left(\frac{w}{w_n}\right)^2 + 1$$

$$\Rightarrow \frac{d}{dw}\left\{\left(\frac{w}{w_n}\right)^4 + (4\zeta^2-2)\left(\frac{w}{w_n}\right)^2 + 1\right\} = 0$$

$$\Rightarrow \left(\frac{4w^3}{w_n^4}\right) + (4\zeta^2-2)\left(\frac{2w}{w_n^2}\right) = 0$$

$$\Rightarrow w\left(w^2 + (2\zeta^2-1)w_n^2\right) = 0$$

$$\Rightarrow w = 0,\ \pm\sqrt{1-2\zeta^2}\,w_n$$

- For $\zeta < \frac{\sqrt{2}}{2}$

$$\Rightarrow \max\{|H(jw)|\} \text{ at } w_{\max} = w_n\sqrt{1-2\zeta^2}$$

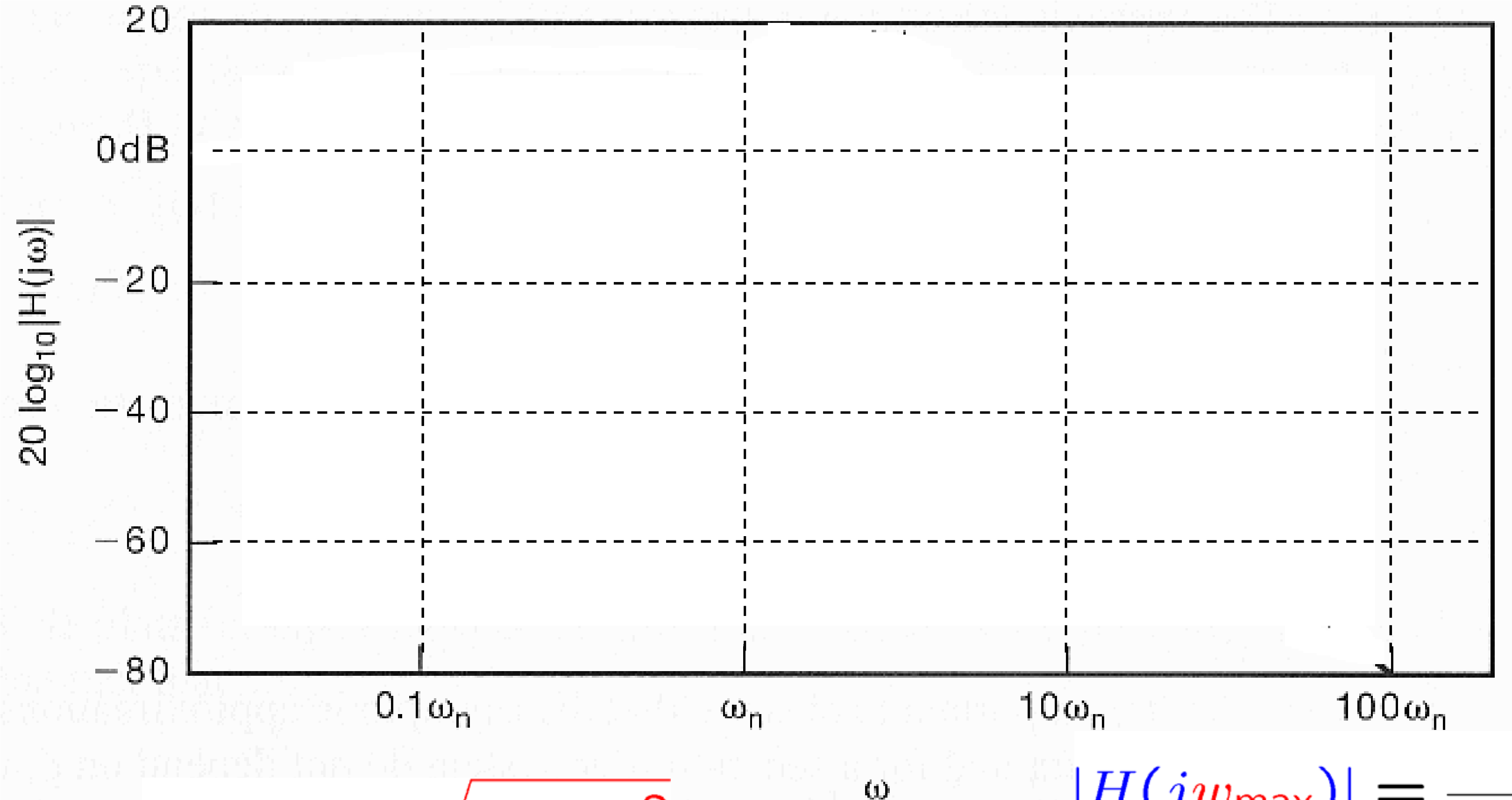
$$|H(jw_{\max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

▪ Second-Order CT Systems:

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

$$\begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

$$Q = \frac{1}{2\zeta}$$



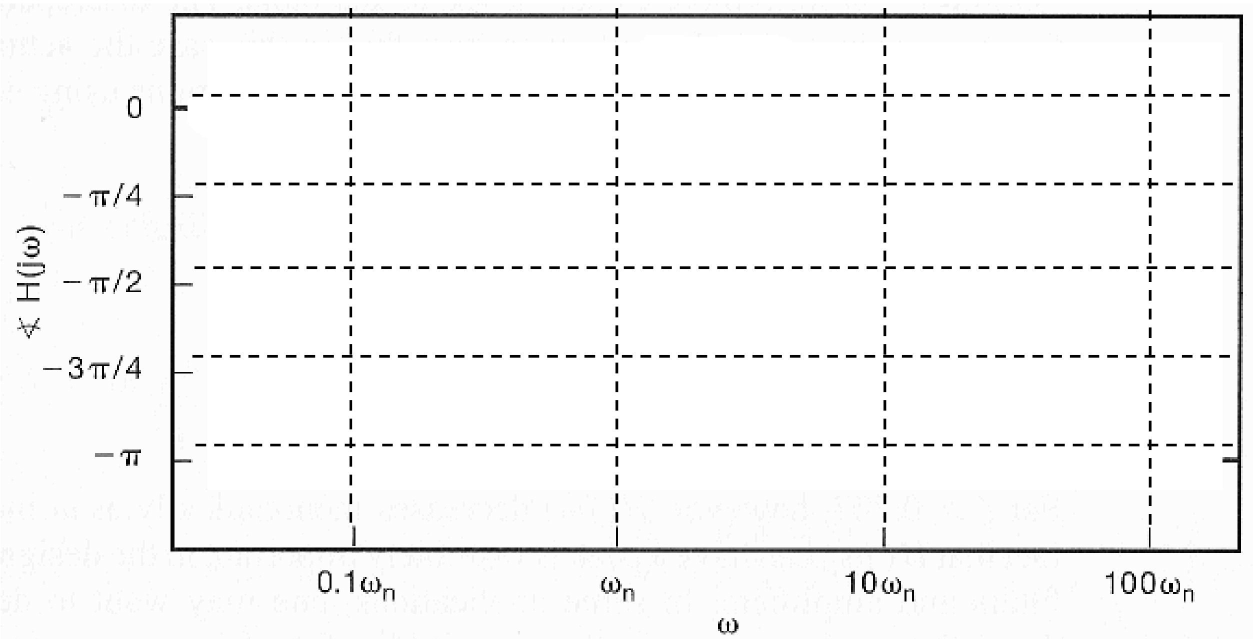
$$w_{\max} = w_n\sqrt{1-2\zeta^2}$$

$$|H(jw_{\max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- **Second-Order CT Systems:** $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

$$\measuredangle H(jw) = -\tan^{-1}\left(\frac{2\zeta(\frac{w}{w_n})}{1-(\frac{w}{w_n})^2}\right)$$

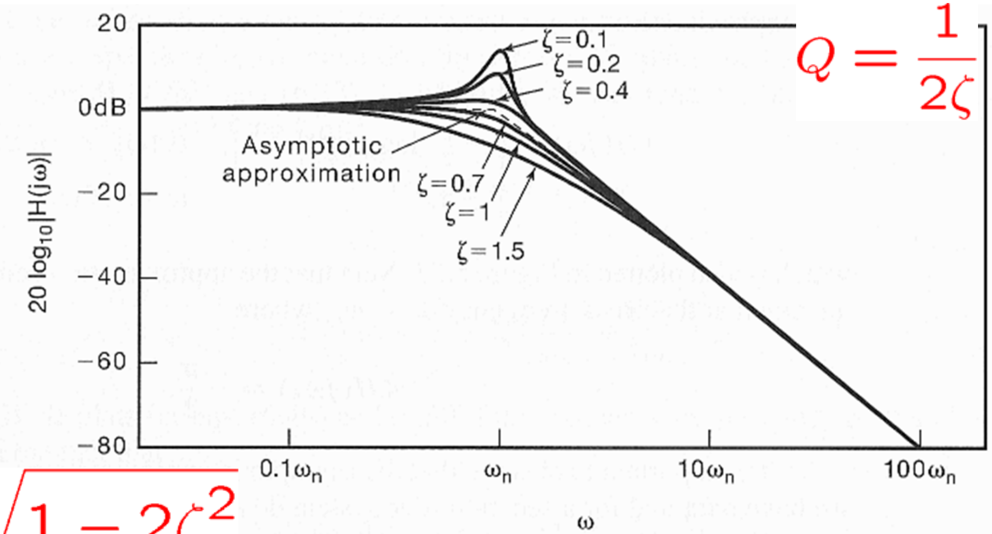
$$\approx \begin{cases} 0 & w \le 0.1 w_n \\ -\pi/2 & w = w_n \\ -(\pi/2)[\log_{10}(\frac{w}{w_n})+1] & 0.1w_n \le w \le 10 w_n \\ -\pi & w \ge 10 w_n \end{cases}$$



- **Second-Order CT Systems:** $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

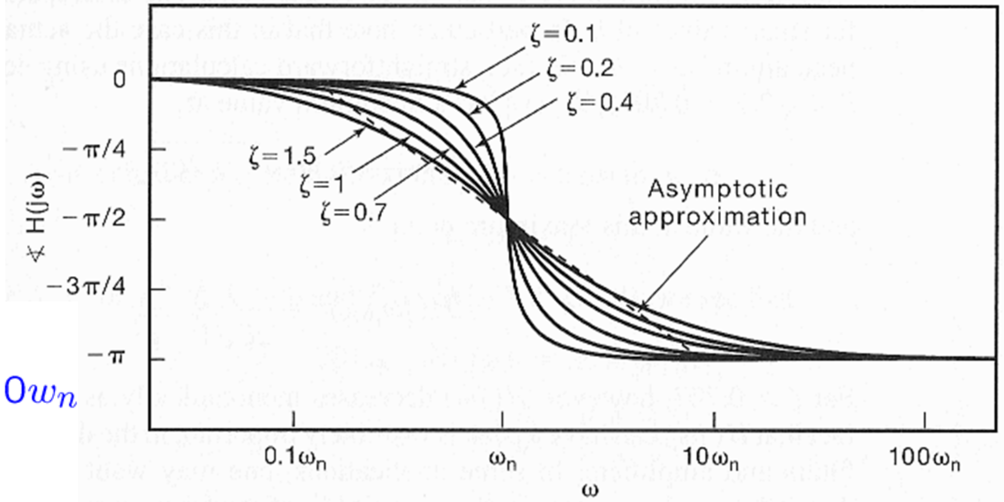


$$Q = \frac{1}{2\zeta}$$

- For $\zeta < \frac{\sqrt{2}}{2}$ $\quad w_{\max} = w_n\sqrt{1 - 2\zeta^2}$

$$\sphericalangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1w_n \\ -(\pi/2)[\log_{10}(w/w_n) + 1] & 0.1w_n \le w \le 10w_n \\ -\pi/2 & w = w_n \\ -\pi & w \ge 10w_n \end{cases}$$



- Example 6.4: (p.457)

$$\hat{H}(jw) = \frac{w_n^2}{(jw)^2 + 2\zeta w_n (jw) + w_n^2}$$

$$H(jw) = \frac{2 \times 10^4}{(jw)^2 + 100(jw) + 10^4}$$

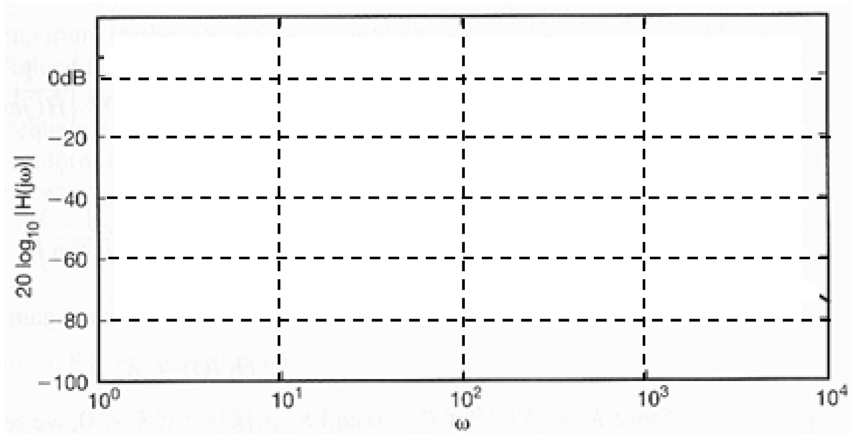
$$H(jw) = 2 \times \hat{H}(jw)$$

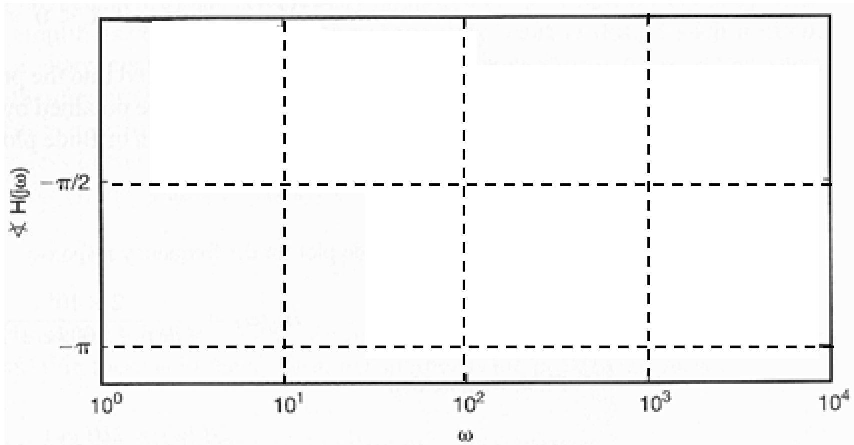
$$\sphericalangle H(jw) = \sphericalangle \hat{H}(jw)$$

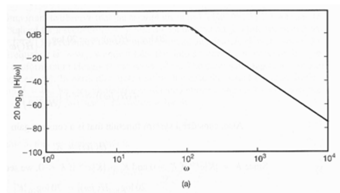
$$\Rightarrow \begin{cases} w_n = 100 \\ \zeta = 1/2 \end{cases}$$

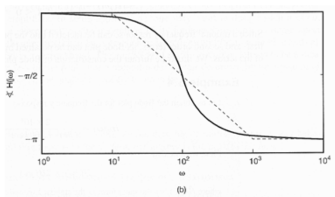
$$\Rightarrow \; 20 \log_{10} |H(jw)| =$$

$$20 \log_{10}(2) + 20 \log_{10} |\hat{H}(jw)|$$





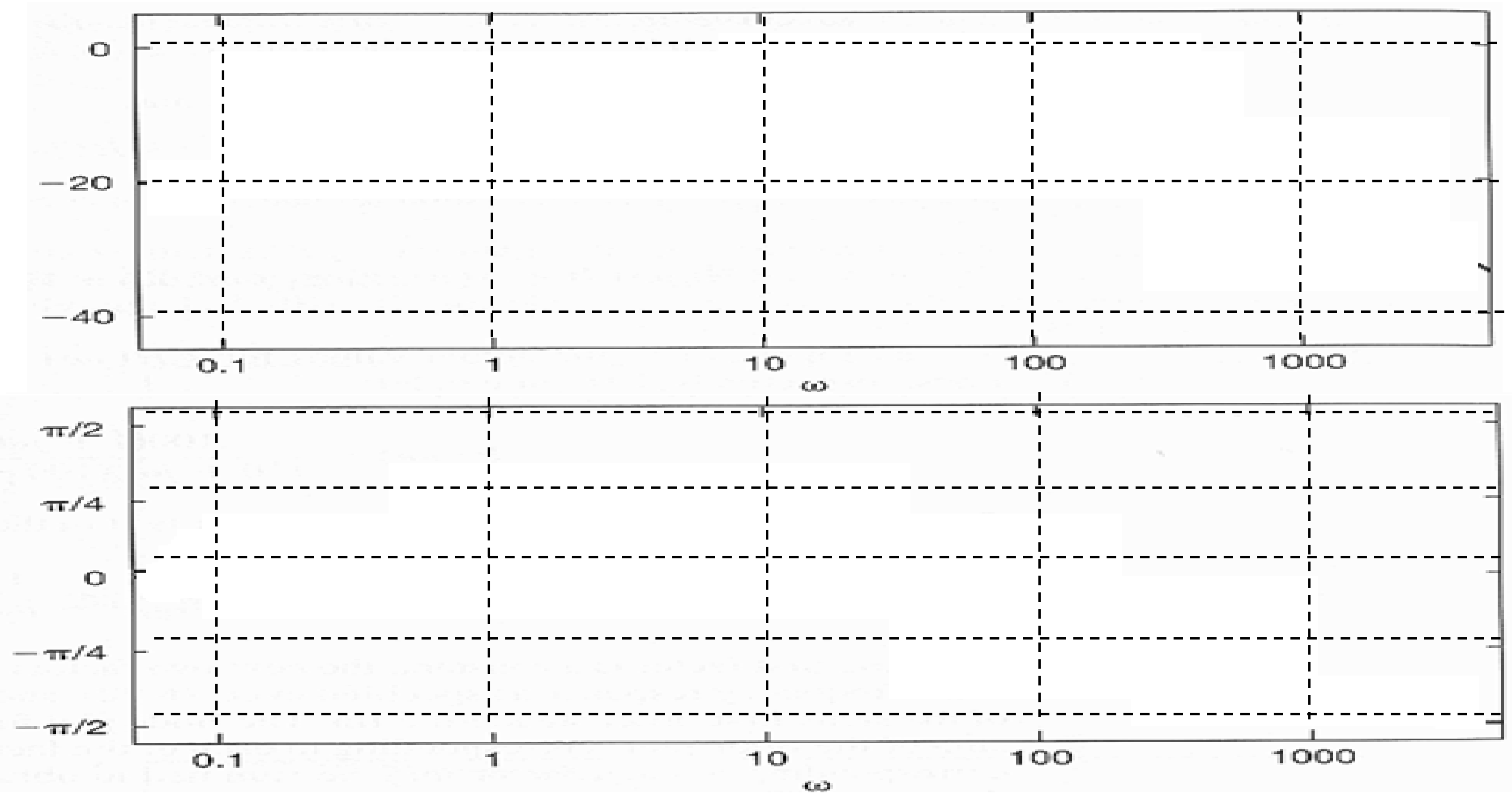




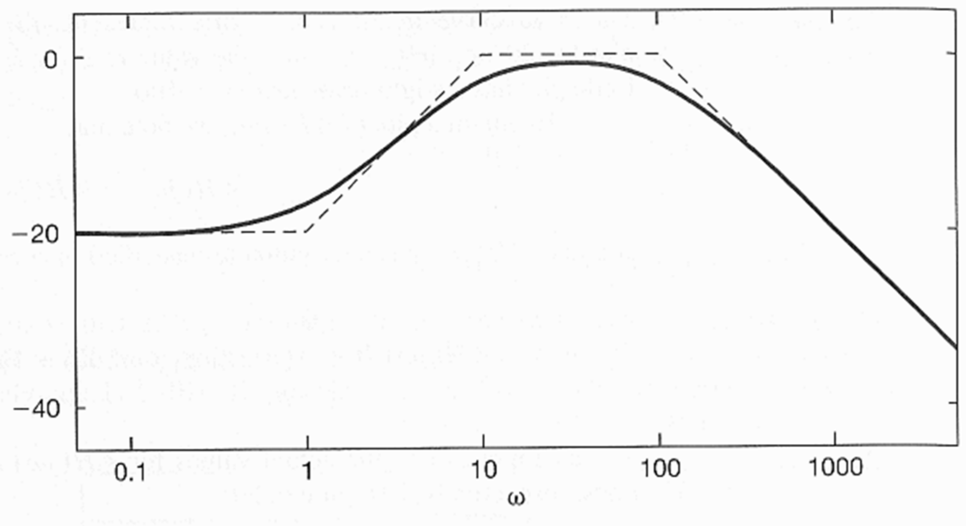
- Example 6.5:

$$H(jw) = \frac{100(1+jw)}{(10+jw)(100+jw)}$$

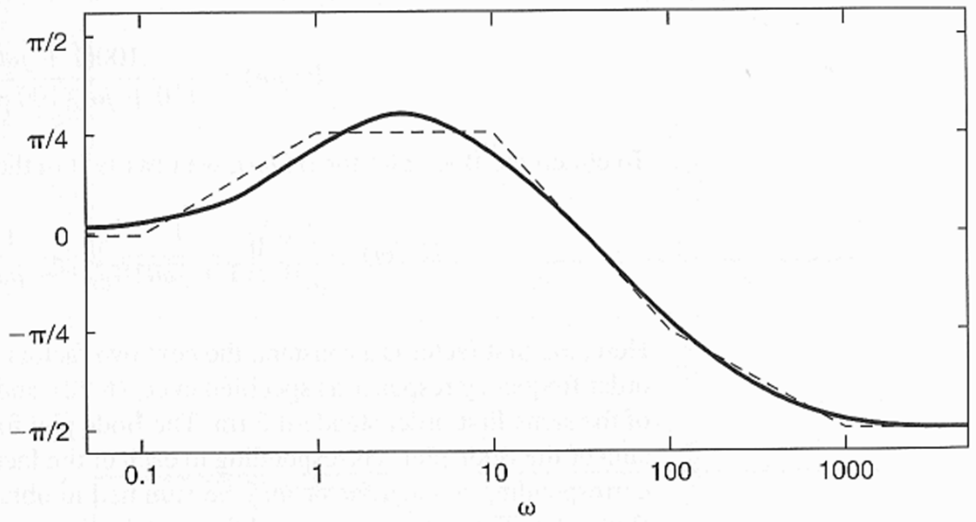
$$= \left(\frac{1}{10}\right)(1+jw)\left(\frac{1}{1+j\frac{w}{10}}\right)\left(\frac{1}{1+j\frac{w}{100}}\right)$$



- Example 6.5:

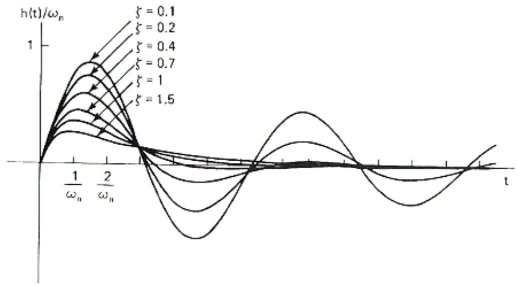


(a)

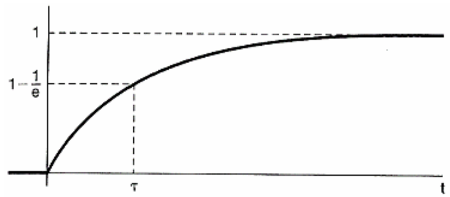


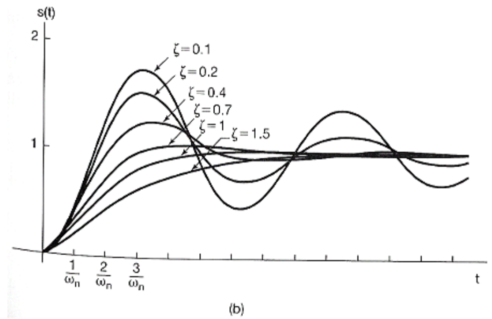
(b)

$h(t)$

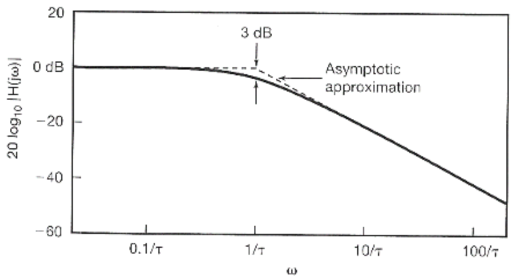


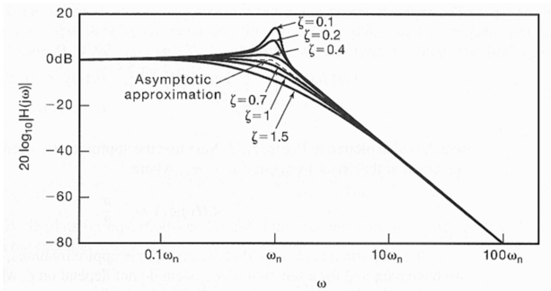
$s(t)$



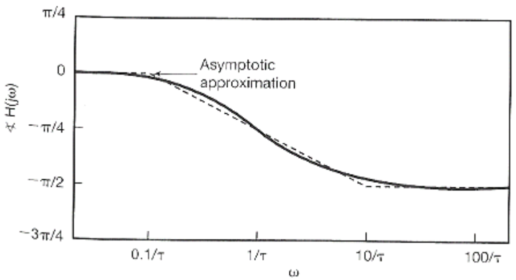


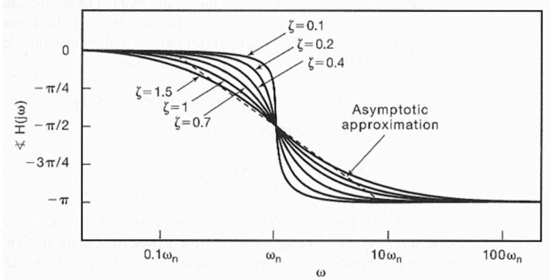
$20\log_{10}|H(jw)|$





$\measuredangle H(jw)$

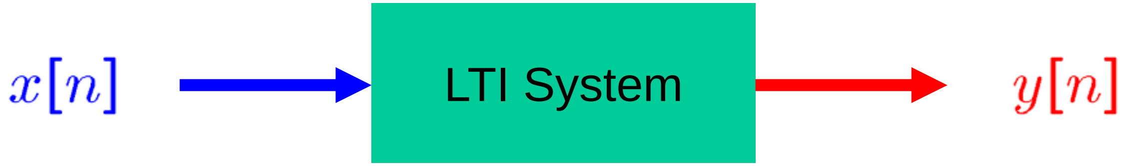




- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems
- p.461 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

- First-Order DT Systems: (p.461)

$$Y(e^{jw}) = X(e^{jw})\ H(e^{jw})$$



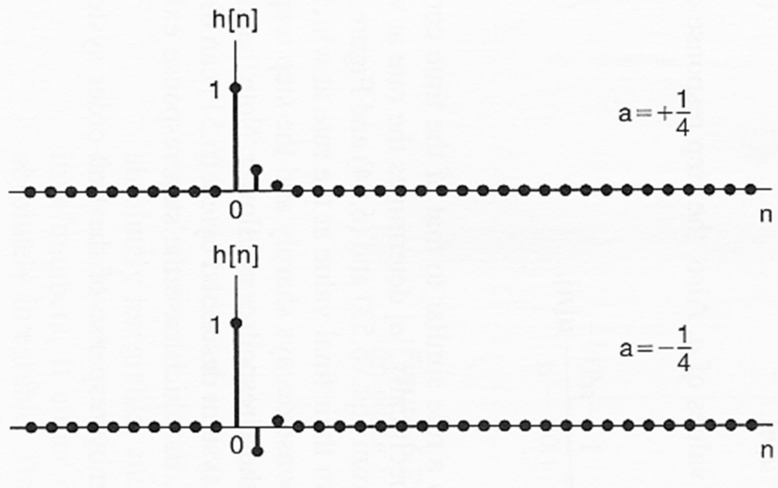
$$y[n] - a\ y[n-1] = x[n] \qquad |a| < 1$$

$$\Rightarrow\quad H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

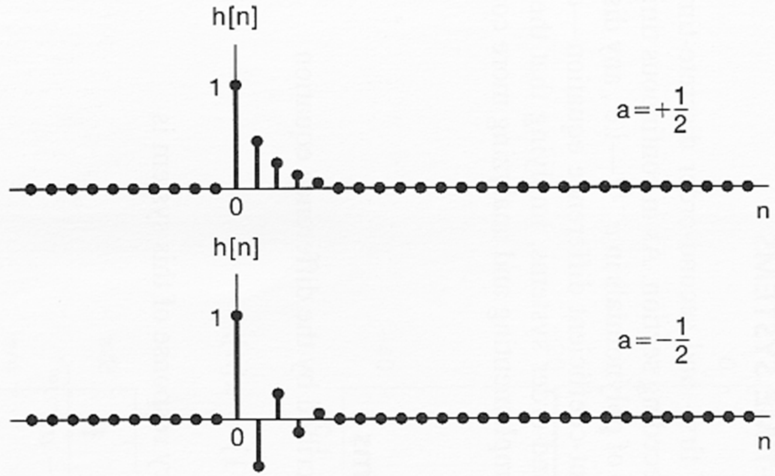
$$\Rightarrow\quad h[n] = a^n\ u[n]$$

$$\Rightarrow\quad s[n] = h[n] * u[n] = \frac{1 - a^{n+1}}{1 - a}\ u[n]$$

- **Impulse Response of First-Order DT Systems:**

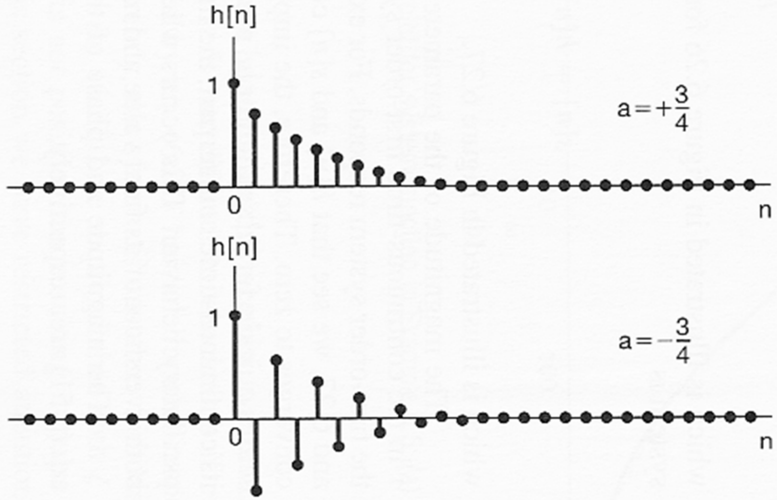


(a)

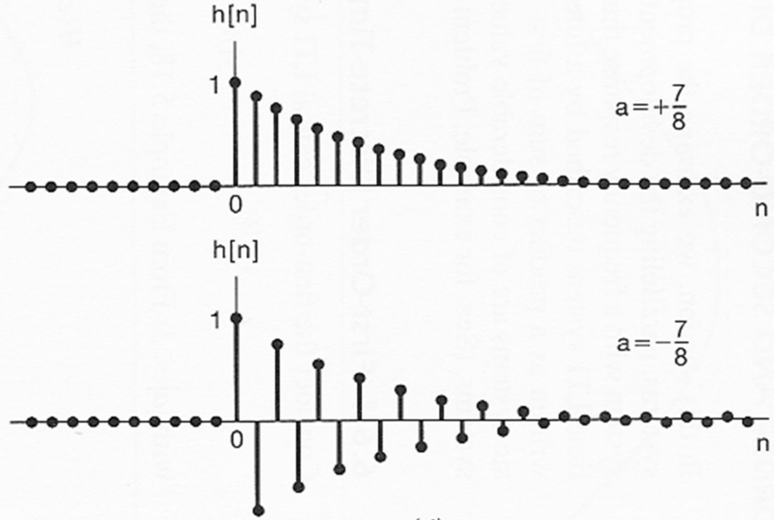


(b)

$$h[n] = a^n \, u[n]$$

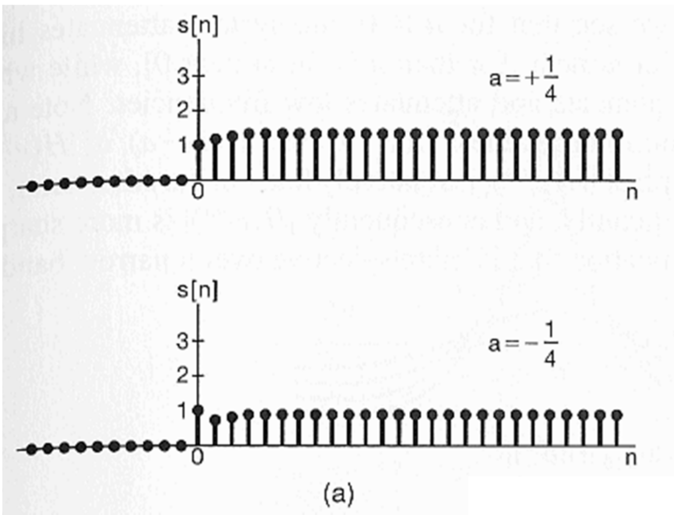


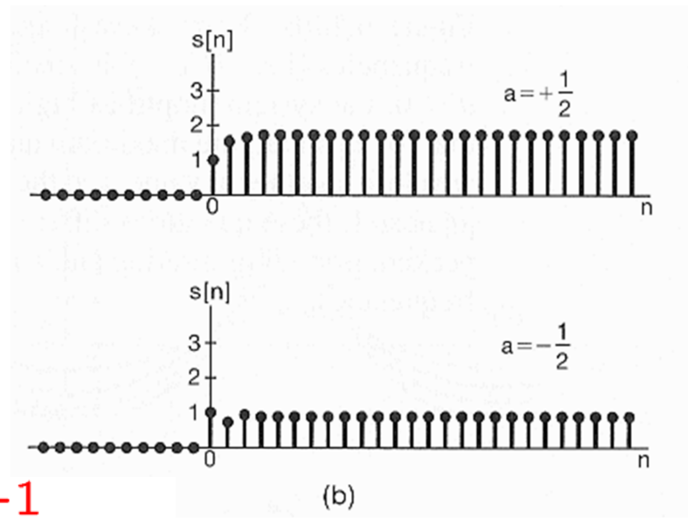
(c)



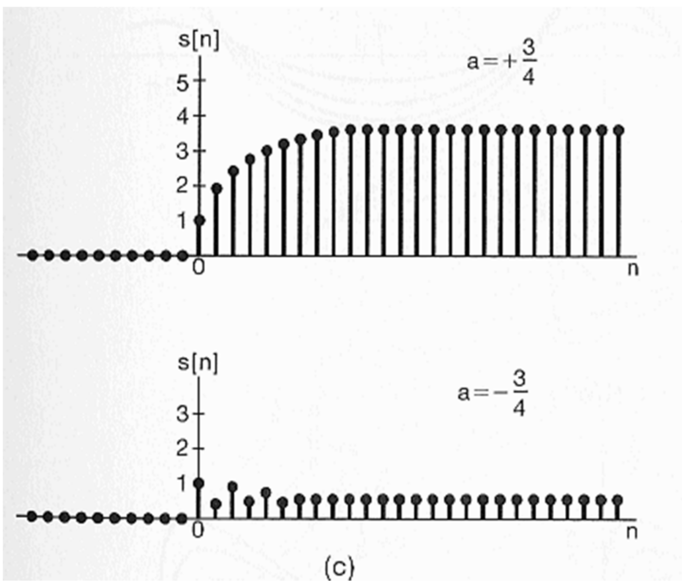
(d)

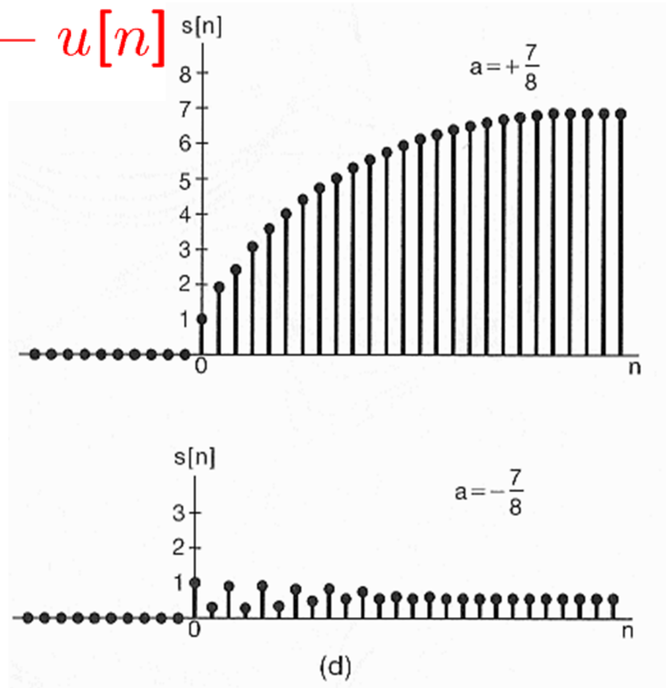
▪ Step Response of First-Order DT Systems:





$$s[n] = \frac{1 - a^{n+1}}{1 - a} u[n]$$





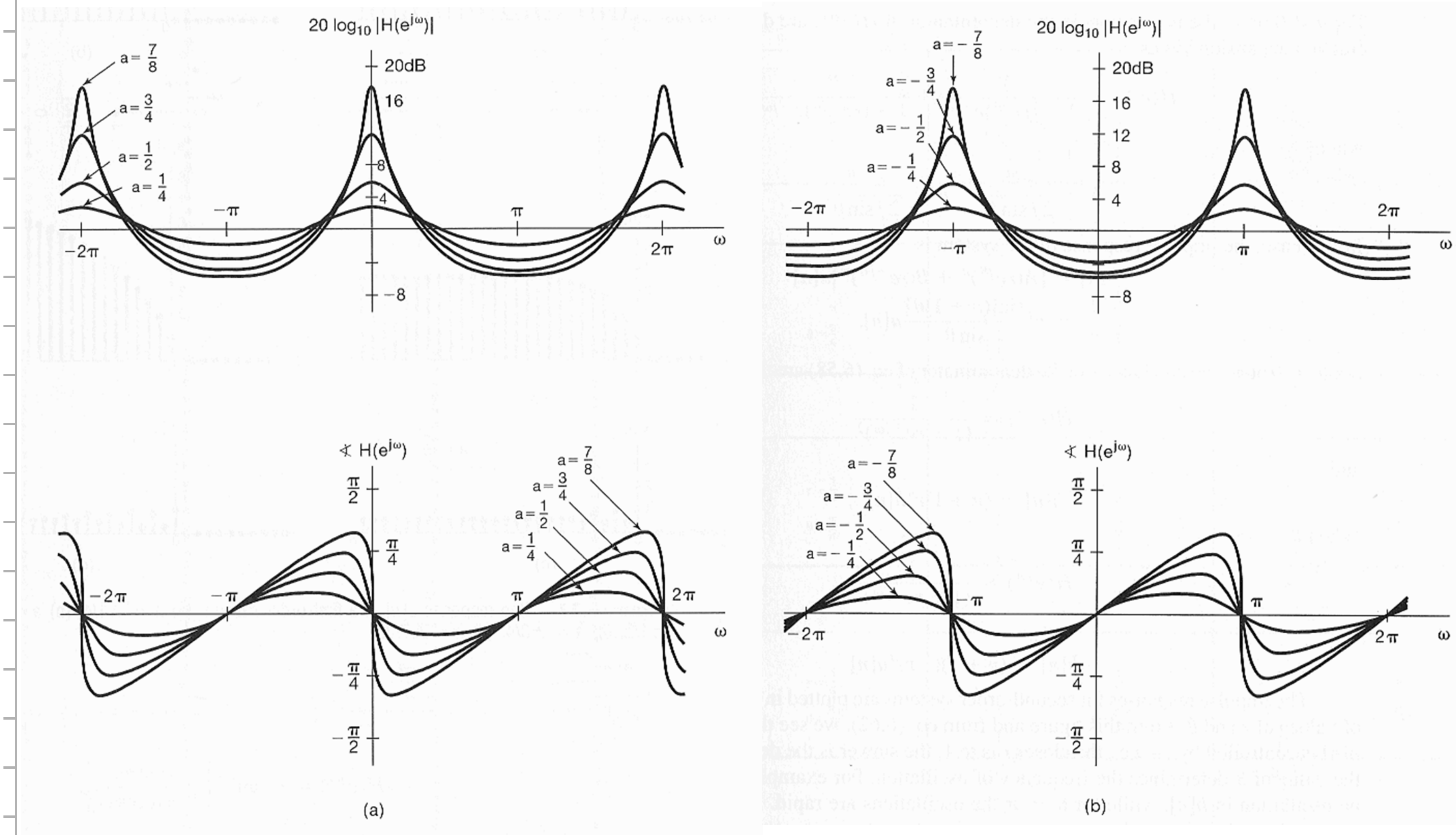
Magnitude & Phase of Frequency Response:

$$H(e^{jw}) = \frac{1}{1 - ae^{-jw}} = \frac{1}{1 - a\cos w + ja\sin w}$$

$$|H(e^{jw})| = \frac{1}{\sqrt{1 + a^2 - 2a\cos w}}$$

$$\sphericalangle H(e^{jw}) = -\tan^{-1}\left[\frac{a\sin w}{1 - a\cos w}\right]$$

- Magnitude & Phase of Frequency Response:



- Second-Order DT Systems: (p.465)

$$0 < r < 1 \text{ and } 0 \leq \theta \leq \pi$$

$$y[n] - 2r\cos(\theta)\, y[n-1] + r^2\, y[n-2] = x[n]$$

$$\Rightarrow \quad H(e^{jw}) = \frac{1}{1 - 2r\cos(\theta)\, e^{-jw} + r^2\, e^{-j2w}}$$

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$= \frac{1}{[1 - (re^{j\theta})e^{-jw}]\ [1 - (re^{-j\theta})e^{-jw}]}$$

Impulse Response of 2nd-Order DT Systems:

- For $\theta = 0$:

$$\Rightarrow \; H(e^{jw}) = \frac{1}{(1 - r\, e^{-jw})^2} \qquad \Rightarrow \; h[n] = (n+1)\,(r)^n\,u[n]$$

- For $\theta = \pi$:

$$\Rightarrow \; H(e^{jw}) = \frac{1}{(1 + r\, e^{-jw})^2} \qquad \Rightarrow \; h[n] = (n+1)\,(-r)^n\,u[n]$$

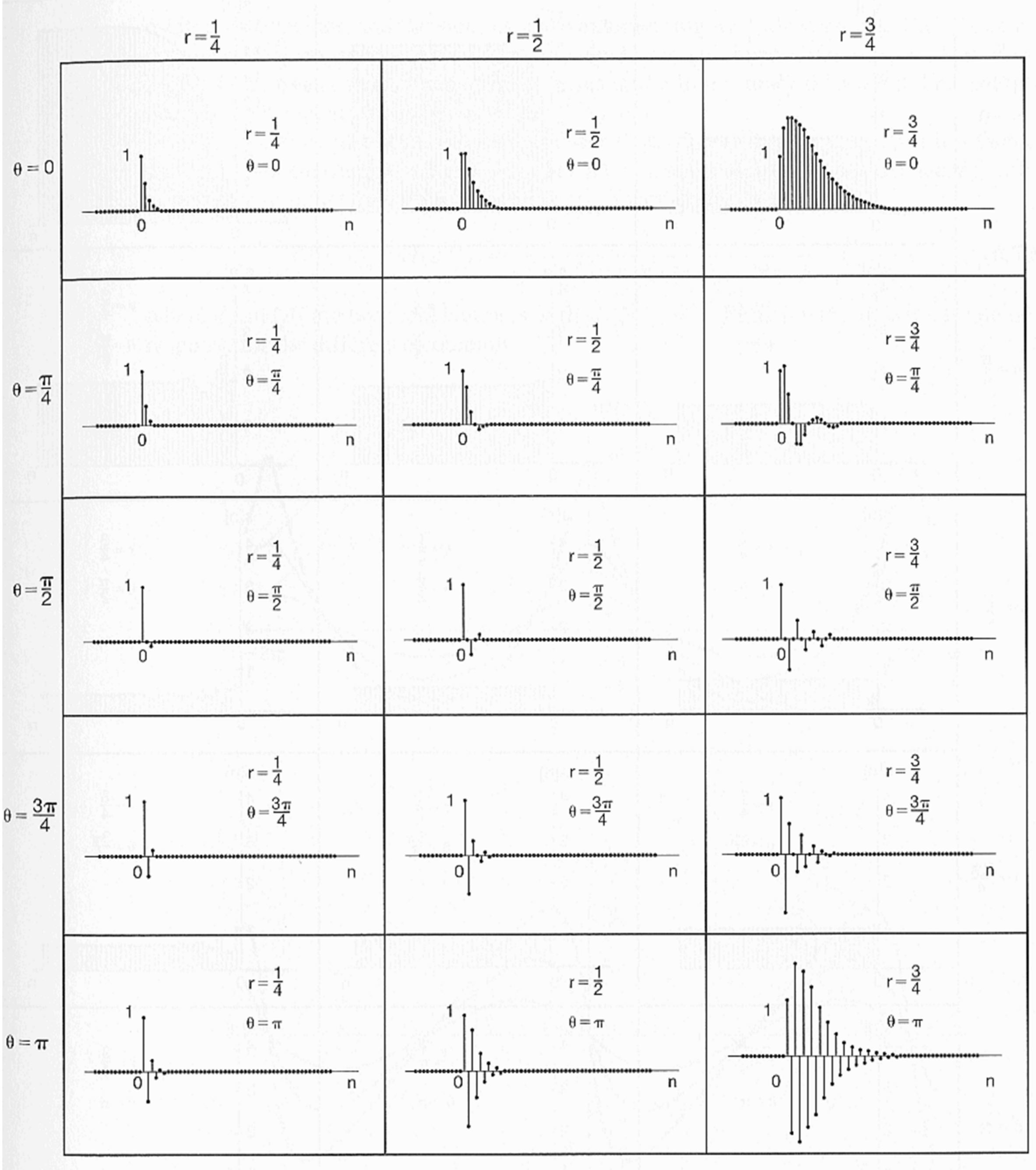
- For $\theta \neq 0$ or π:

$$\Rightarrow \; H(e^{jw}) = \frac{A}{1 - (re^{j\theta})e^{-jw}} + \frac{B}{1 - (re^{-j\theta})e^{-jw}}$$

$$A = \frac{e^{j\theta}}{2j\sin(\theta)}$$

$$B = -\frac{e^{-j\theta}}{2j\sin(\theta)}$$

$$\Rightarrow \; h[n] = \left[A(re^{j\theta})^n + B(re^{-j\theta})^n\right]\; u[n] \quad = r^n \frac{\sin[(n+1)\theta]}{\sin(\theta)} u[n]$$



$$h[n] = (n+1)\ (r)^n\ u[n]$$

$$r^n \frac{\sin[(n+1)\theta]}{\sin(\theta)} u[n]$$

$$h[n] = (n+1)\ (-r)^n\ u[n]$$

Step Response of 2nd-Order DT Systems:

- For $\theta = 0$:

$$s[n] = \left[\frac{1}{(r-1)^2} - \frac{r}{(r-1)^2} r^n + \frac{r}{r-1}(n+1) r^n\right] u[n]$$

- For $\theta = \pi$:

$$s[n] = \left[\frac{1}{(r+1)^2} + \frac{r}{(r+1)^2}(-r)^n + \frac{r}{r+1}(n+1)(-r)^n\right] u[n]$$

- For $\theta \neq 0$ or π:

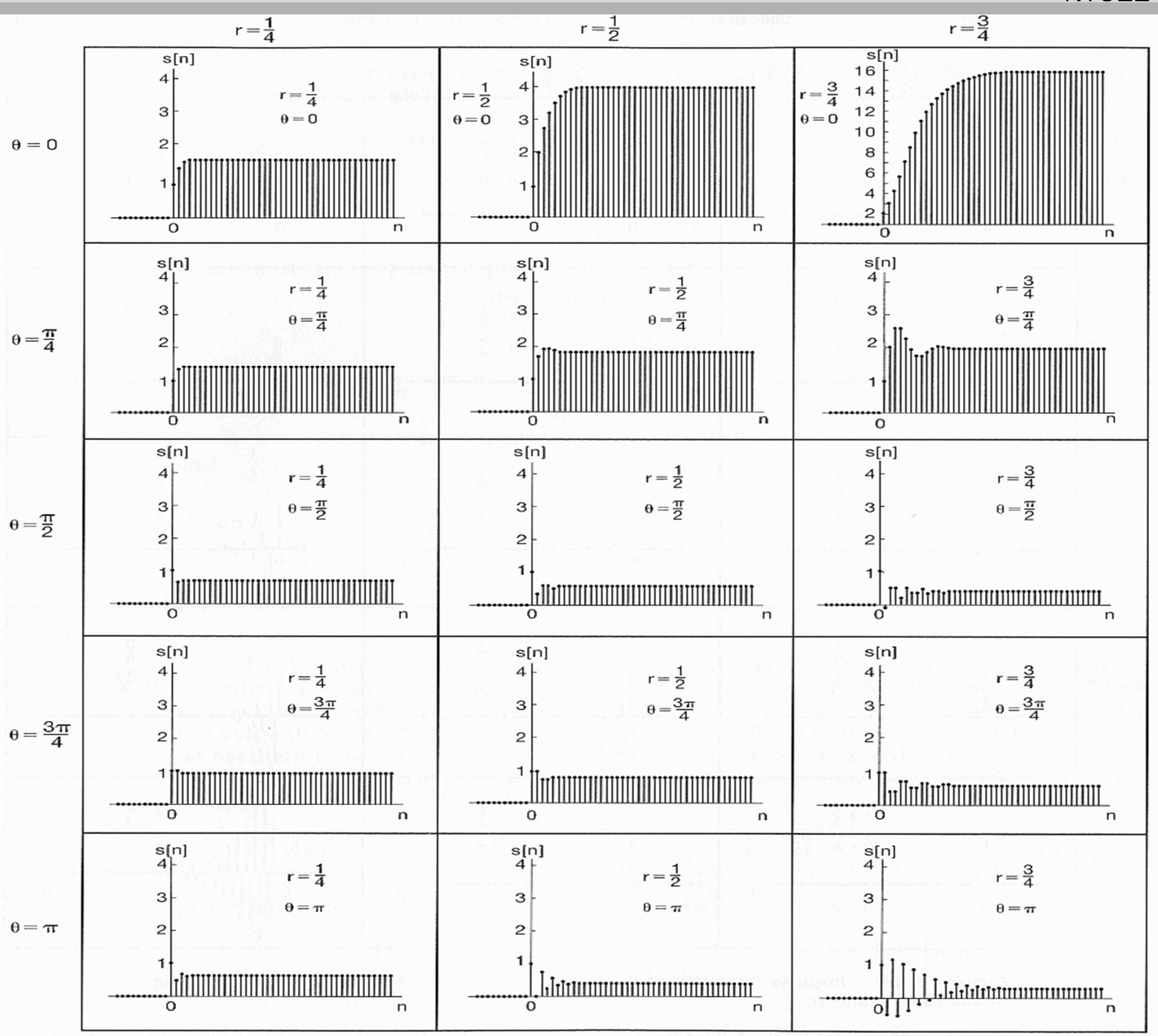
$$A = \frac{e^{j\theta}}{2j\sin(\theta)}$$

$$B = -\frac{e^{-j\theta}}{2j\sin(\theta)}$$

$$s[n] = h[n] * u[n]$$

$$= \left[A\left(\frac{1-(re^{j\theta})^{n+1}}{1-re^{j\theta}}\right) + B\left(\frac{1-(re^{-j\theta})^{n+1}}{1-re^{-j\theta}}\right)\right] u[n]$$

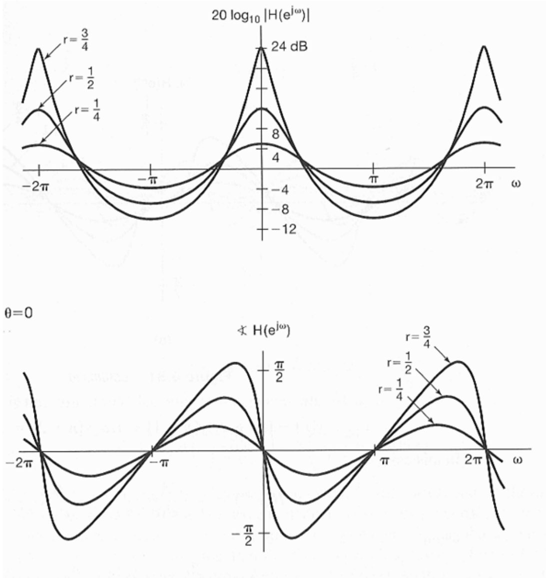
First-Order & Second-Order DT Systems



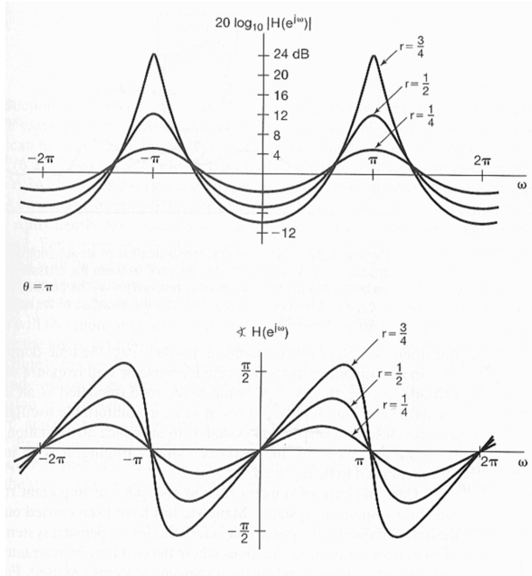
* Note: The plot for $r=\frac{3}{4}$, $\theta=0$ has a different scale from the others.

- Magnitude & Phase of Frequency Response:

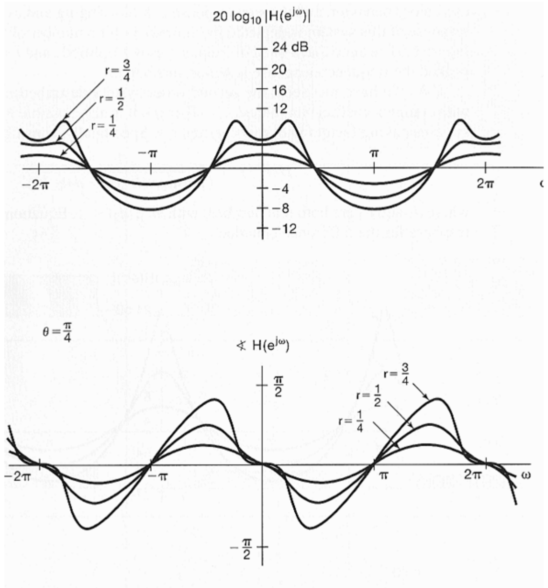
$$H(e^{jw}) = \frac{1}{1 - 2r\cos(\theta)\, e^{-jw} + r^2\, e^{-j2w}}$$



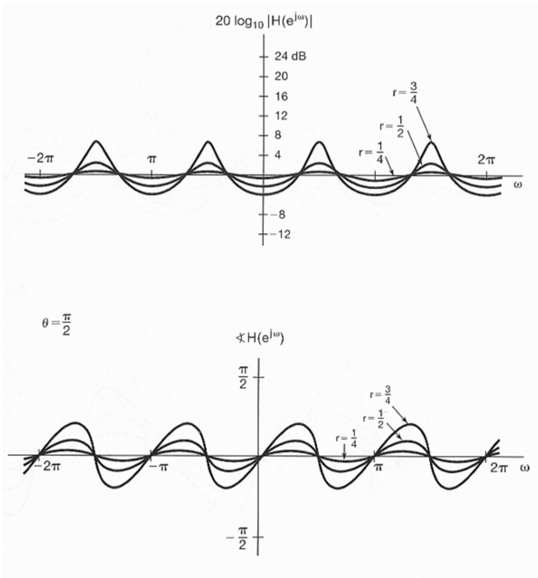
(a)



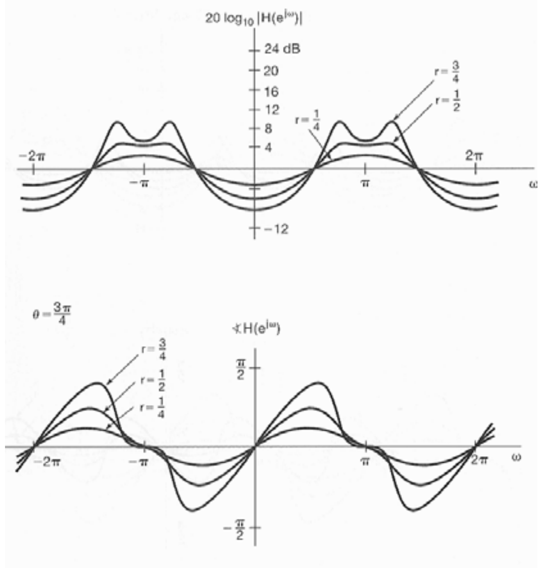
(e)



(b)



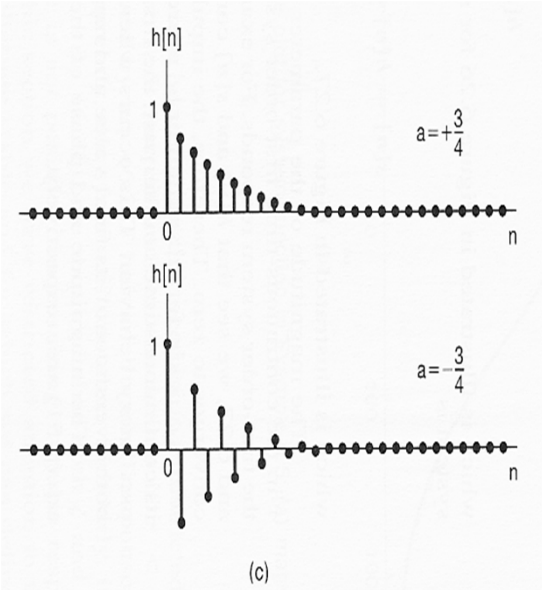
(c)

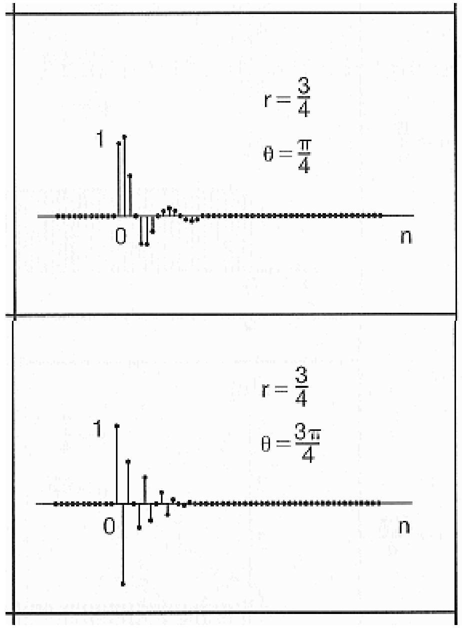


(d)

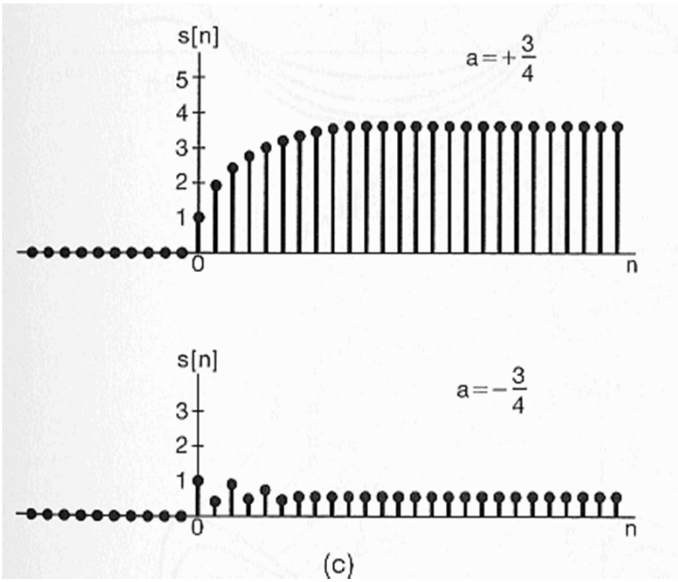
First-Order & Second-Order DT Systems

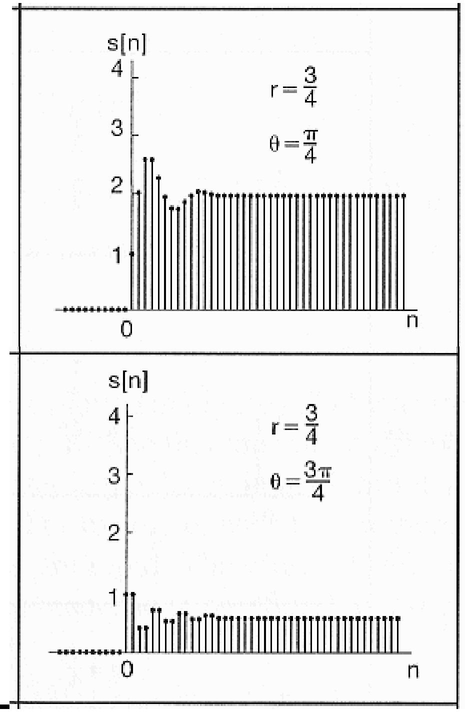
$h[n]$





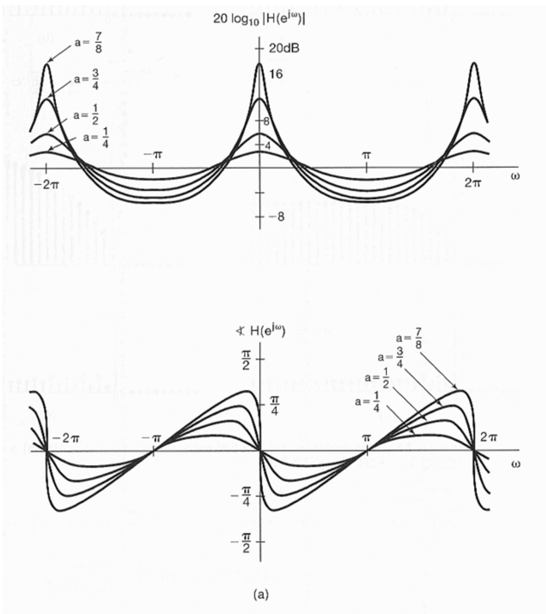
$s[n]$



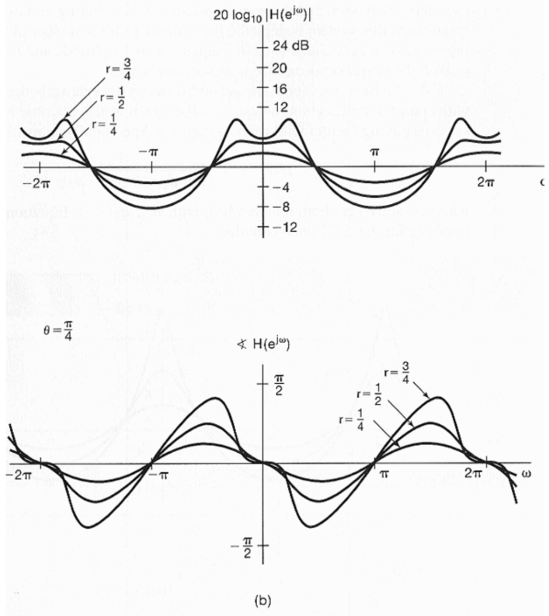


$20\log_{10}|H(e^{jw})|$

$\sphericalangle H(e^{jw})$



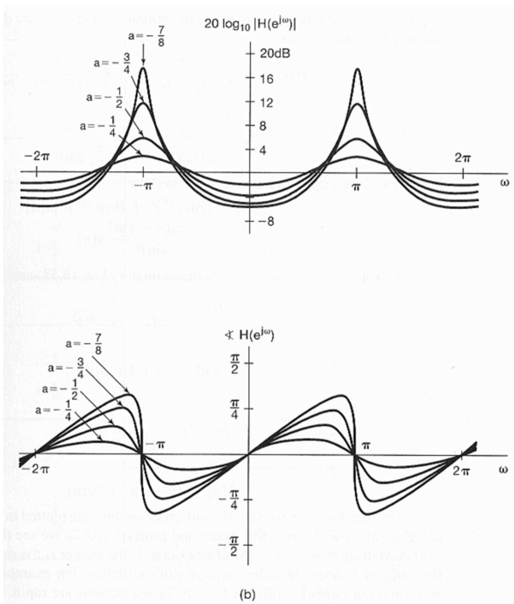
(a)



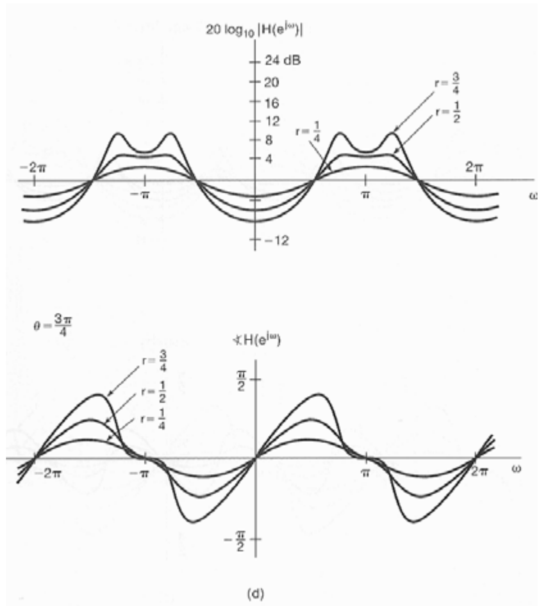
(b)

$20\log_{10}|H(e^{jw})|$

$\sphericalangle H(e^{jw})$



(b)



(d)

- The Magnitude-Phase Representation of the Fourier Transform (p.423)
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
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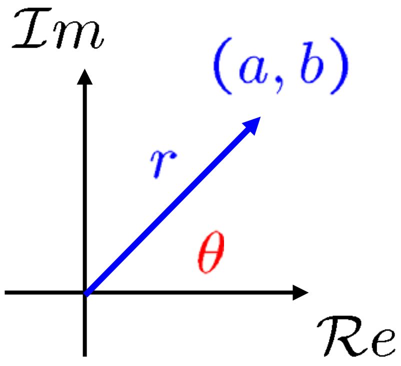
▪ Magnitude & Phase Representation:



$$a + jb \;\Rightarrow\; \begin{cases} r = \sqrt{a^2+b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = re^{j\theta}$$

$$|a+jb| = \sqrt{a^2+b^2} \qquad \left|\frac{1}{a+jb}\right| = \frac{1}{\sqrt{a^2+b^2}}$$

$$\sphericalangle\, a+jb = \tan^{-1}\left(\frac{b}{a}\right) \qquad \sphericalangle\, \frac{1}{a+jb} = -\tan^{-1}\left(\frac{b}{a}\right)$$

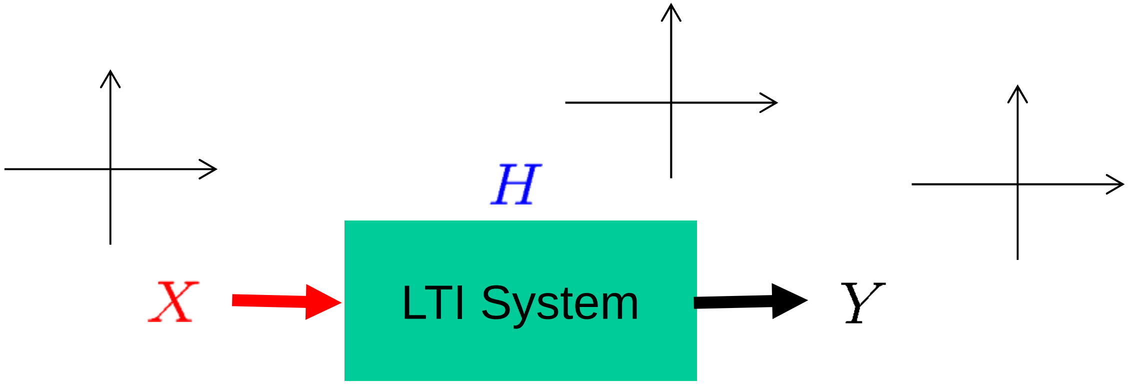
$$X(jw) = \mathcal{R}e\{X(jw)\} + j\ \mathcal{I}m\{X(jw)\} = \left|X(jw)\right| e^{j \sphericalangle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} = \left|X(e^{jw})\right| e^{j \sphericalangle X(e^{jw})}$$

$\left|X(jw)\right|$ or $\left|X(e^{jw})\right|$: magnitude

$\sphericalangle\, X(jw)$ or $\sphericalangle\, X(e^{jw})$: phase angle

Magnitude & Phase Distortions:



$$Y(jw) = X(jw)\ H(jw)$$

$$Y(e^{jw}) = X(e^{jw})\ H(e^{jw})$$

$$\Rightarrow\ |Y(jw)| = |X(jw)|\ |H(jw)|$$

$$|Y(e^{jw})| = |X(e^{jw})|\ |H(e^{jw})|$$

magnitude distortion

$$\Rightarrow\ \sphericalangle Y(jw) = \sphericalangle X(jw) + \sphericalangle H(jw)$$

$$\sphericalangle Y(e^{jw}) = \sphericalangle X(e^{jw}) + \sphericalangle H(e^{jw})$$

phase distortion

$|H(jw)|$ or $|H(e^{jw})|$: gain of the system

$\sphericalangle H(jw)$ or $\sphericalangle H(e^{jw})$: phase shift of the system

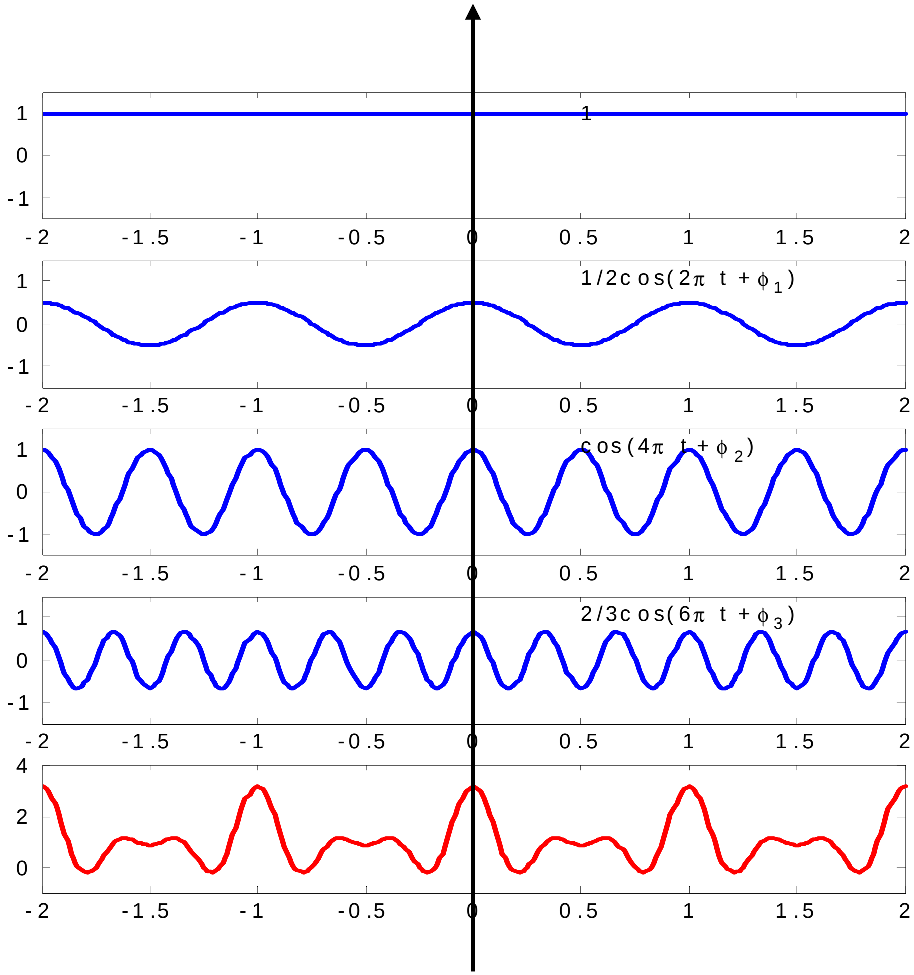
- **Magnitude & Phase Angle:** $A\cos(w_0 t+\phi) = \frac{A}{2}e^{j\phi}e^{jw_0 t} + \frac{A}{2}e^{-j\phi}e^{-jw_0 t}$

$$x(t) = 1\cos(0\pi t+\phi_0)+\frac{1}{2}\cos(2\pi t+\phi_1)+1\cos(4\pi t+\phi_2)+\frac{2}{3}\cos(6\pi t+\phi_3)$$

ss3-17

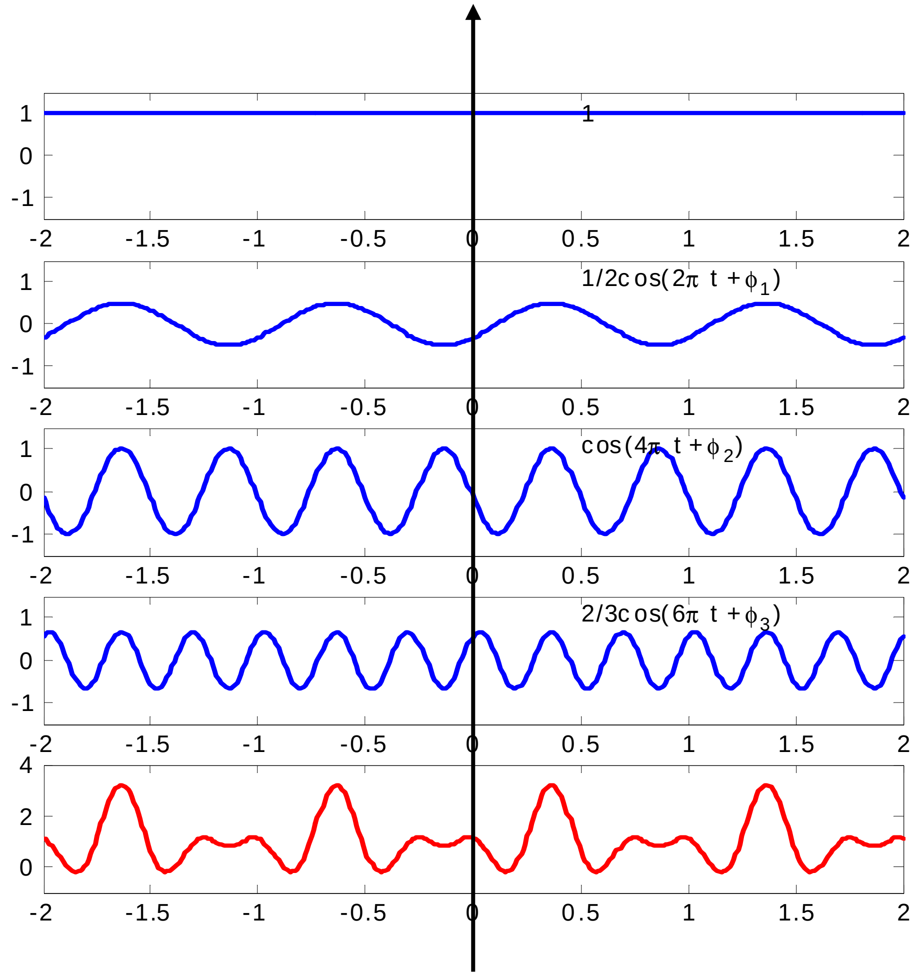
ss3-18

$$\begin{cases}\phi_1 = 0 \ (rad)\\ \phi_2 = 0 \ (rad)\\ \phi_3 = 0 \ (rad)\end{cases}$$



- **Magnitude & Phase Angle:** $A\cos(w_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{jw_0 t} + \frac{A}{2} e^{-j\phi} e^{-jw_0 t}$

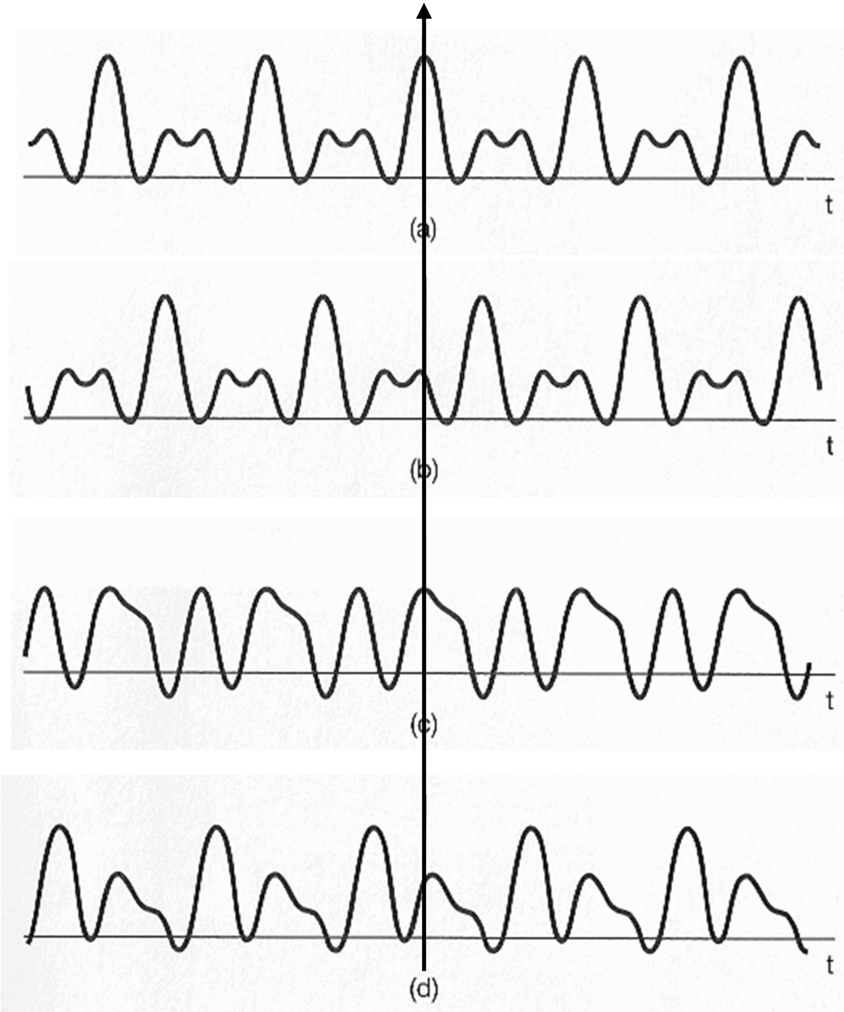
$$x(t) = 1\cos(0\pi t + \phi_0) + \frac{1}{2}\cos(2\pi t + \phi_1) + 1\cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$



$$\begin{cases} \phi_1 = 4 \ (rad) \\ \phi_2 = 8 \ (rad) \\ \phi_3 = 12 \ (rad) \end{cases}$$

- **Magnitude & Phase Angle:** $A\cos(w_0 t+\phi) = \frac{A}{2}\,e^{j\phi}\,e^{jw_0 t} + \frac{A}{2}\,e^{-j\phi}\,e^{-jw_0 t}$

$$x(t) = 1\cos(0\pi t+\phi_0)+\frac{1}{2}\cos(2\pi t+\phi_1)+1\cos(4\pi t+\phi_2)+\frac{2}{3}\cos(6\pi t+\phi_3)$$



$$\begin{cases}\phi_1 = 0 \ (rad)\\ \phi_2 = 0 \ (rad)\\ \phi_3 = 0 \ (rad)\end{cases}$$

$$\begin{cases}\phi_1 = 4 \ (rad)\\ \phi_2 = 8 \ (rad)\\ \phi_3 = 12 \ (rad)\end{cases}$$

$$\begin{cases}\phi_1 = 6 \ (rad)\\ \phi_2 = -2.7 \ (rad)\\ \phi_3 = 0.93 \ (rad)\end{cases}$$

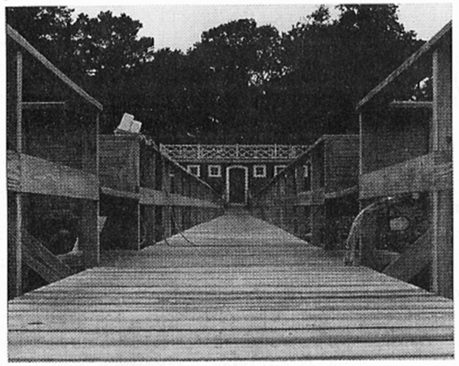
$$\begin{cases}\phi_1 = 1.2 \ (rad)\\ \phi_2 = 4.1 \ (rad)\\ \phi_3 = -7.02 \ (rad)\end{cases}$$

▪ Magnitude & Phase Angle in Images:

$$x(t_1, t_2) \xleftrightarrow{\mathcal{F}} X(jw_1, jw_2) \qquad \left|X(jw_1, jw_2)\right| e^{j \angle X(jw_1, jw_2)}$$

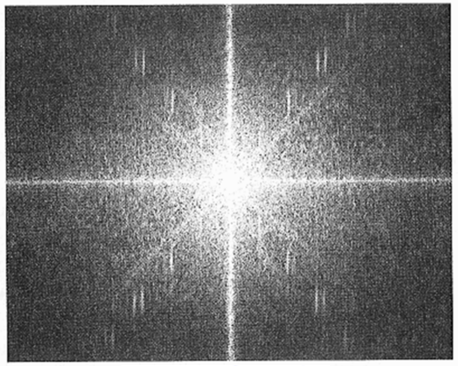
$$x[n_1, n_2] \xleftrightarrow{\mathcal{F}} X(e^{jw_1}, e^{jw_2}) \qquad \left|X(e^{jw_1}, e^{jw_2})\right| e^{j \angle X(e^{jw_1}, e^{jw_2})}$$

$x[n_1, n_2]$



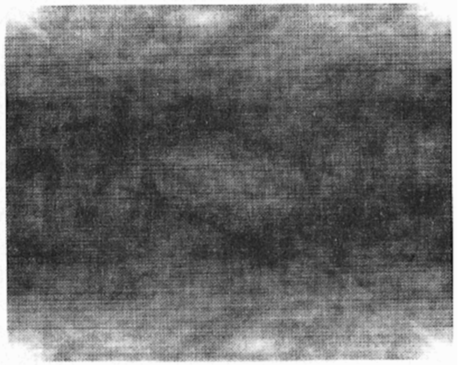
image

$|X|$



magnitude

$|X|$ $\quad \angle X$

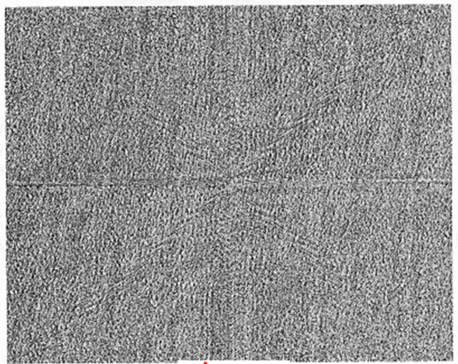


mag + zero pha

$y[n_1, n_2]$

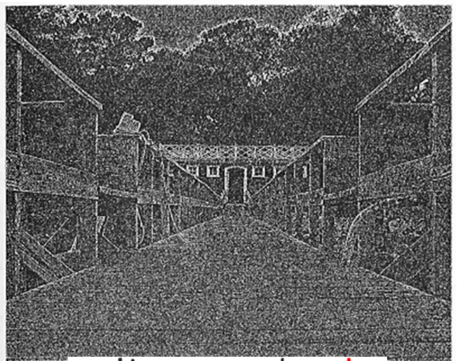
another image

$\angle X$



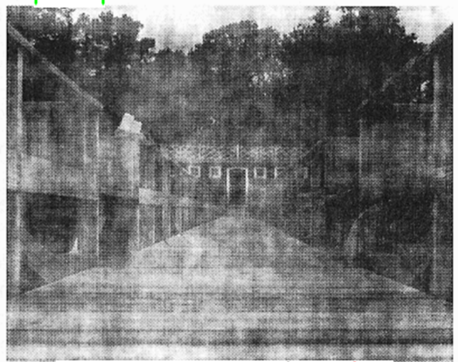
phase

$|X|$ $\quad \angle X$



unit mag + pha

$|Y|$ $\quad \angle X$



new mag + pha

- The Magnitude-Phase Representation of the Fourier Transform
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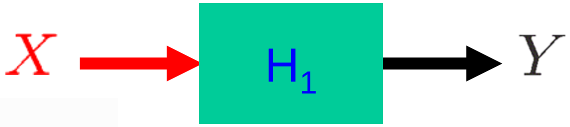
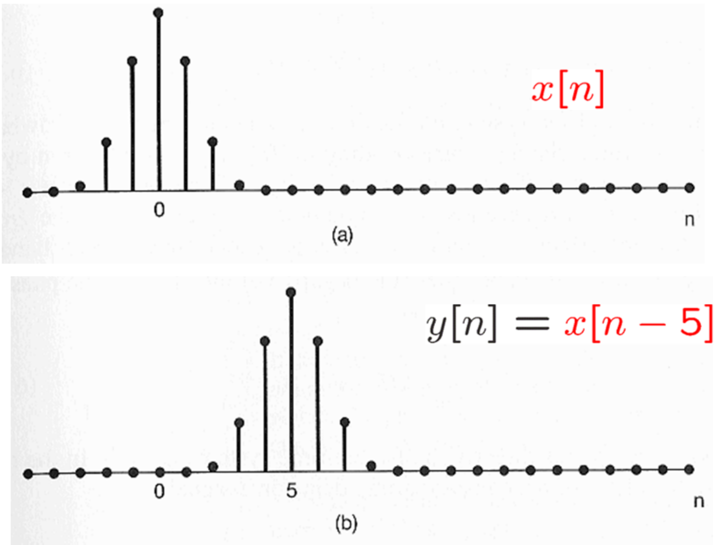
- Linear Phase:





$n_0 = 5$

- $H_1(e^{jw}) = e^{-jwn_0}$

$$\Rightarrow \begin{cases} |H_1(e^{jw})| & = 1 \\ \sphericalangle |H_1(e^{jw})| & = -wn_0 \end{cases}$$

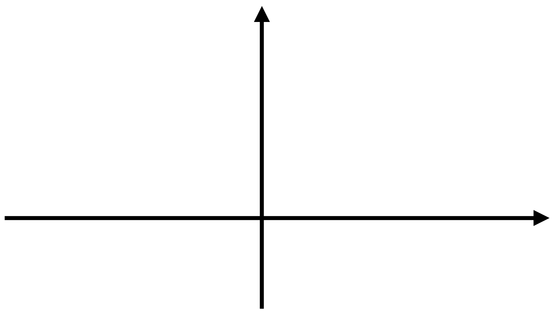
$$Y_1(e^{jw}) = H_1(e^{jw})\ X(e^{jw})$$

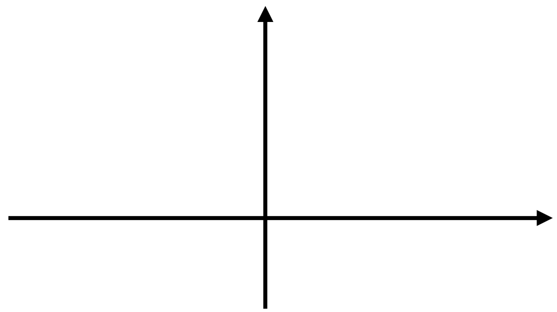
$$= e^{-jwn_0} \quad X(e^{jw})$$

$$\Rightarrow \ y[n] = x[n - n_0]$$

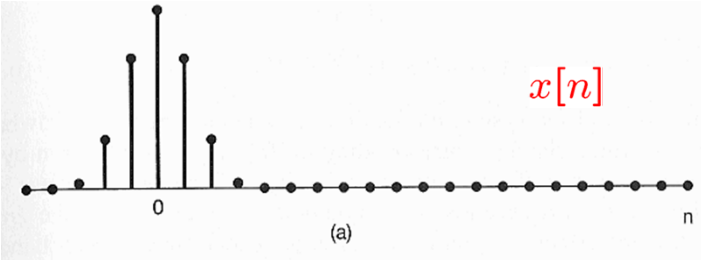
$\sphericalangle H_1(e^{jw}) =$

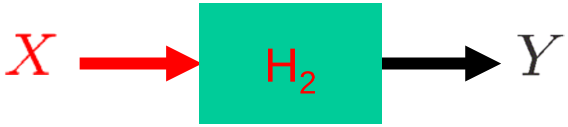
$$-\frac{d}{dw}\left\{\sphericalangle H_1(e^{jw})\right\} =$$



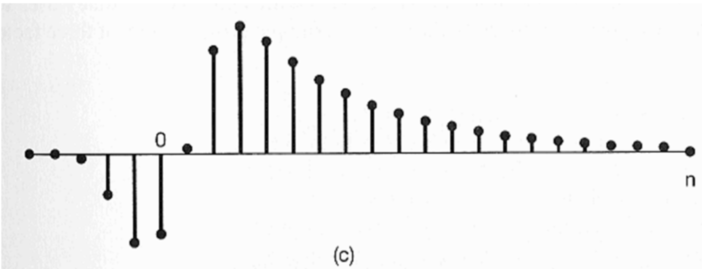


- Non-Linear Phase:





$$H_2(e^{jw}) = e^{jf(w)}$$

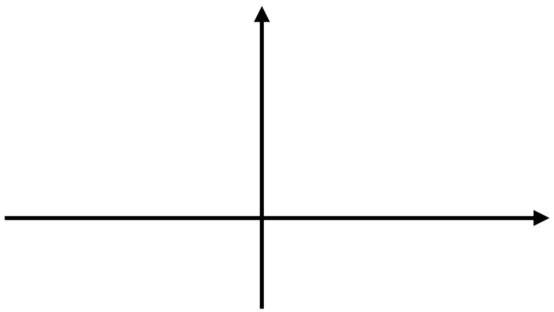


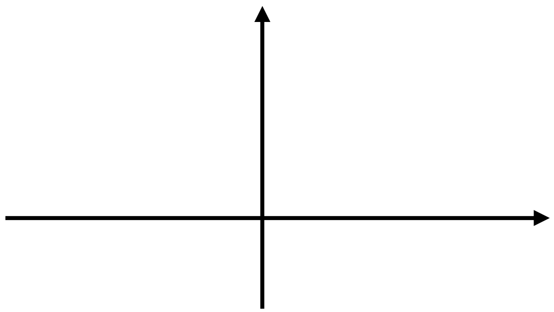
- $Y_2(e^{jw}) = H_2(e^{jw}) \, X(e^{jw})$

$$= e^{jf(w)} \quad X(e^{jw})$$

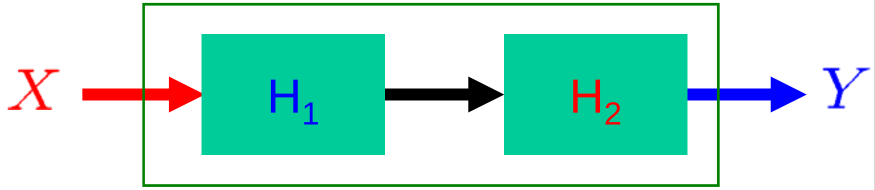
$\sphericalangle H_2(e^{jw}) =$

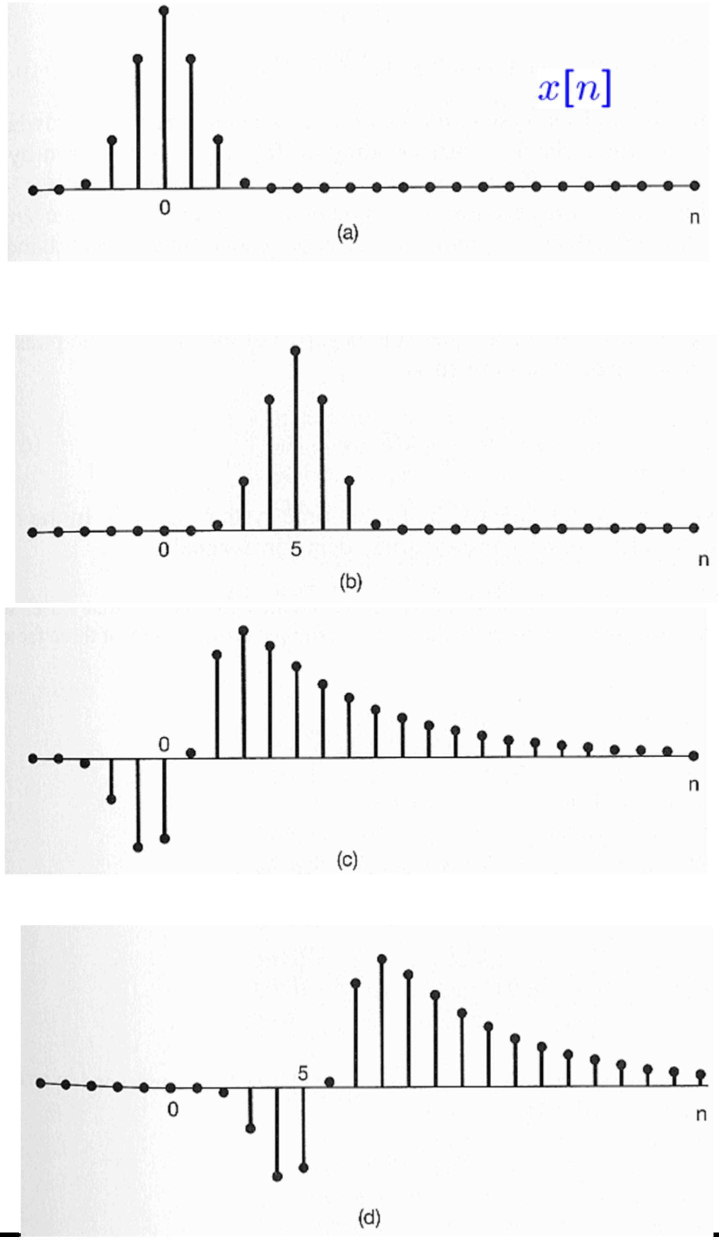
$-\dfrac{d}{dw}\left\{ \sphericalangle H_2(e^{jw}) \right\} =$





▪ Linear & Non-Linear Phase:





- $$Y_3(e^{jw}) = H_2(e^{jw})\ H_1(e^{jw})\ X(e^{jw})$$

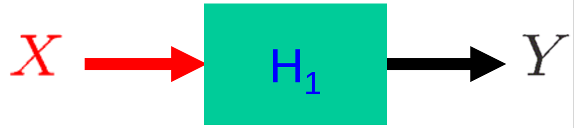
$$H_3(e^{jw}) = H_2(e^{jw}) H_1(e^{jw})$$

$$= H_2(e^{jw}) e^{-jwn_0}$$

$$= e^{j(f(w) - wn_0)}$$

$$-\frac{d}{dw}\left\{ \measuredangle H_3(e^{jw}) \right\} =$$

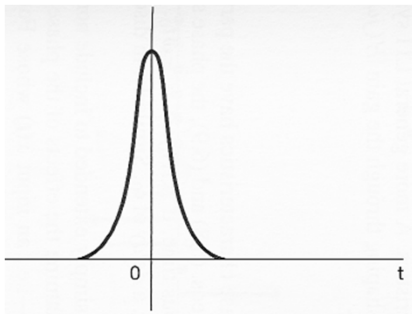
- **Linear Phase:**



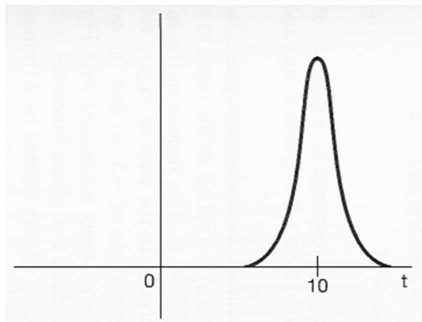
$$H_1(jw) = e^{-jwt_0}$$

$$\Rightarrow \begin{cases} |H_1(jw)| & = & 1 \\ \sphericalangle\, |H_1(jw)| & = & -wt_0 \end{cases}$$

$$\Rightarrow \; y(t) = x(t - t_0)$$



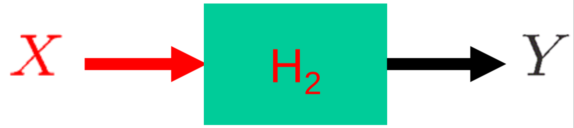
$x(t)$



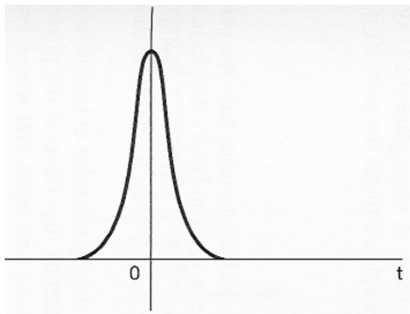
$y(t) = x(t-10)$

$t_0 = 10$

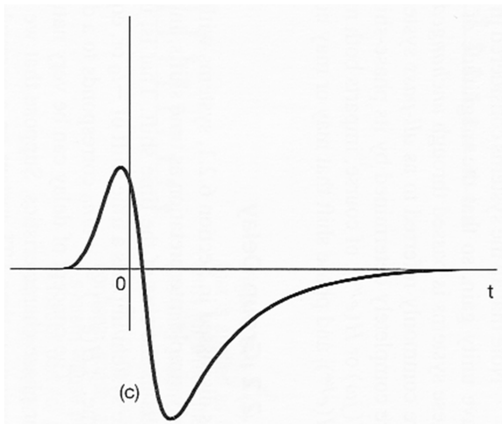
- Non-Linear Phase:



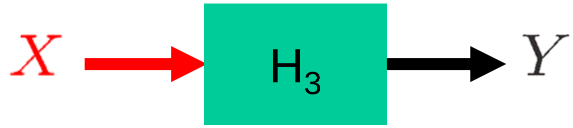
$$H_2(jw) = e^{jf(w)}$$



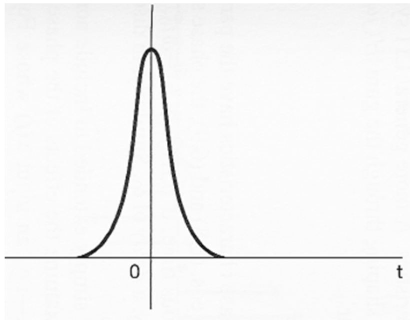
$x(t)$



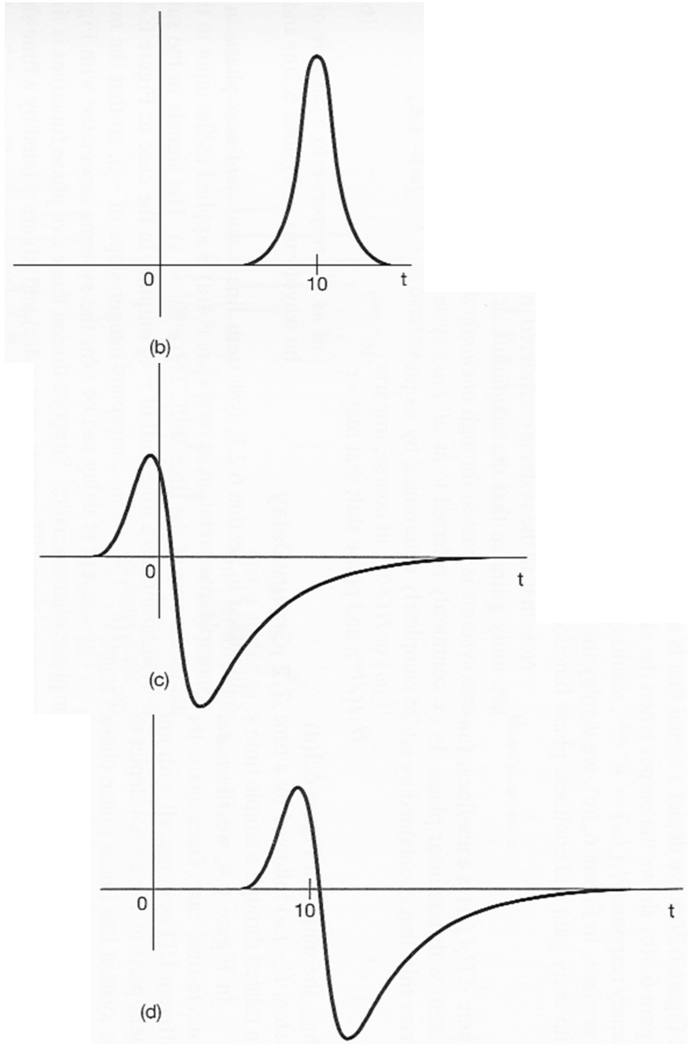
- Linear & Non-Linear Phase:



$$H_3(jw) = H_2(jw)\ H_1(jw) = H_2(jw)e^{-jwt_0} \quad = e^{j(f(w) - wt_0)}$$



$x(t)$



Group Delay & Phase:

- Linear Phase & Delay:

$$H_1(jw) = e^{-jwt_0} \quad \Rightarrow \quad y(t) = x(t - t_0) \quad \Rightarrow \quad \text{delay} = t_0$$

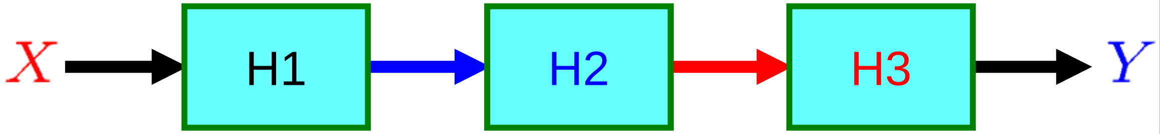
$$H_1(e^{jw}) = e^{-jwn_0} \quad \Rightarrow \quad y[n] = x[n - n_0] \quad \Rightarrow \quad \text{delay} = n_0$$

- Nonlinear Phase & Group Delay

$$H_2(jw) = e^{jf(w)} \quad \Rightarrow \quad \tau(w) = -\frac{d}{dw}\left\{ \measuredangle H_2(jw) \right\}$$

$$= -\frac{d}{dw}\left\{ f(w) \right\}$$

- Example 6.1:



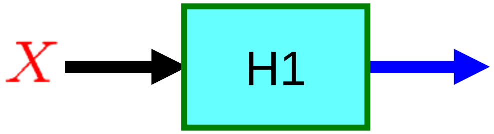
$$H(jw) = H_1(jw)\ H_2(jw)\ H_3(jw)$$

$$H_i(jw) = \frac{(w_i)^2 + (jw)^2 - 2\zeta_i w_i (jw)}{(w_i)^2 + (jw)^2 + 2\zeta_i w_i (jw)} = \frac{1 + (\frac{jw}{w_i})^2 - 2j\zeta_i(\frac{w}{w_i})}{1 + (\frac{jw}{w_i})^2 + 2j\zeta_i(\frac{w}{w_i})}$$

$$\Rightarrow \begin{cases} |H_i(jw)| &= 1 \\ \measuredangle H_i(jw) &= -\ 2\ \text{arctan}\left[\frac{2\zeta_i(\frac{w}{w_i})}{1-(\frac{w}{w_i})^2}\right] \end{cases}$$

$$\Rightarrow \begin{cases} |H(jw)| &= 1 \\ \measuredangle H(jw) &= \measuredangle H_1(jw) + \measuredangle H_2(jw) + \measuredangle H_3(jw) \end{cases}$$

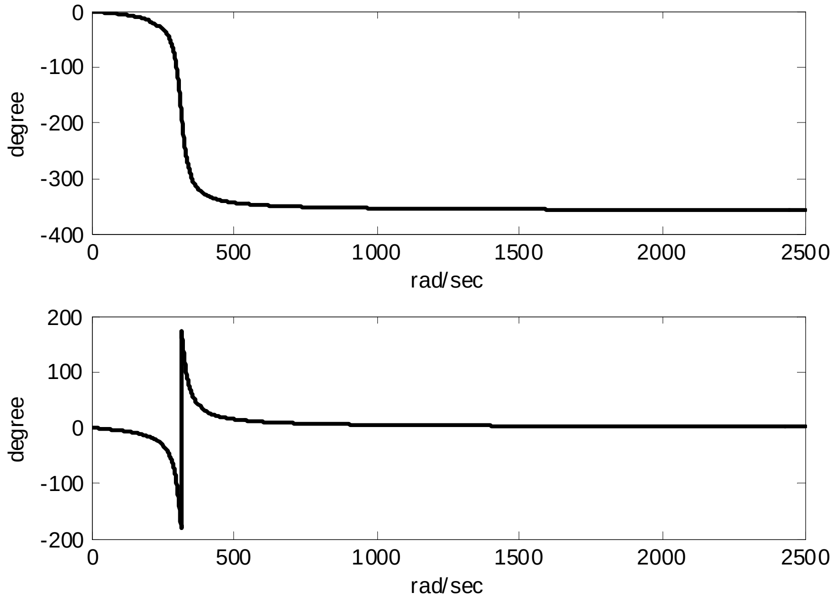
$$\Rightarrow\ \tau(w) = -\frac{d}{dw}\left\{\measuredangle H(jw)\right\}$$

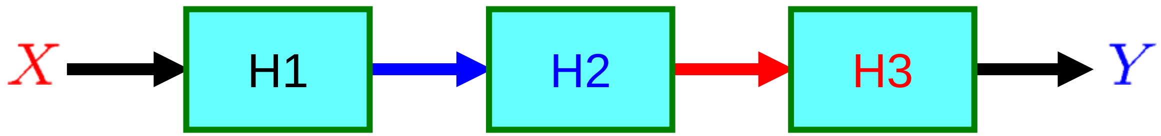


$$H_1(jw) = \frac{1 + (j\frac{w}{w_1})^2 - 2j\zeta_1(\frac{w}{w_1})}{1 + (j\frac{w}{w_1})^2 + 2j\zeta_1(\frac{w}{w_1})}$$

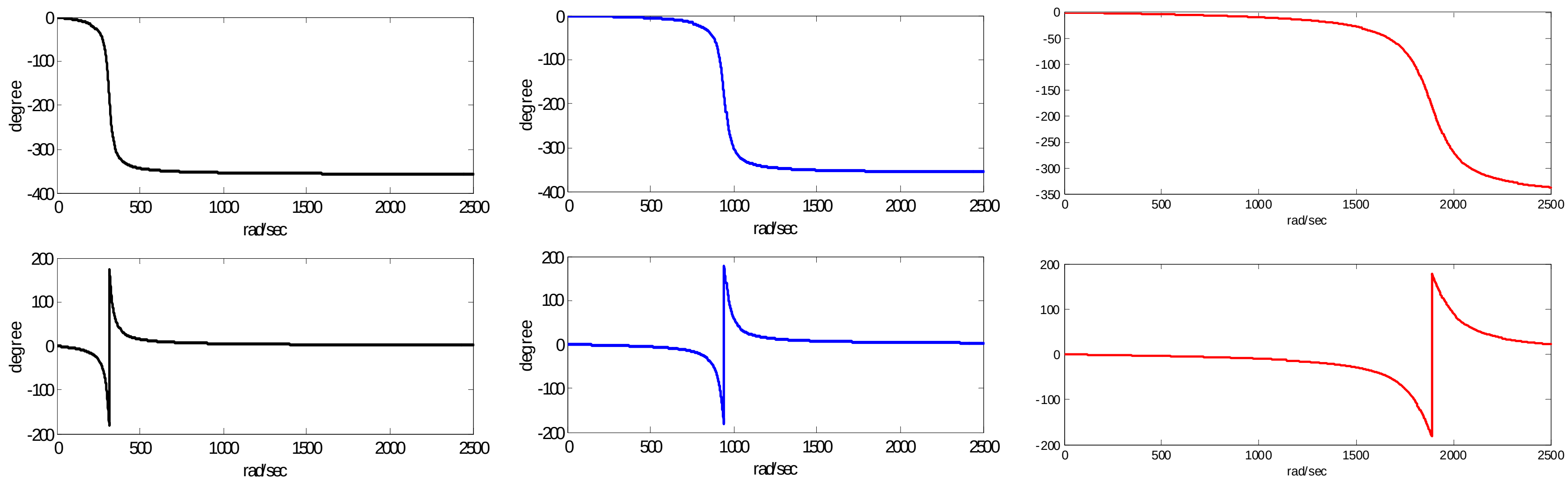
$$\begin{cases} w_1 & = & 315 & \text{rad/sec} \\ \zeta_1 & = & 0.066 & \\ f_1 & \approx & 50 & \text{Hz} \end{cases}$$

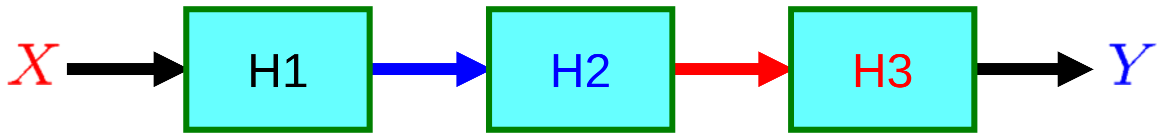
$$\Rightarrow \begin{cases} |H_1(jw)| & = & 1 \\ \measuredangle H_1(jw) & = & -2 \ \arctan \left[\frac{2\zeta_1(\frac{w}{w_1})}{1-(\frac{w}{w_1})^2} \right] \end{cases}$$





$$\begin{cases} w_1 = 315 \quad \text{rad/sec} \\ w_2 = 943 \quad \text{rad/sec} \\ w_3 = 1888 \quad \text{rad/sec} \end{cases} \qquad \begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

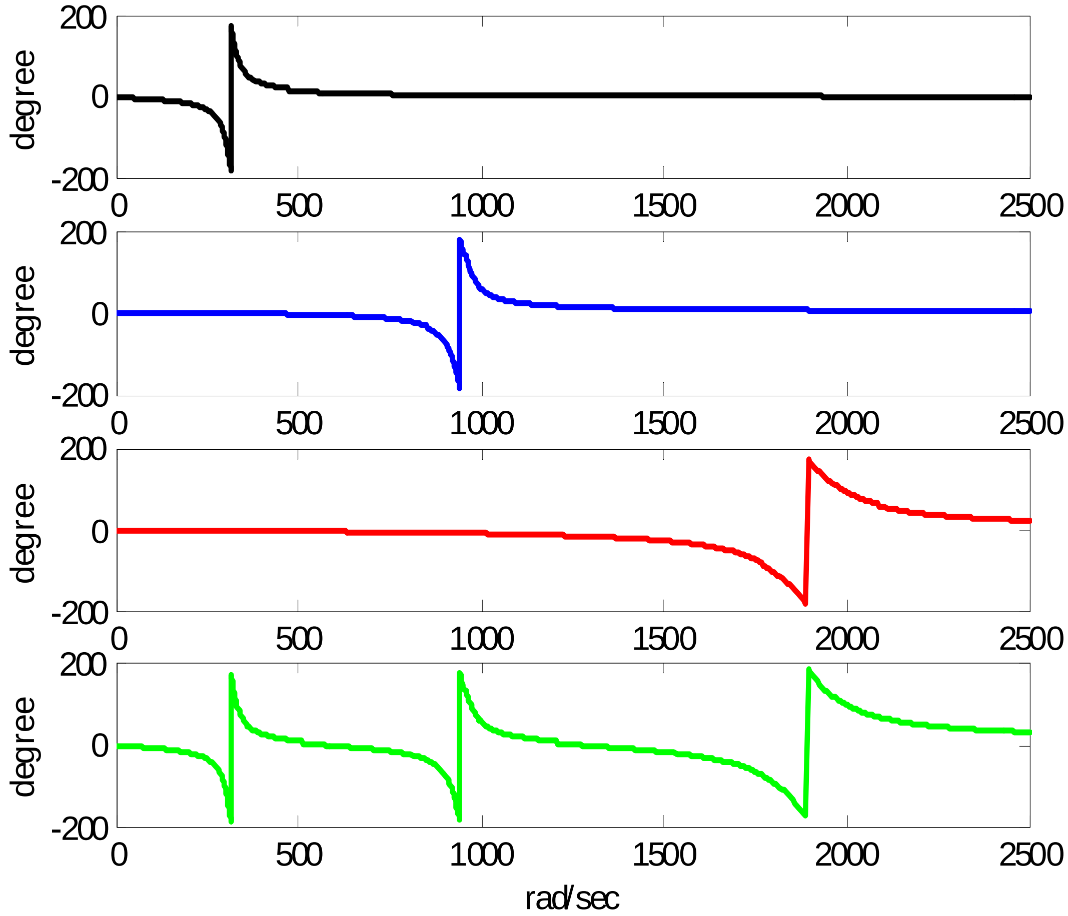




$$\begin{cases} |H(jw)| &= 1 \\ \measuredangle H(jw) &= \measuredangle H_1(jw) + \measuredangle H_2(jw) + \measuredangle H_3(jw) \end{cases}$$

$$\begin{cases} w_1 = 315 \quad \text{rad/sec} \\ w_2 = 943 \quad \text{rad/sec} \\ w_3 = 1888 \quad \text{rad/sec} \end{cases}$$

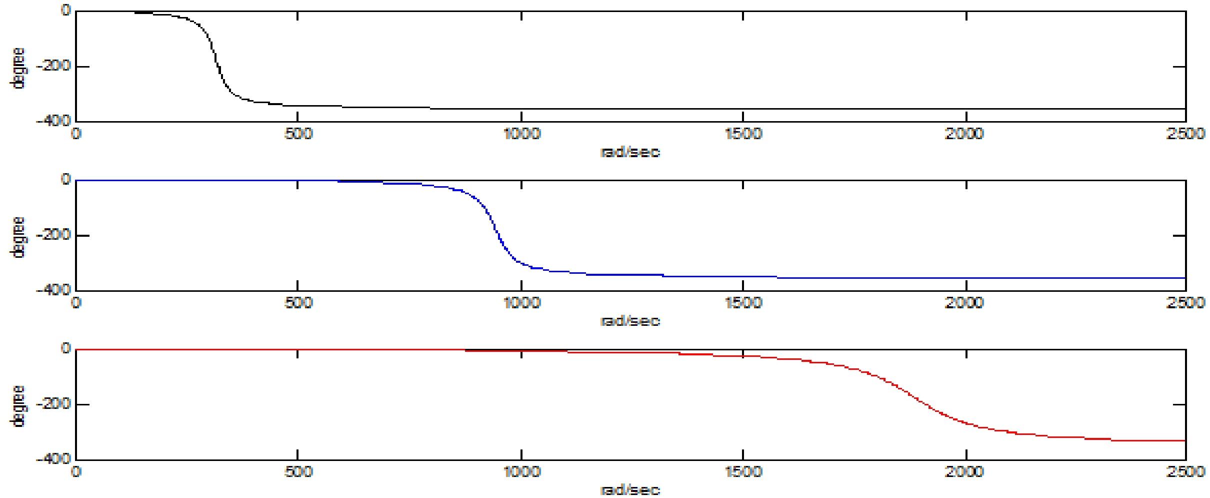
$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

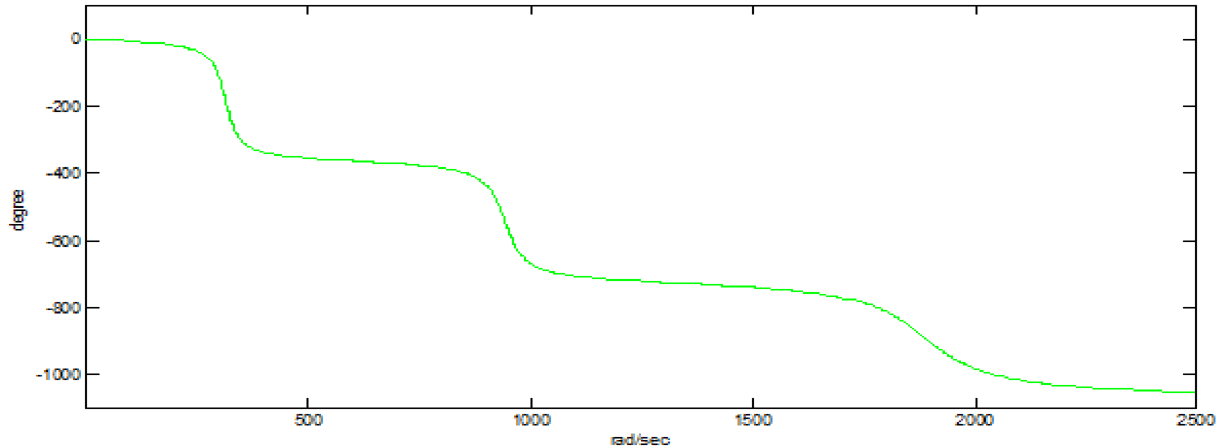


$$\begin{cases} |H(jw)| &= 1 \\ \sphericalangle H(jw) &= \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

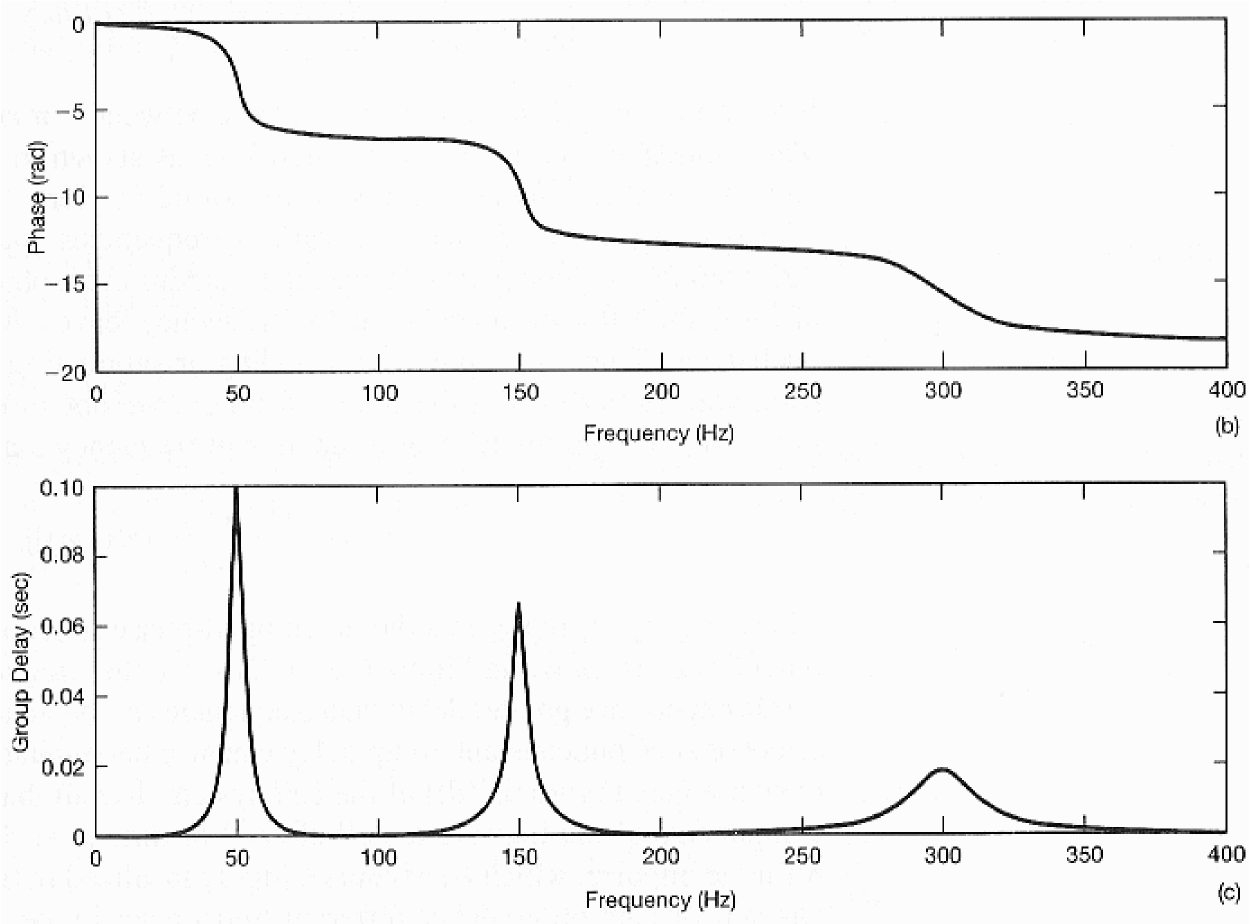
$$\begin{cases} w_1 &= 315 \quad \text{rad/sec} \\ w_2 &= 943 \quad \text{rad/sec} \\ w_3 &= 1888 \quad \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 &= 0.066 \\ \zeta_2 &= 0.033 \\ \zeta_3 &= 0.058 \end{cases}$$





$$\tau(w) = -\frac{d}{dw}\left\{ \sphericalangle\, H(jw) \right\}$$



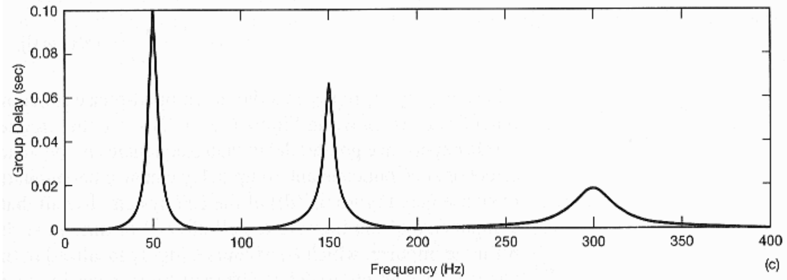
$$\begin{cases} w_1 & = & 315 & \text{rad/sec} \\ w_2 & = & 943 & \text{rad/sec} \\ w_3 & = & 1888 & \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 & = & 0.066 \\ \zeta_2 & = & 0.033 \\ \zeta_3 & = & 0.058 \end{cases}$$

$$w_i = 2\pi f_i$$

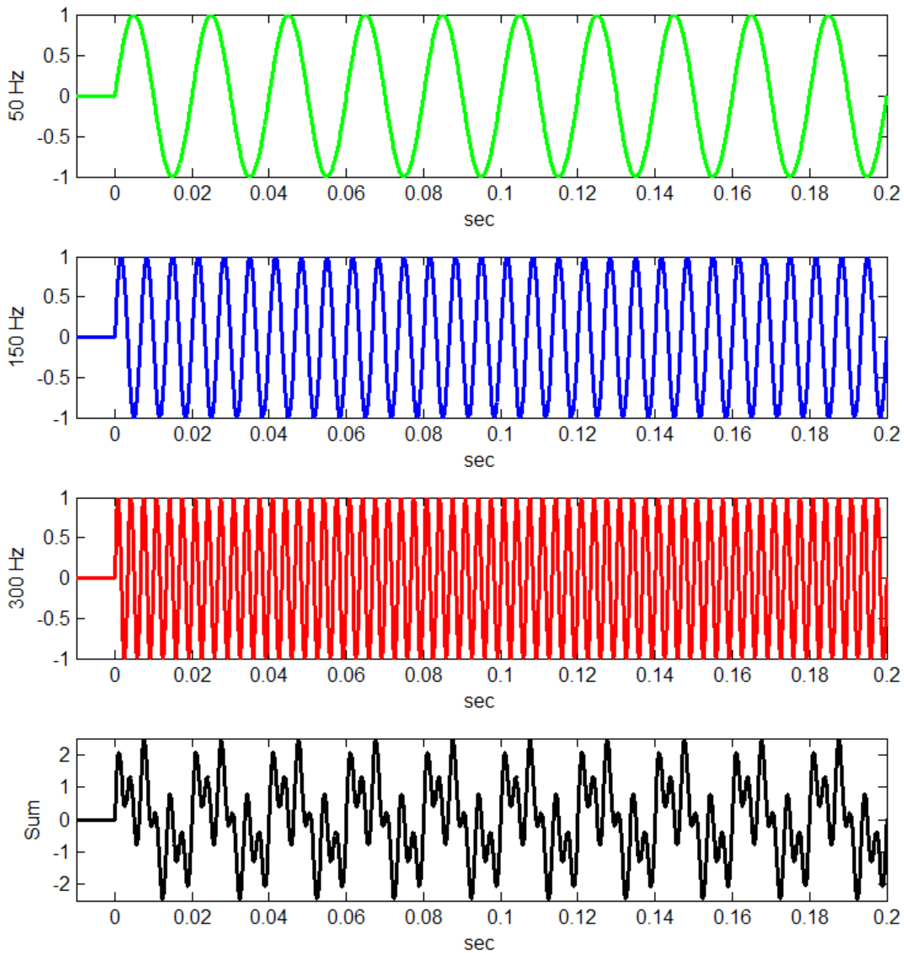
$$\begin{cases} f_1 & \approx & 50 & \text{Hz} \\ f_2 & \approx & 150 & \text{Hz} \\ f_3 & \approx & 300 & \text{Hz} \end{cases}$$

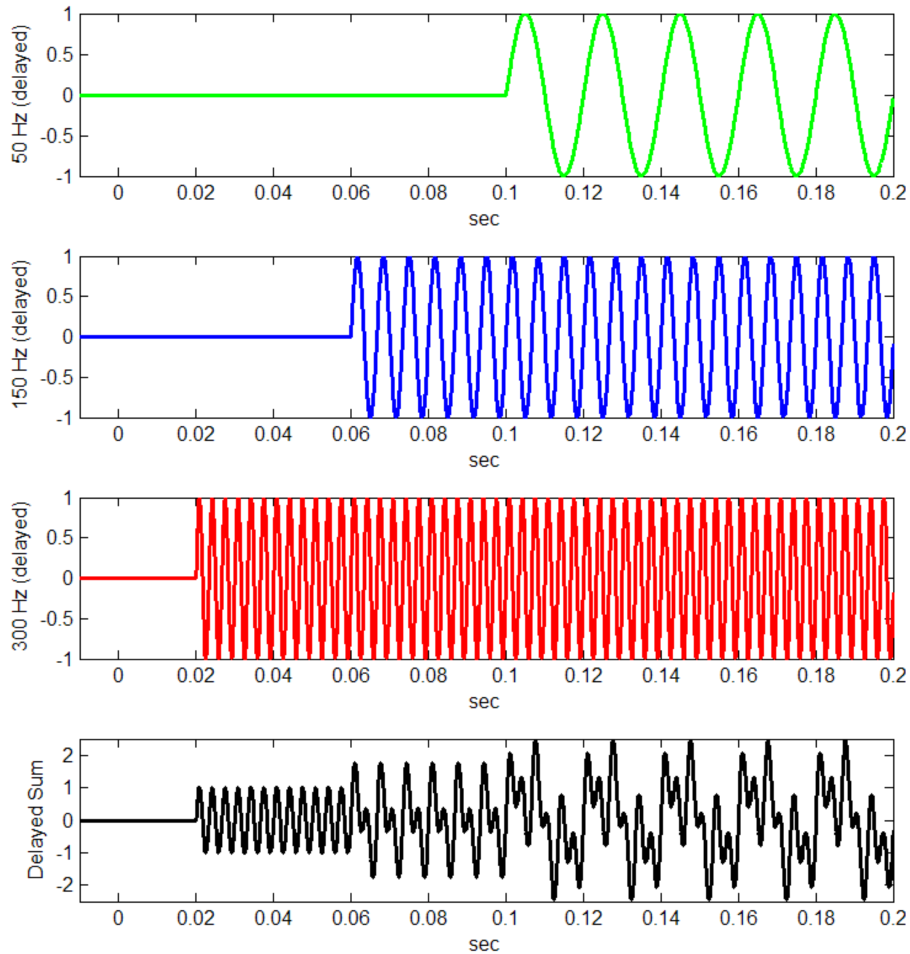
Magnitude-Phase Representation of Freq Resp of LTI Systems



$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

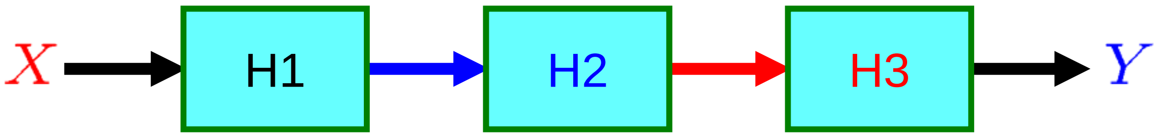
$$y(t) = y_1(t) + y_2(t) + y_3(t)$$



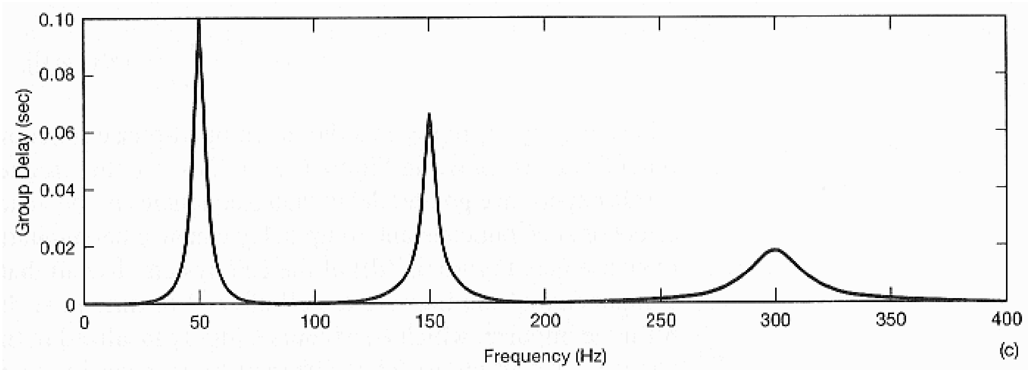


$$x(t) = \delta(t)$$

$$X(jw) = 1, \ \forall w$$



$$\begin{cases} |H(jw)| &= 1 \\ \sphericalangle H(jw) &= \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

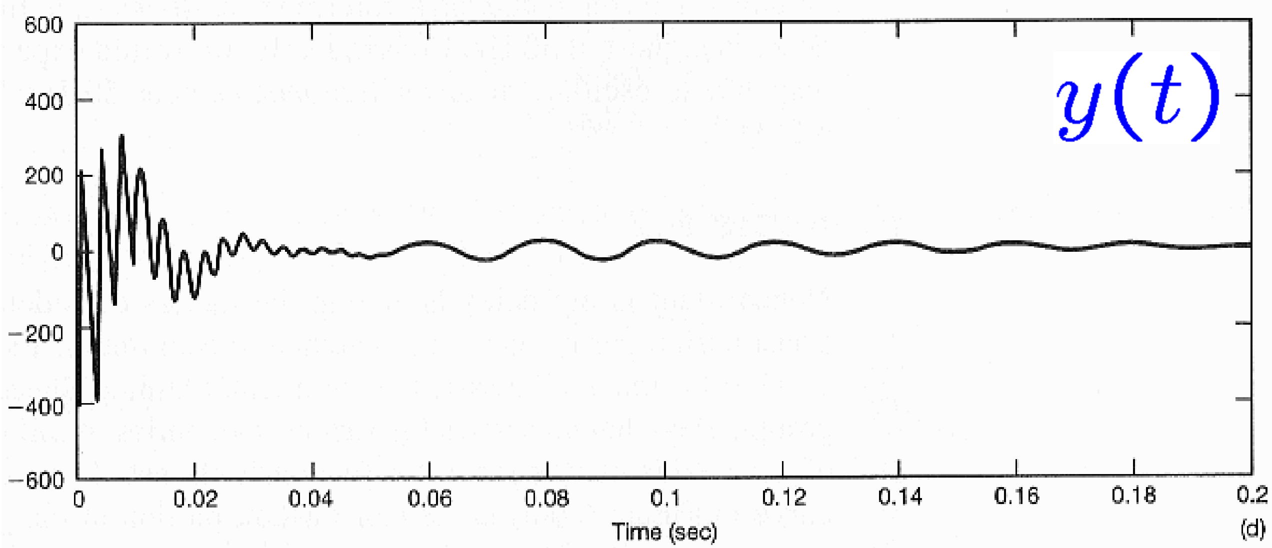


$$\begin{cases} w_1 &= 315 \quad \text{rad/sec} \\ w_2 &= 943 \quad \text{rad/sec} \\ w_3 &= 1888 \quad \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 &= 0.066 \\ \zeta_2 &= 0.033 \\ \zeta_3 &= 0.058 \end{cases}$$

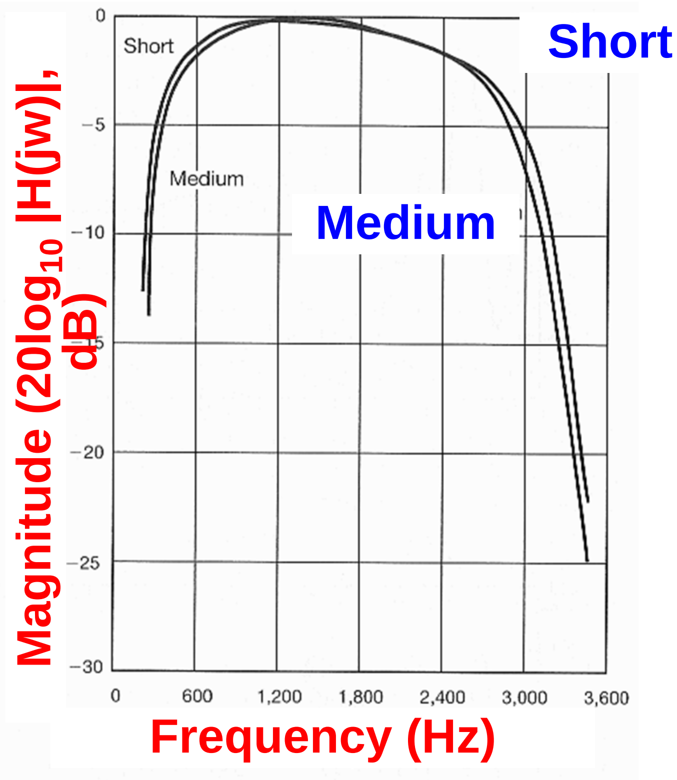
$$w_i = 2\pi f_i$$

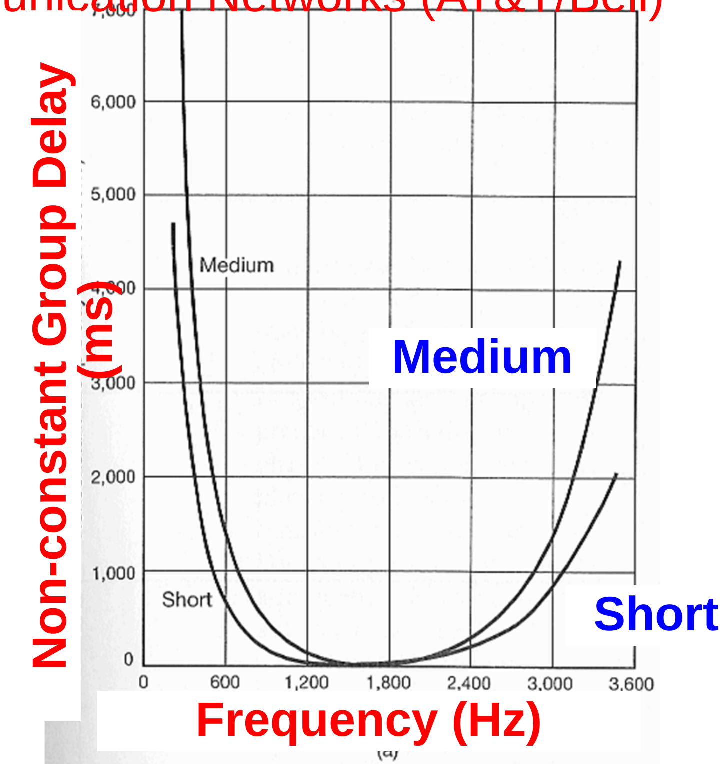
$$\begin{cases} f_1 &\approx 50 \quad \text{Hz} \\ f_2 &\approx 150 \quad \text{Hz} \\ f_3 &\approx 300 \quad \text{Hz} \end{cases}$$

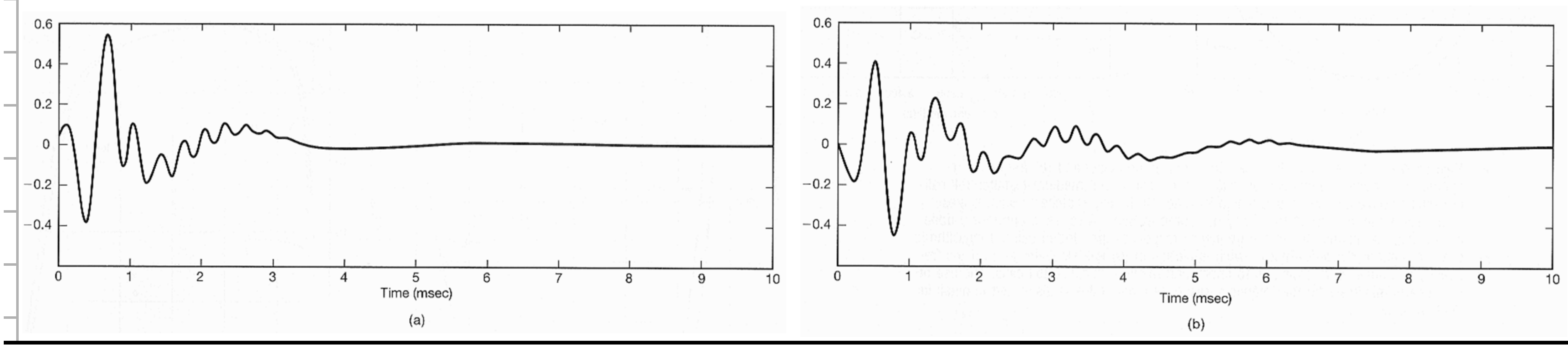


- [Example 6.2:]

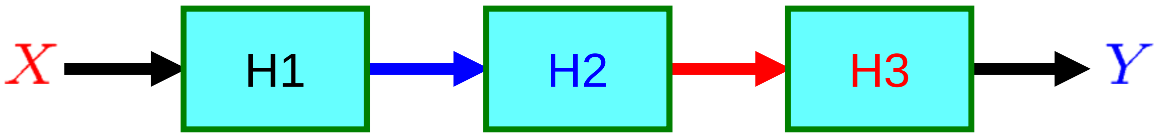
Analog Transmission Performance on the Switched Telecommunication Networks (AT&T/Bell)

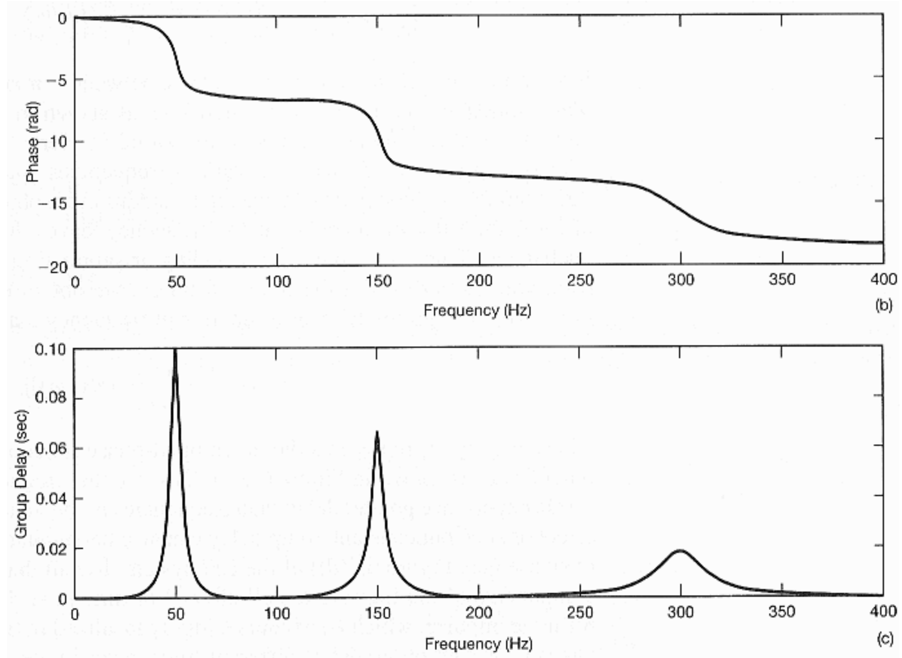






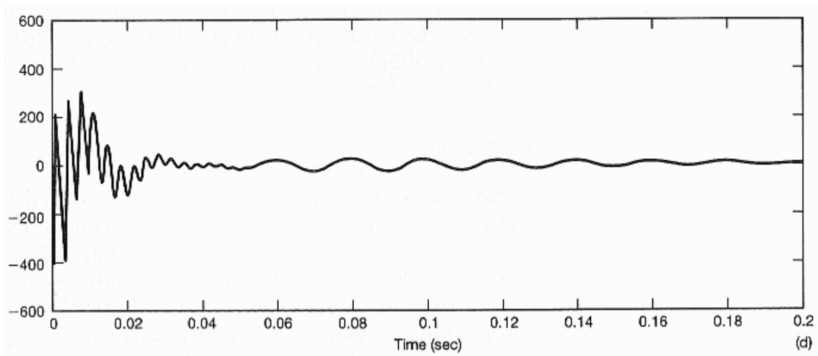
Phase Distortion and Group Delay





$$\tau(w) = -\frac{d}{dw}\left\{\sphericalangle H(jw)\right\}$$

$$x(t) = \delta(t)$$



- The Magnitude-Phase Representation of the Fourier Transform
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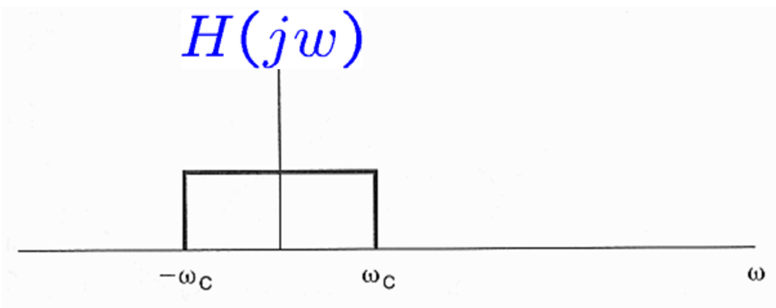
- Ideal Lowpass Filters:

$$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}$$

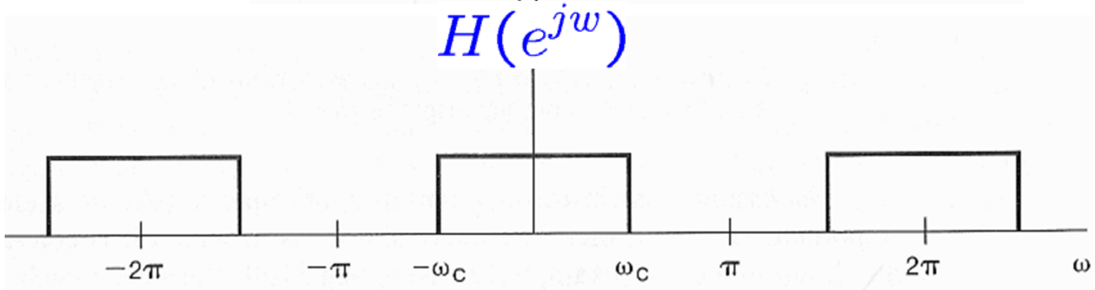
$$H(e^{jw}) = \begin{cases} 1, & |w| \le w_c \\ 0, & w_c < |w| \le \pi \end{cases}$$

–unit gain

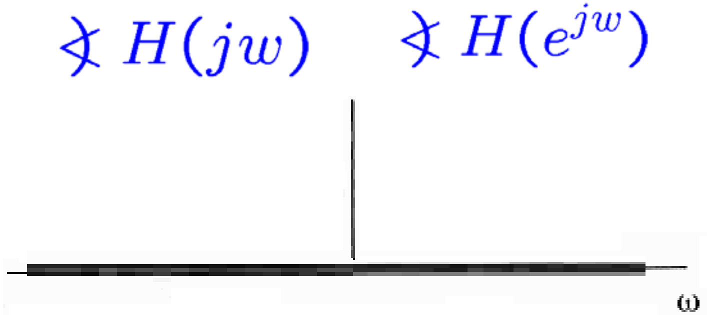
–zero phase distortion



(a)



(b)



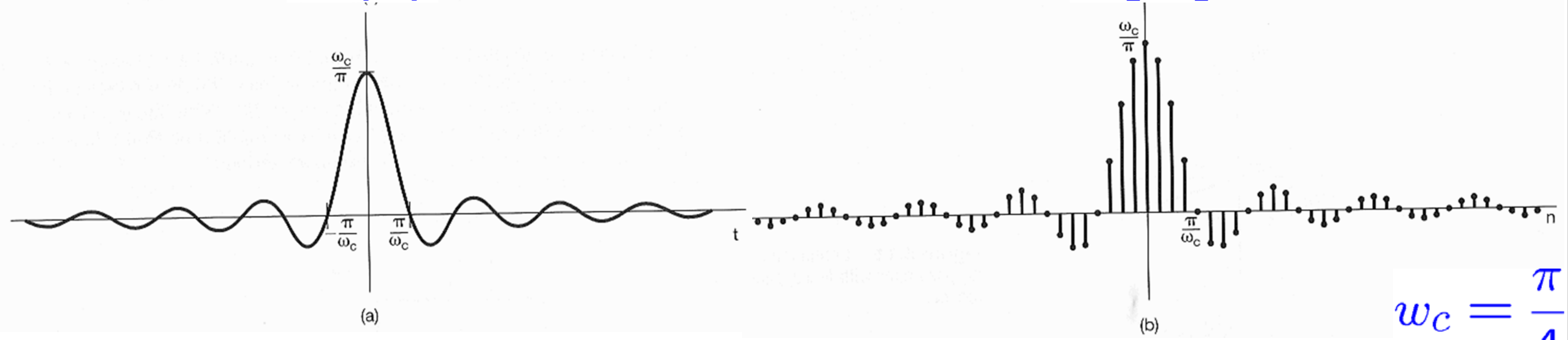
- Ideal Lowpass Filters:

$$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}$$

$$H(e^{jw}) = \begin{cases} 1, & |w| \le w_c \\ 0, & w_c < |w| \le \pi \end{cases}$$

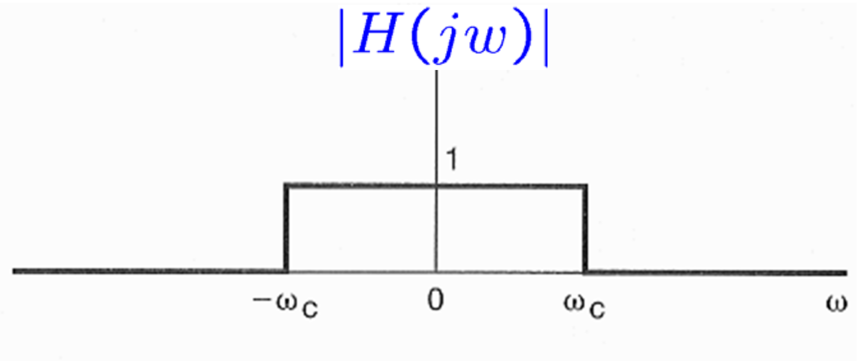
$$\Rightarrow \begin{cases} h(t) & = & \dfrac{\sin w_c t}{\pi t} \\ h[n] & = & \dfrac{\sin w_c n}{\pi n} \end{cases}$$

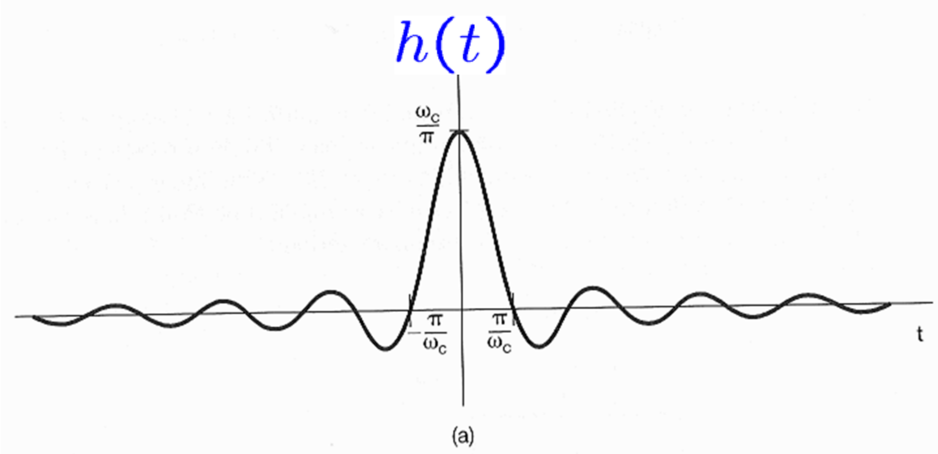
$h(t)$ $h[n]$

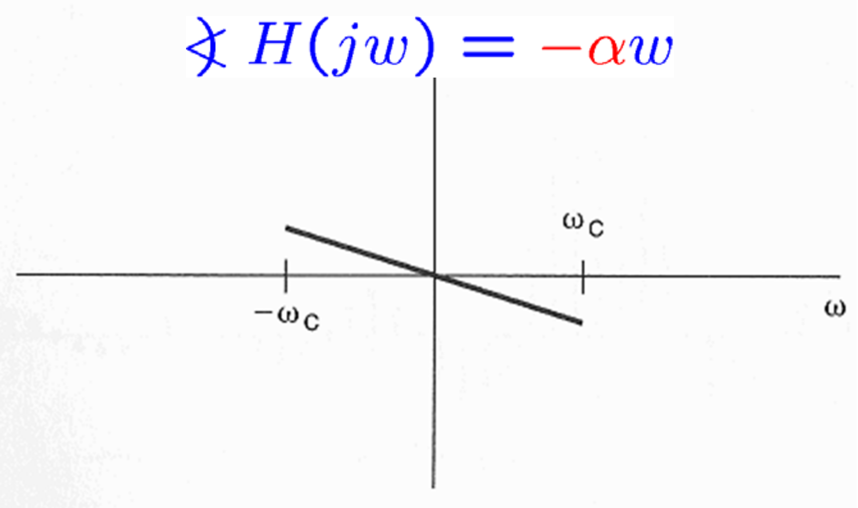


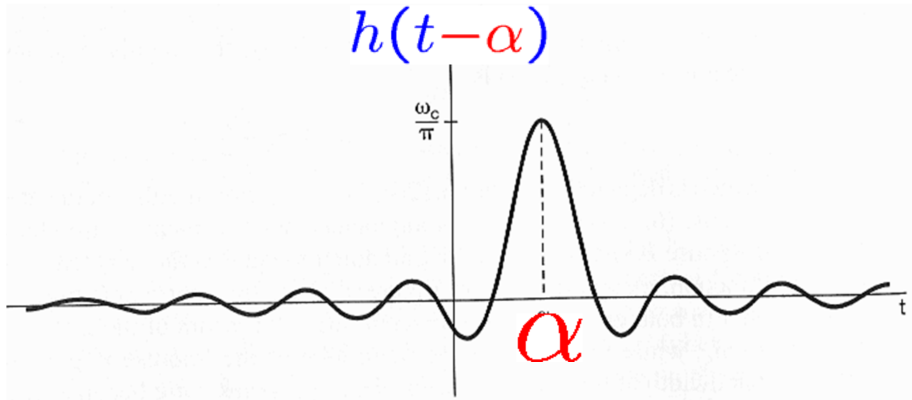
$w_c = \dfrac{\pi}{4}$

- Ideal Lowpass Filters with Linear Phase:



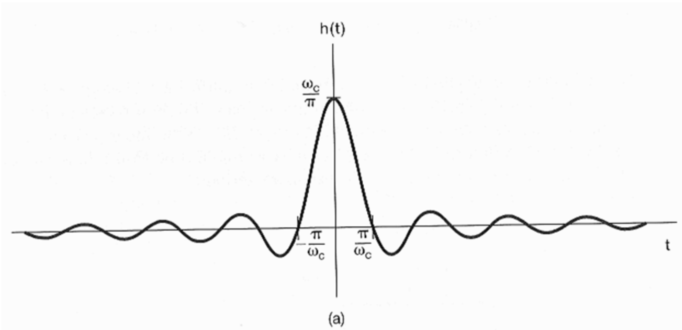


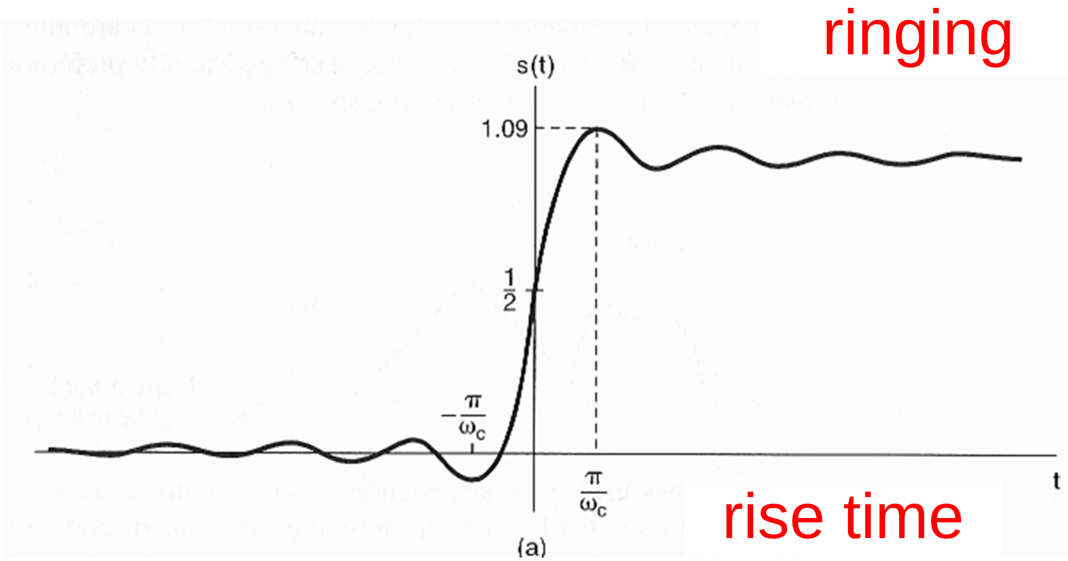




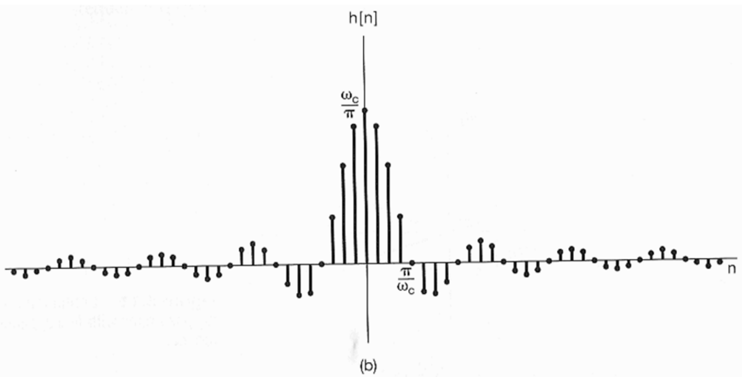
Step Response of Ideal Lowpass Filters:

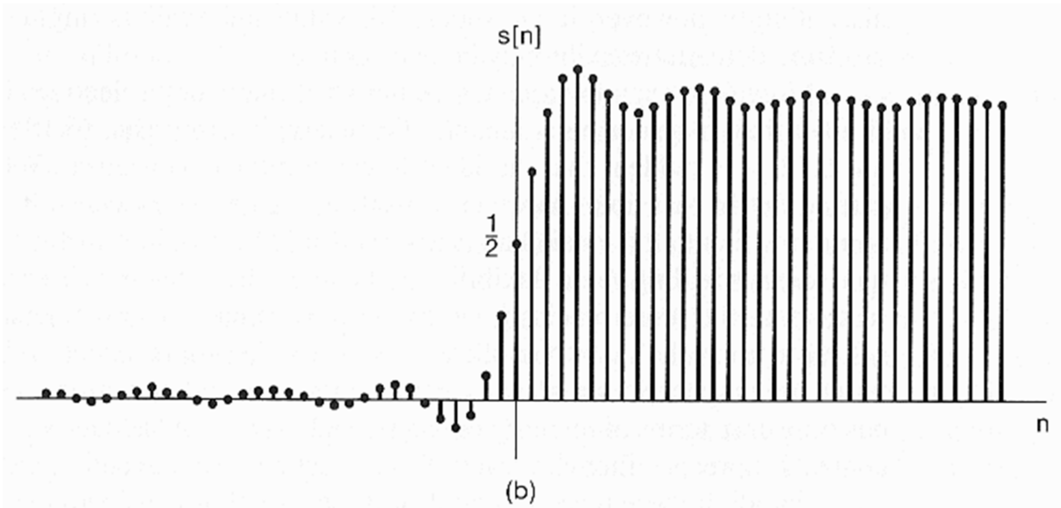
$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$





$$s[n] = \sum_{m=-\infty}^{n} h[m]$$





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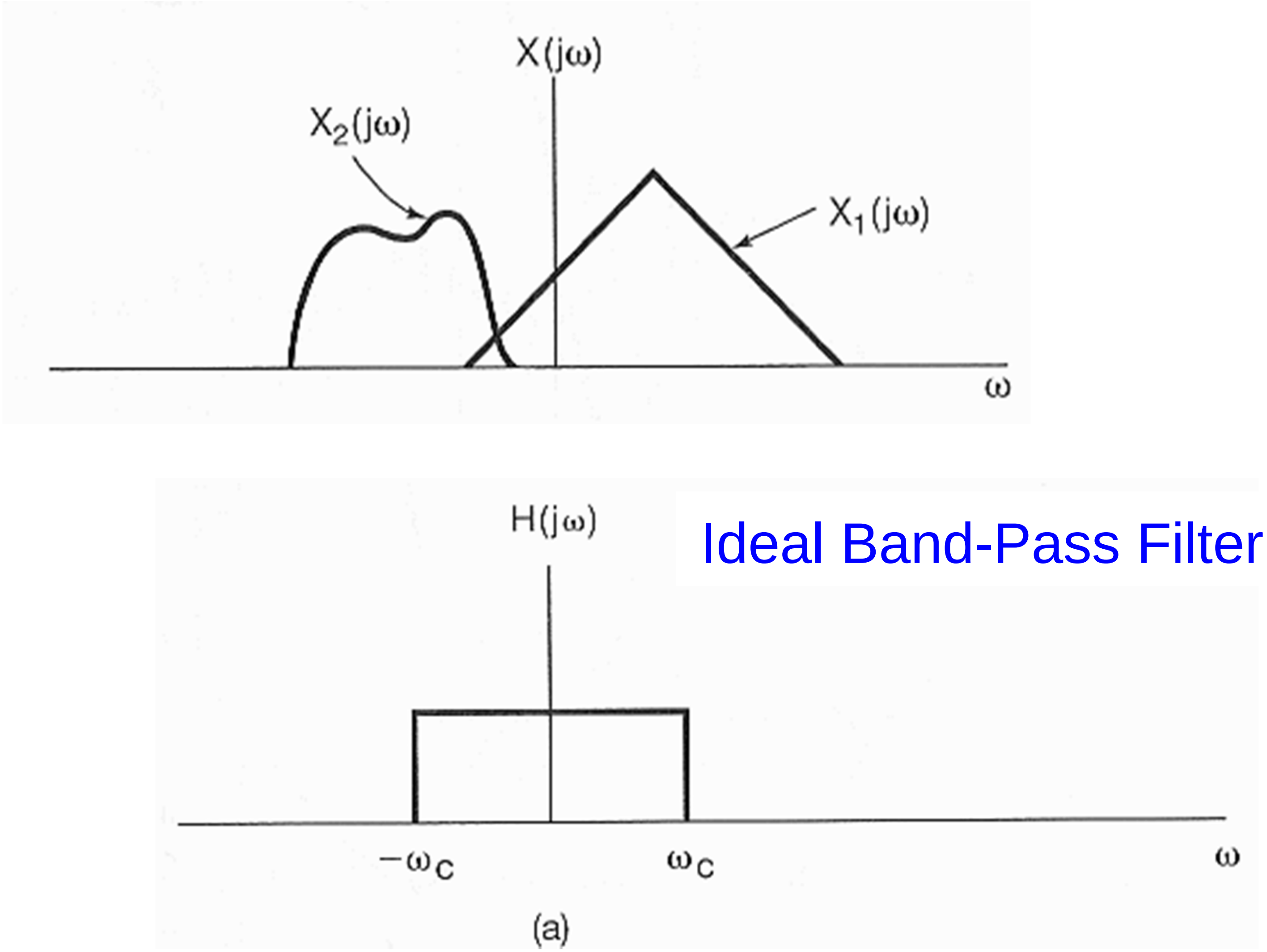
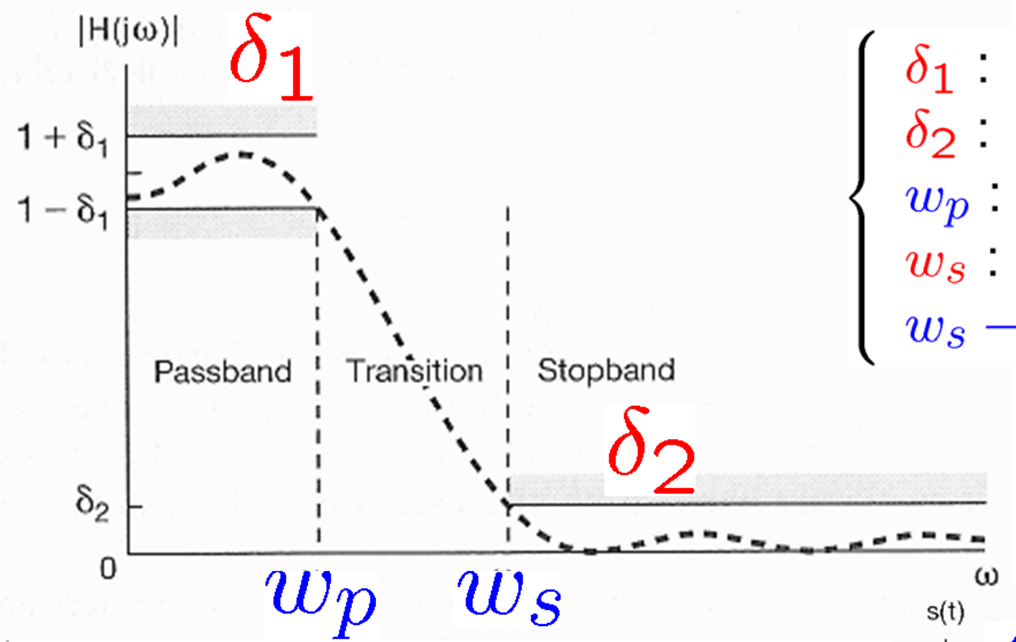
- Overlapping Spectra:



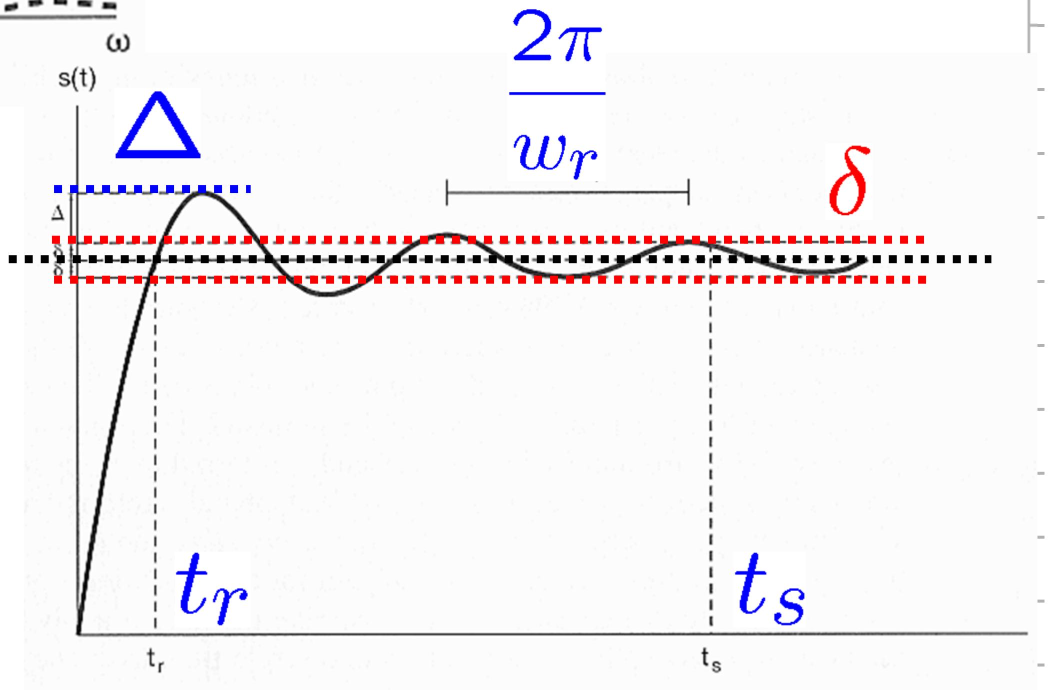
Ideal Band-Pass Filter ?

Desired Filter Characteristics:

$X \rightarrow \boxed{H} \rightarrow Y$



δ_1 : allowable passband ripple
δ_2 : allowable stopband ripple
w_p : passband edge
w_s : stopband edge
$w_s - w_p$: transition band

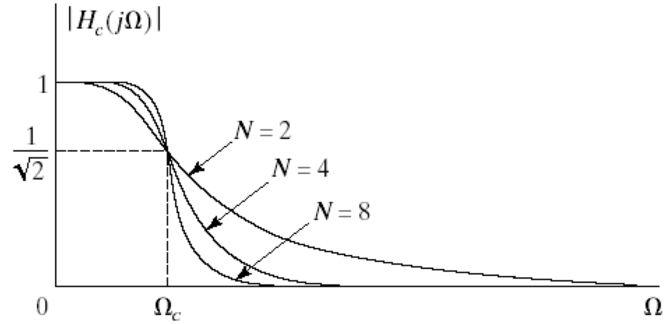


Δ : overshoot
δ : steady-state error
w_r : ringing frequency
t_r : rise time
t_s : settling time

Three Frequently Used Filters:

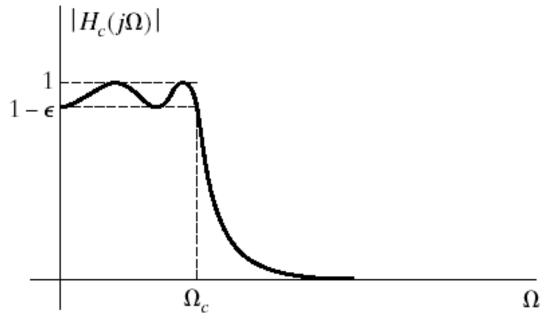
- Butterworth, Chebyshev, Elliptic filters

$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$



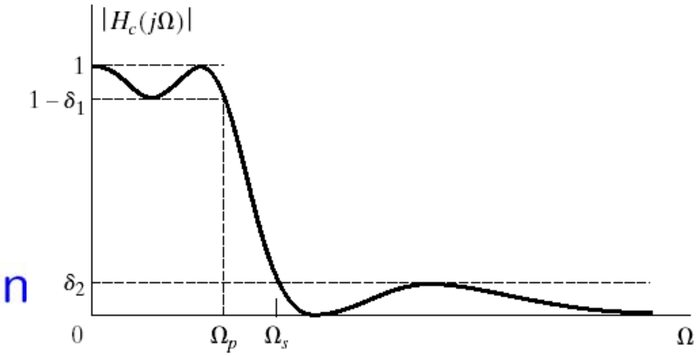
$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\epsilon^2 V_N^2\left(\frac{\Omega}{\Omega_c}\right)}$$

$$V_N(x) = \cos\left(N \cos^{-1} x\right)$$



$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\epsilon^2 U_N^2(\Omega)}$$

$U_N(x)$: Jacabian elliptic function



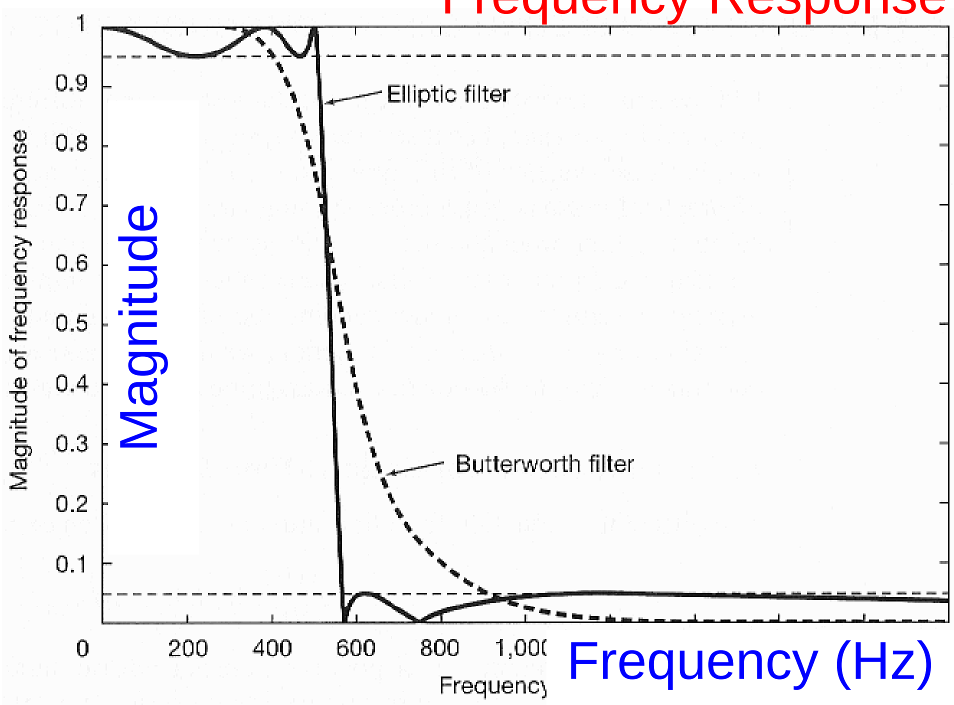
Example 6.3: Two Frequently Used Filters:

- Butterworth filter
- Elliptic filter

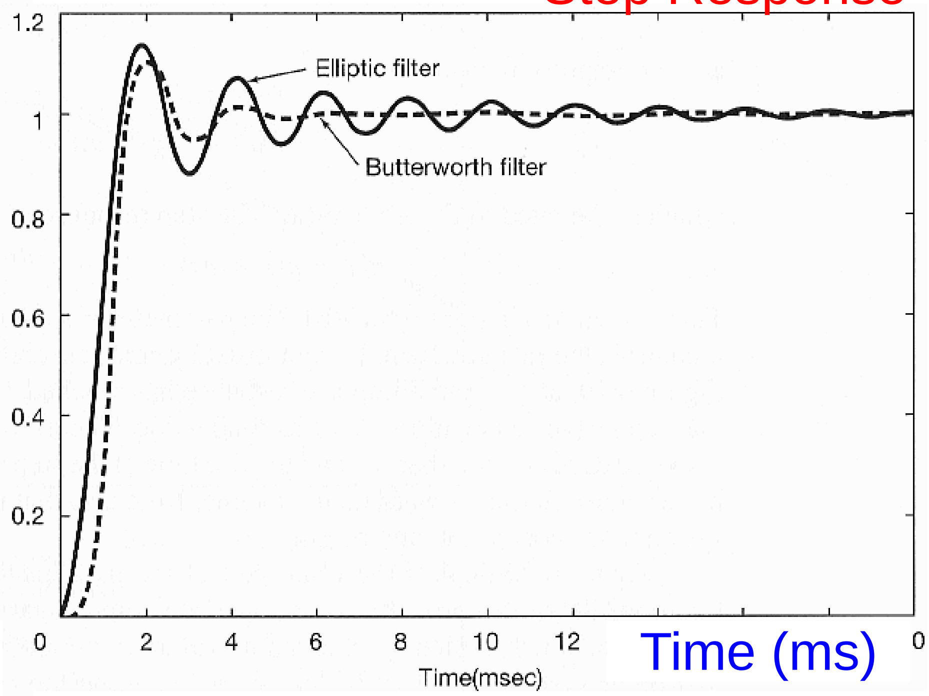
Fifth-order rational frequency response

Cutoff frequency = 500 Hz

Frequency Response



Step Response



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(p.472)

DT Non-recursive Filters:

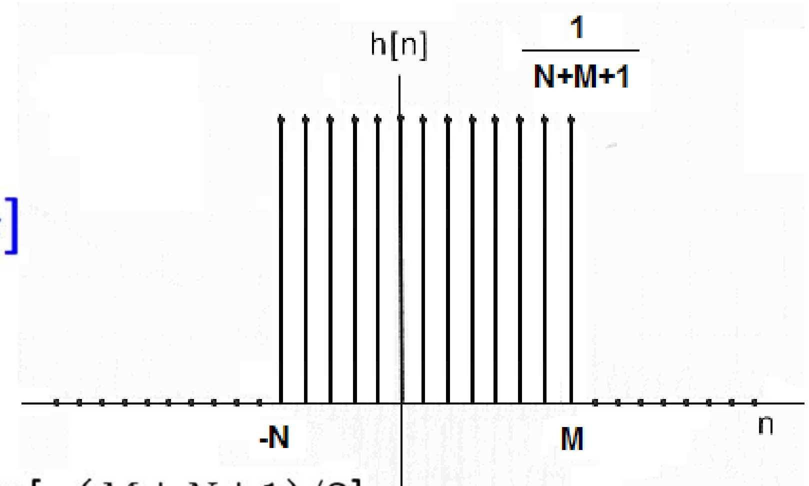
- Recursive or infinite impulse response (IIR) filters
  - > Impossible to design a causal, recursive filter with exactly linear phase (related to time delay)

$$y[n] - a\, y[n-1] = x[n] \qquad |a| < 1 \quad \Rightarrow H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

- Non-recursive or finite impulse response (FIR) filters
  - > Can have exactly linear phase (related to time delay)

ss3-105

$$y[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} x[n-k]$$

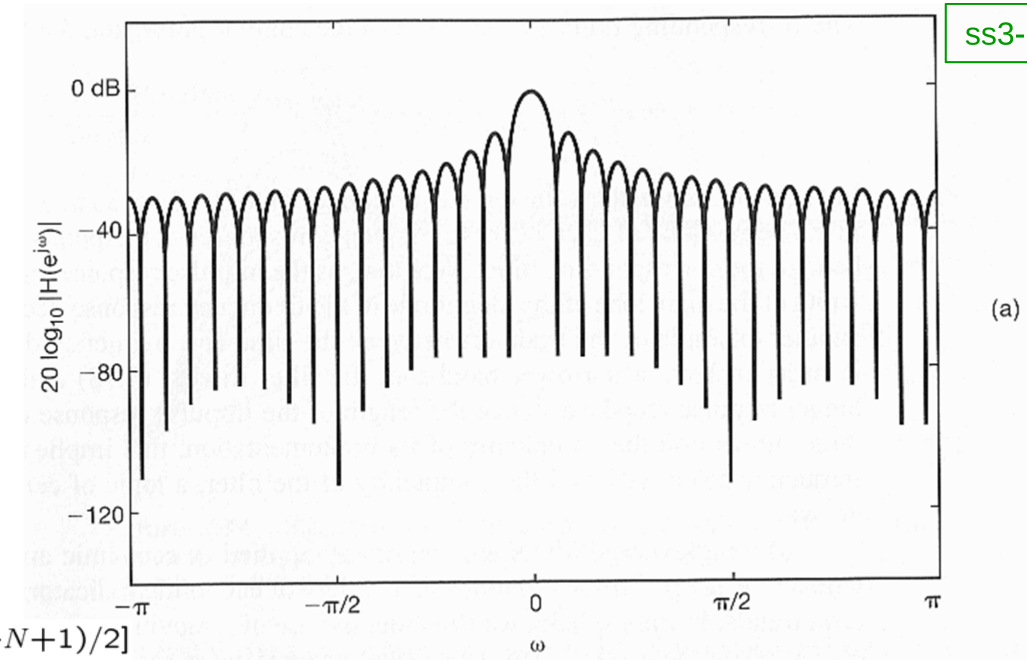


$$\Rightarrow H(e^{jw}) = \frac{1}{N+M+1} e^{jw[(N-M)/2]} \frac{\sin[w(M+N+1)/2]}{\sin(w/2)}$$

ss3-106

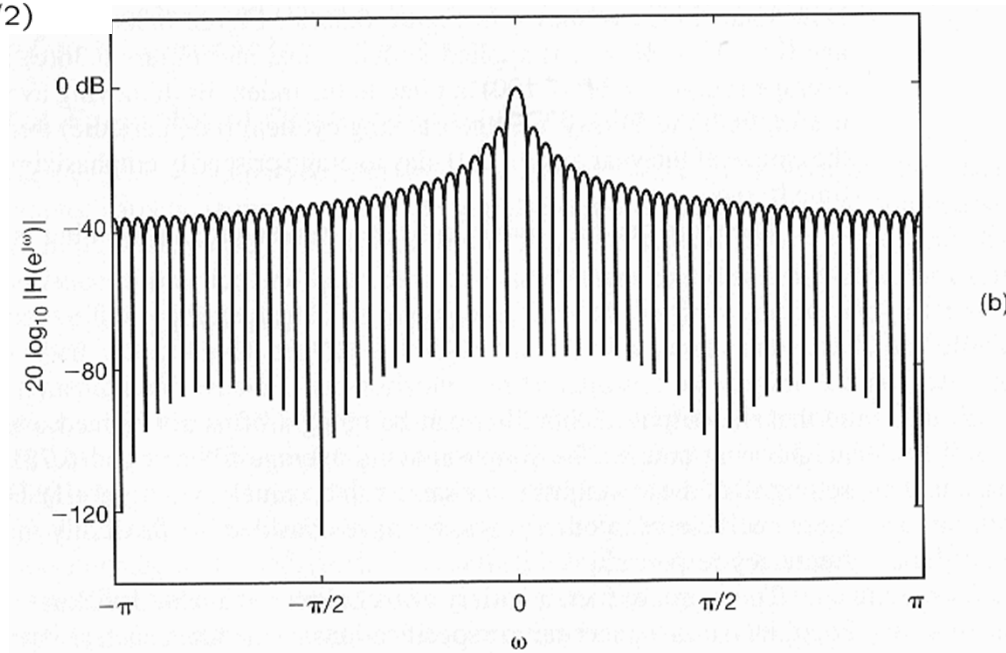
Log-Magnitude Plots:

$$N + M + 1 = 33$$



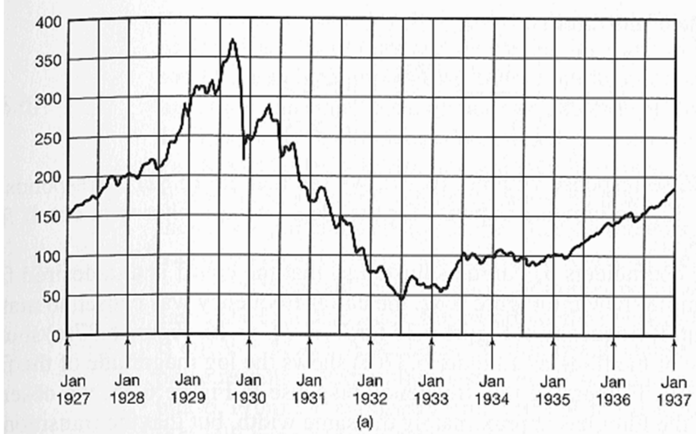
$$H(e^{jw}) = \frac{1}{N+M+1} e^{jw[(N-M)/2]} \frac{\sin[w(M+N+1)/2]}{\sin(w/2)}$$

$$N + M + 1 = 65$$

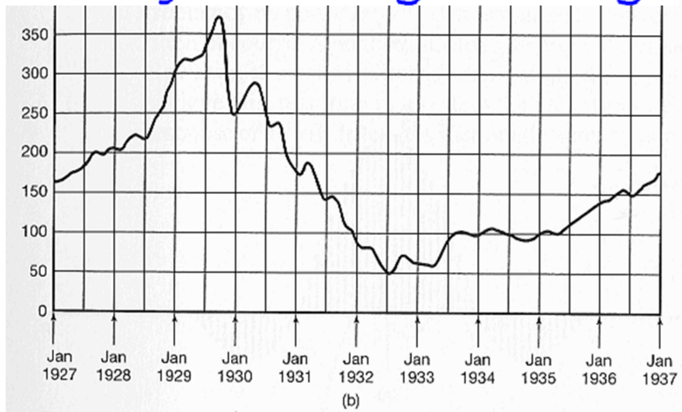


- Lowpass Filtering on Dow Jones Weekly Stock Market Index:

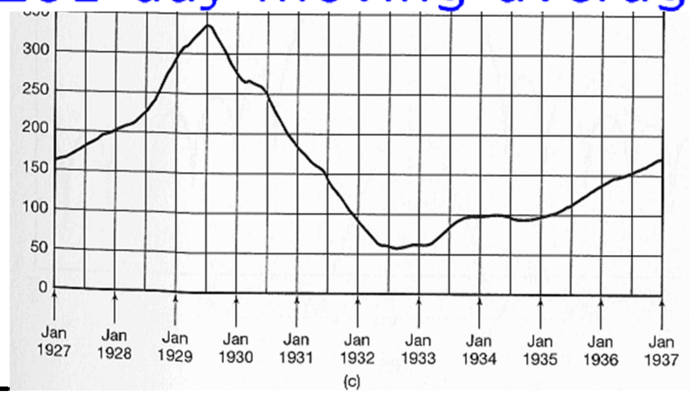
http://big5.jrj.com.cn/



51-day moving average



201-day moving average

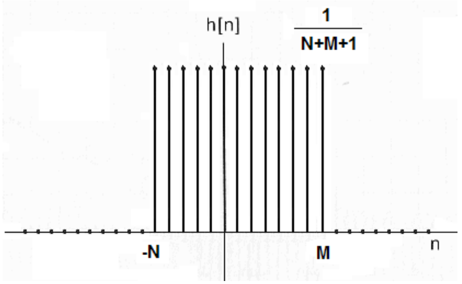


General Form of DT Non-recursive Filters:

ss3-104
ss3-108

$$y[n] = \sum_{k=-N}^{M} b_k \, x[n-k]$$

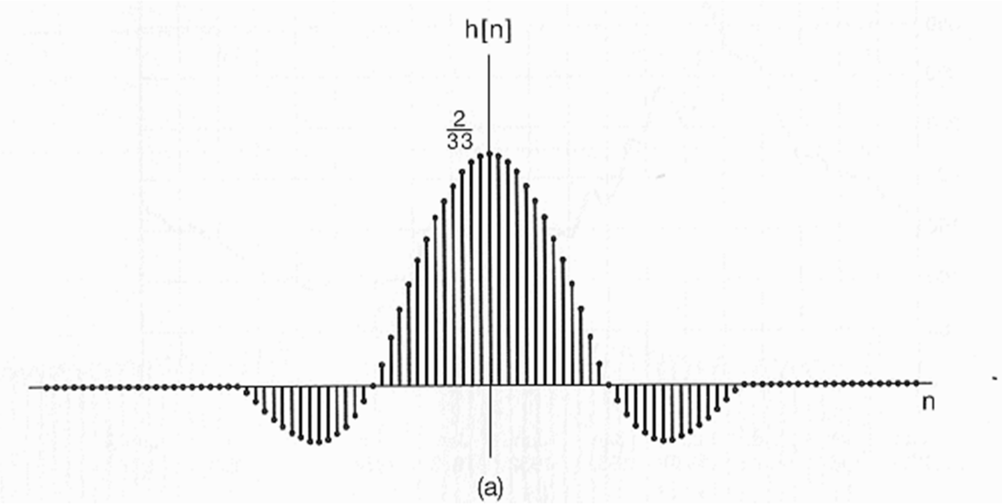
$$y[n] = \sum_{k=-N}^{M} \frac{1}{N+M+1} x[n-k]$$



- Let $N = M = 16$:

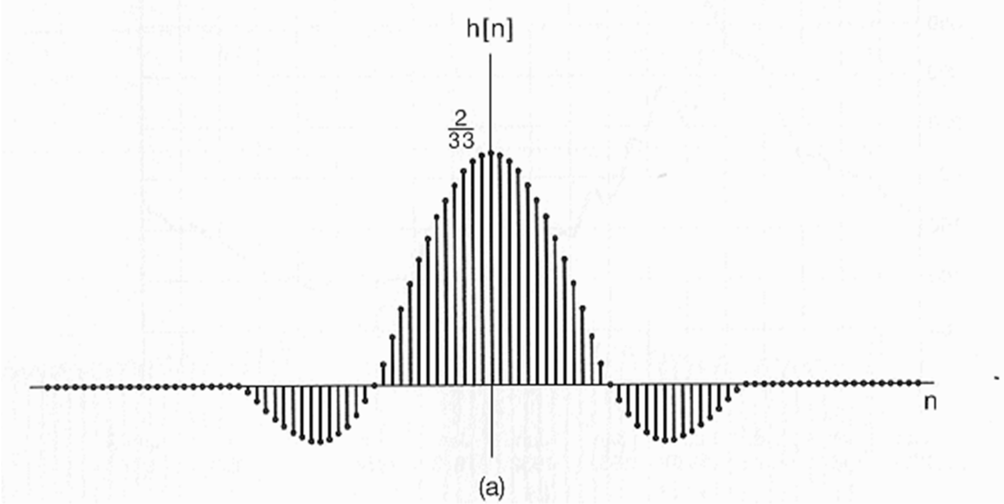
$$b_k = \begin{cases} \dfrac{\sin(2\pi k/33)}{\pi k}, & |k| \le 32 \\ 0, & |k| > 32 \end{cases}$$

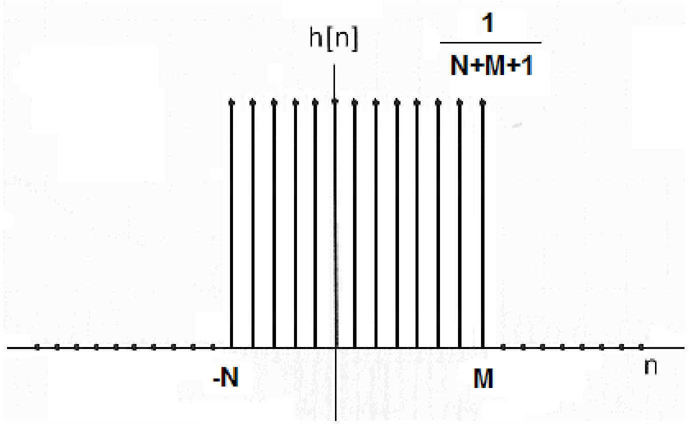
$$h[n] = \begin{cases} \dfrac{\sin(2\pi n/33)}{\pi n}, & |n| \le 32 \\ 0, & |n| > 32 \end{cases}$$

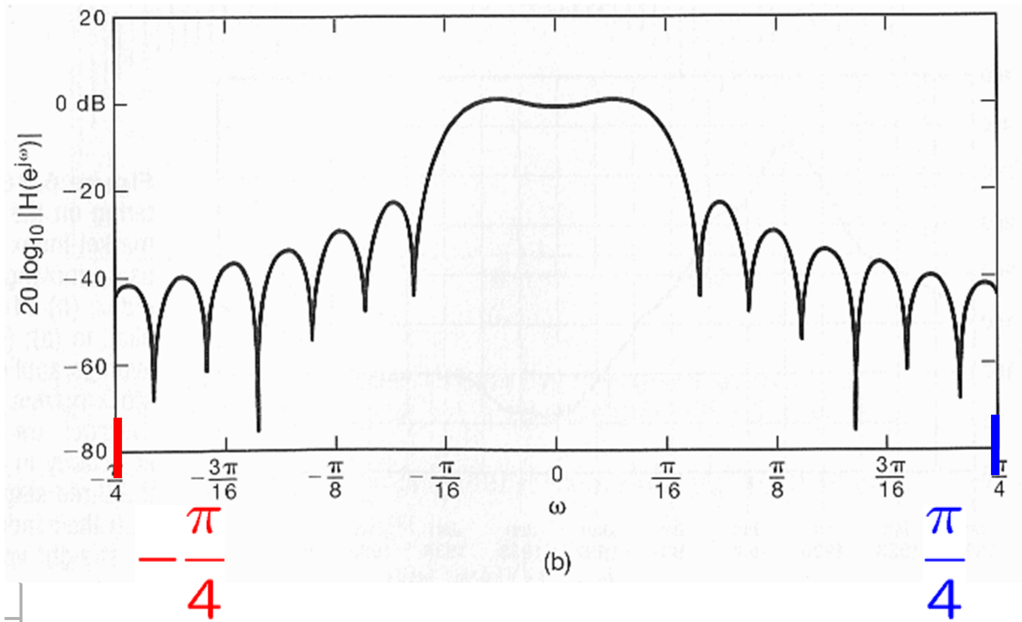


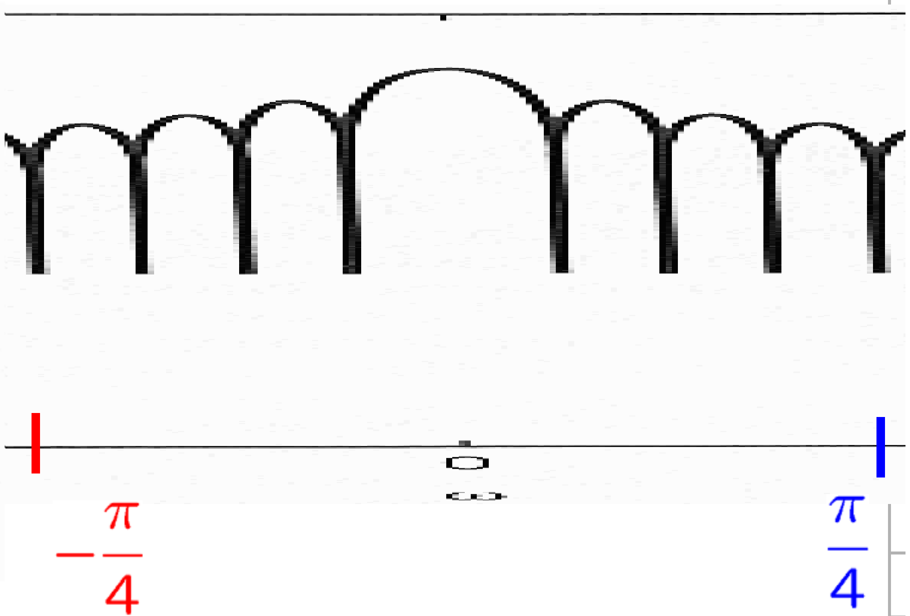
▪ General Form of DT Non-recursive Filters:

ss6-82

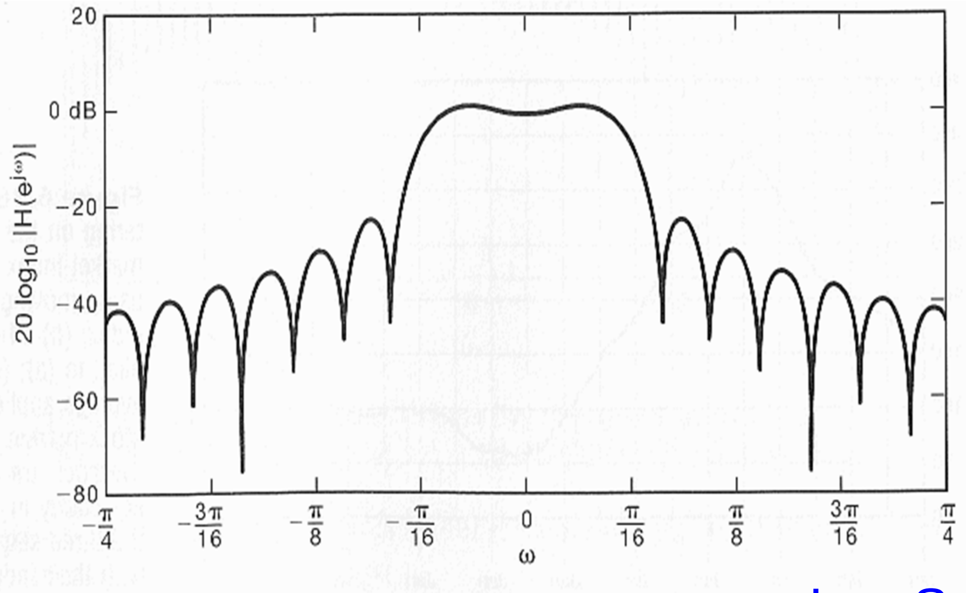




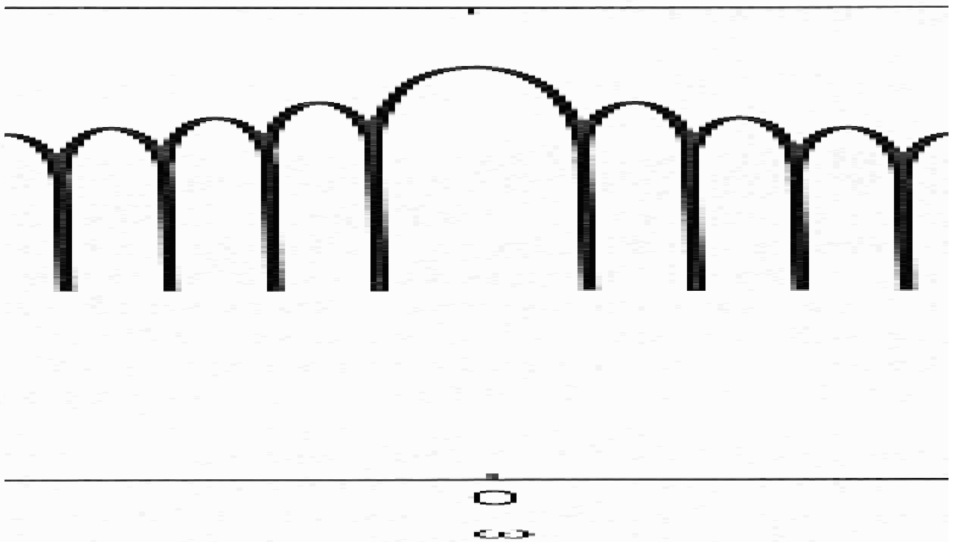




Comparison on a Linear Amplitude Scale:

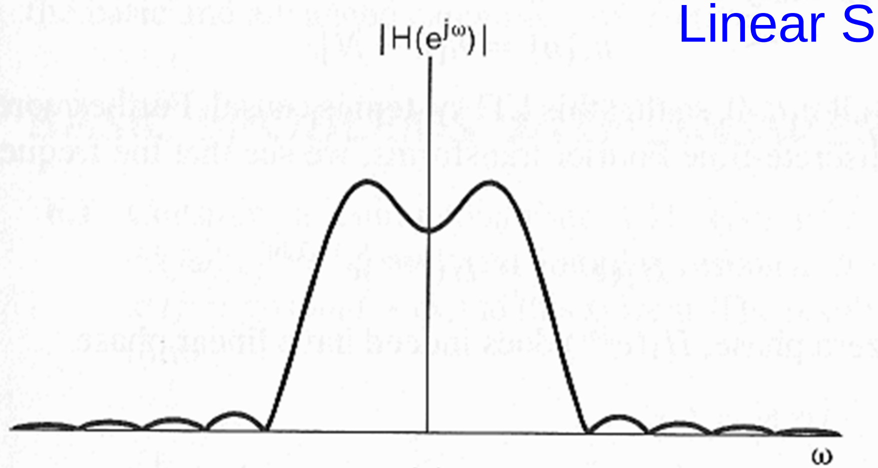


(b)

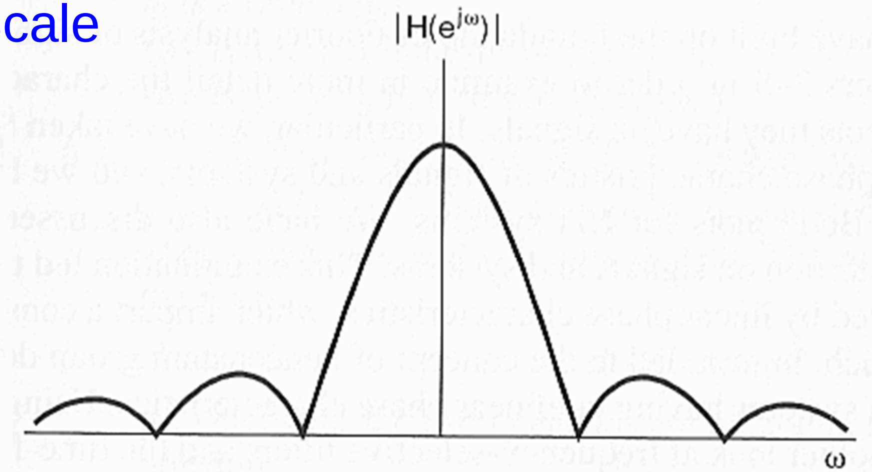


Log Scale

Linear Scale



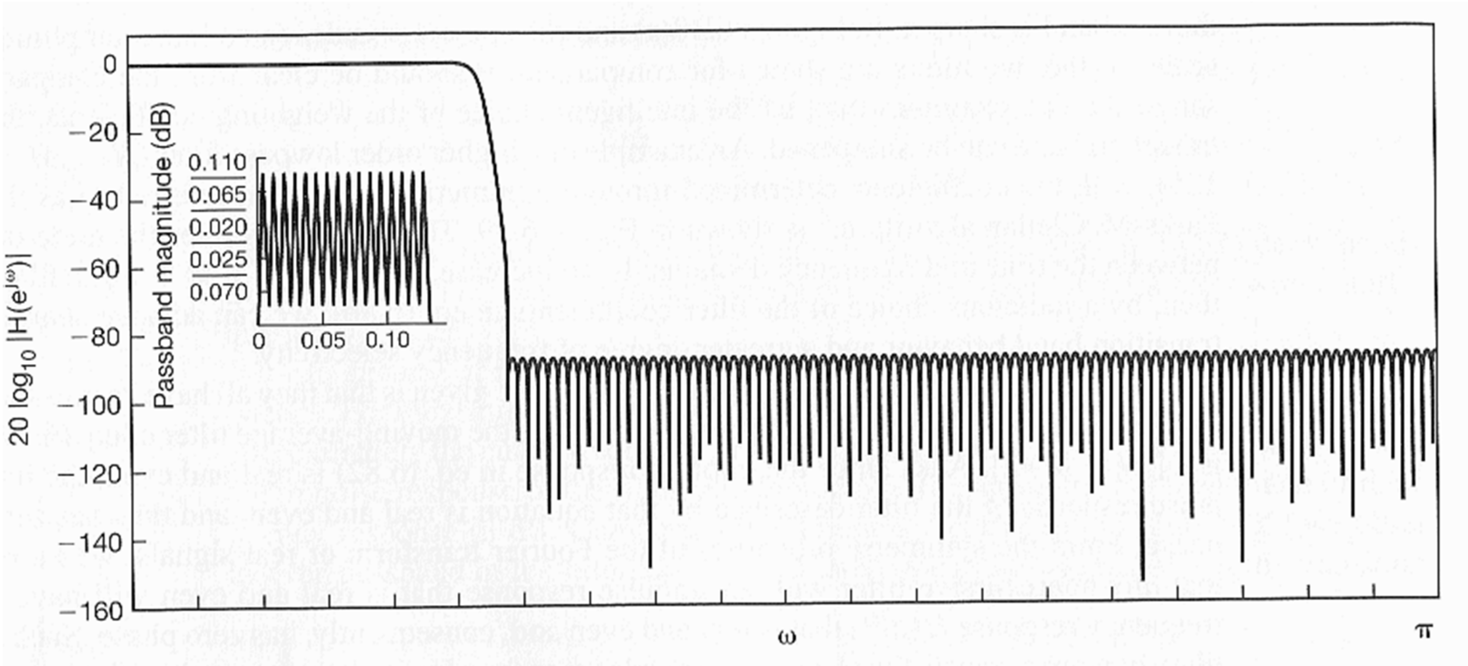
(a)



(b)

Lowpass Non-recursive Filter with 251 Coefficients:

$$y[n] = \sum_{k=-N}^{M} b_k \, x[n-k]$$



Coefficients determined by the Parks-McClellan algorithm

$$\frac{d}{dt}y(t) + ay(t) = ax(t)$$

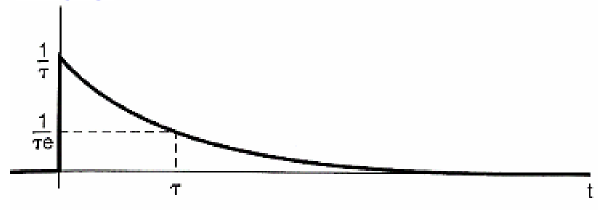
$$\Rightarrow \quad H(jw) = \frac{a}{jw + a}$$

$$\Rightarrow \quad H(s) = \frac{a}{s + a}$$

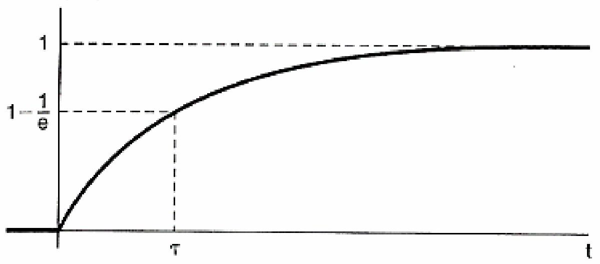
$$h(t) = a\, e^{-at}\, u(t)$$

$$s(t) = \left[1 - e^{-at}\right]\, u(t)$$

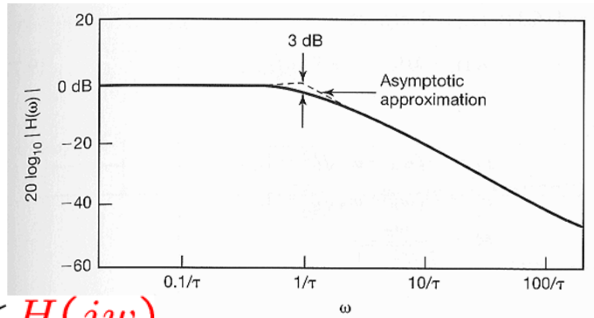
$h(t)$



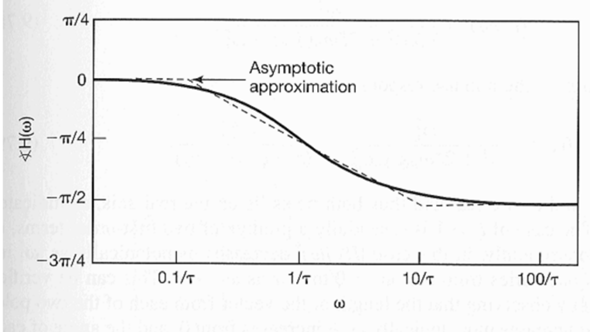
$s(t)$



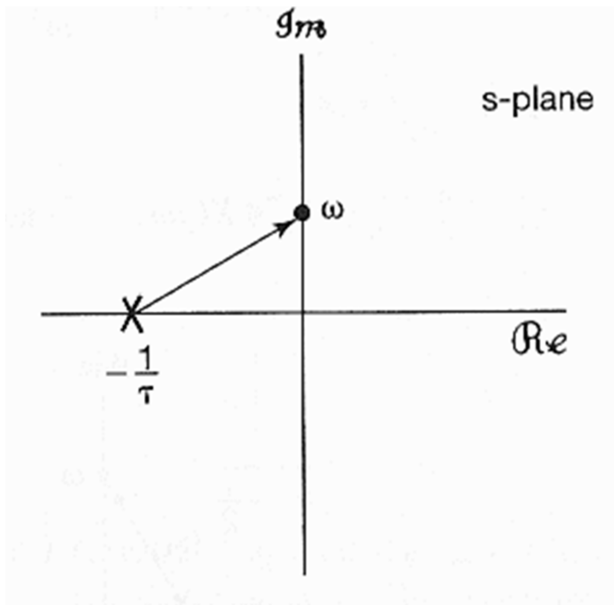
$20 \log_{10} |H(jw)|$



$\measuredangle H(jw)$



$H(s)$



$$y[n] - a\,y[n-1] = x[n] \qquad |a| < 1$$

$$\Rightarrow \; H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

$$\Rightarrow \; H(z) = \frac{z}{z-a}, \quad |z| > |a|$$

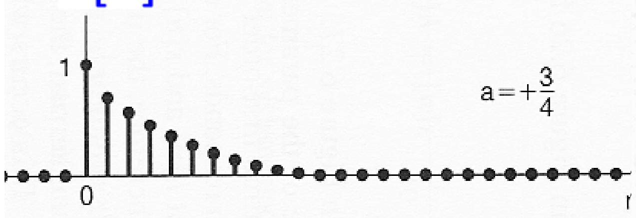
$$\Rightarrow \; h[n] = a^n\,u[n]$$

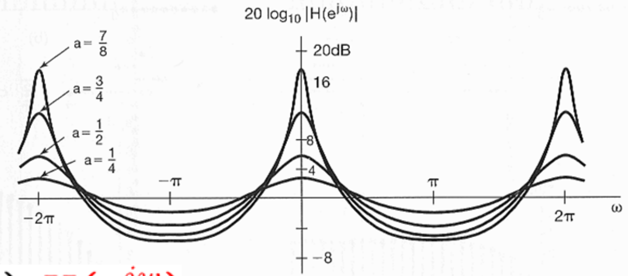
$$\Rightarrow \; s[n] = \frac{1 - a^{n+1}}{1-a}\,u[n]$$

$H(z)$

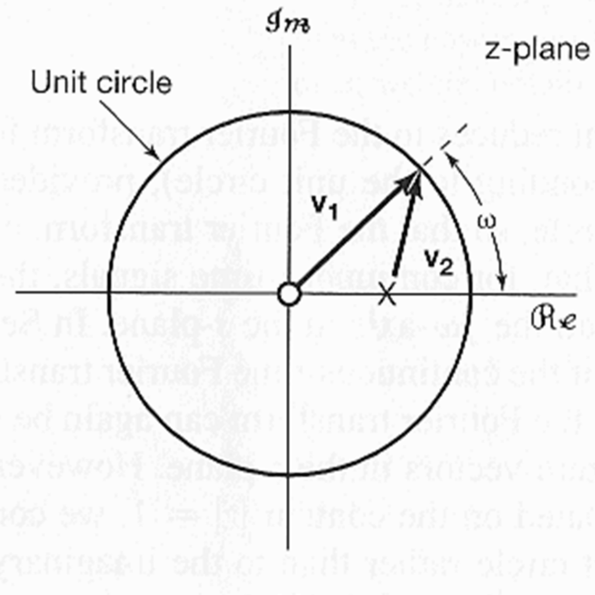
$20\log_{10}|H(e^{jw})|$

$h[n]$



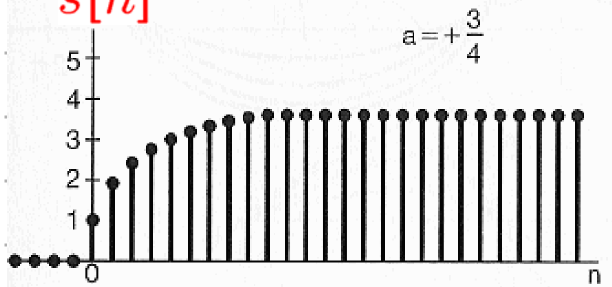


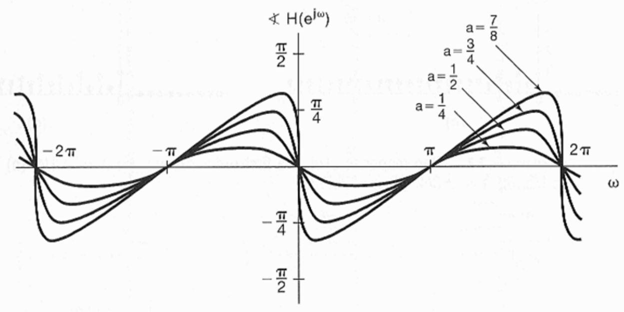
$\measuredangle H(e^{jw})$



(a)

$s[n]$



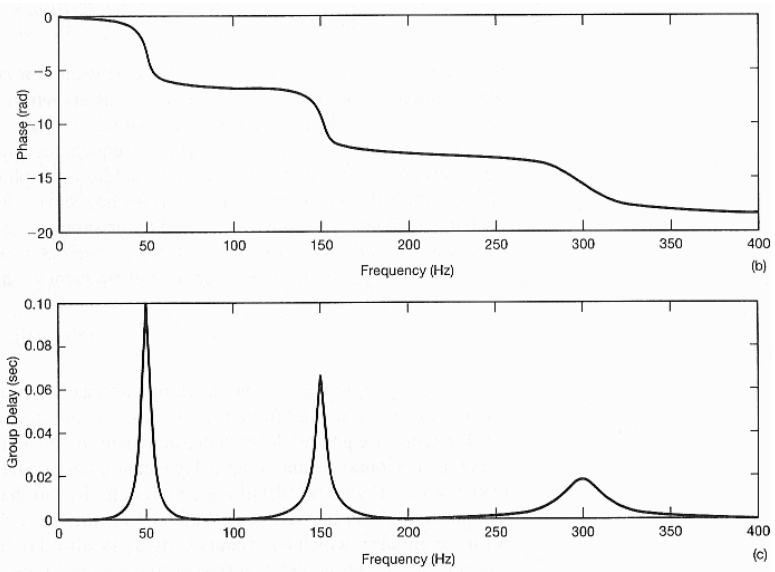


(a)

$$x(t) = \delta(t)$$

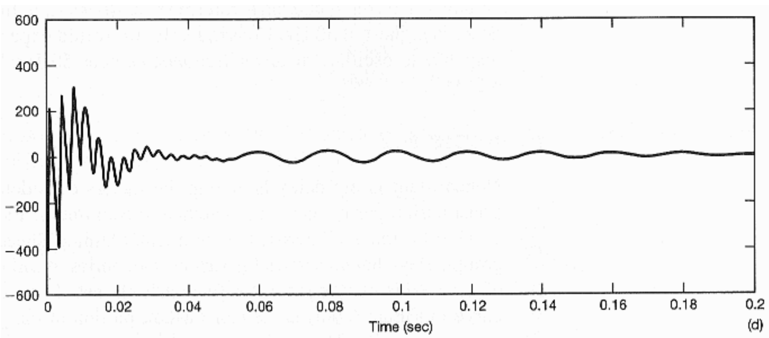
$$X(jw) = 1, \ \forall w$$

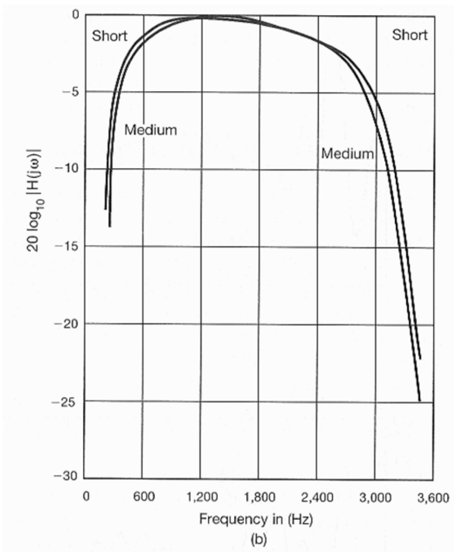
$$X \to \boxed{H1} \to \boxed{H2} \to \boxed{H3} \to Y$$

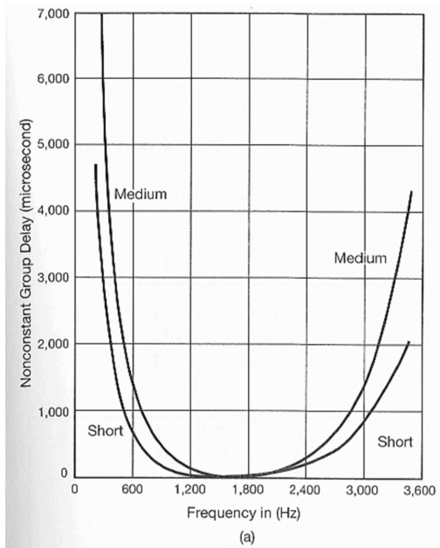


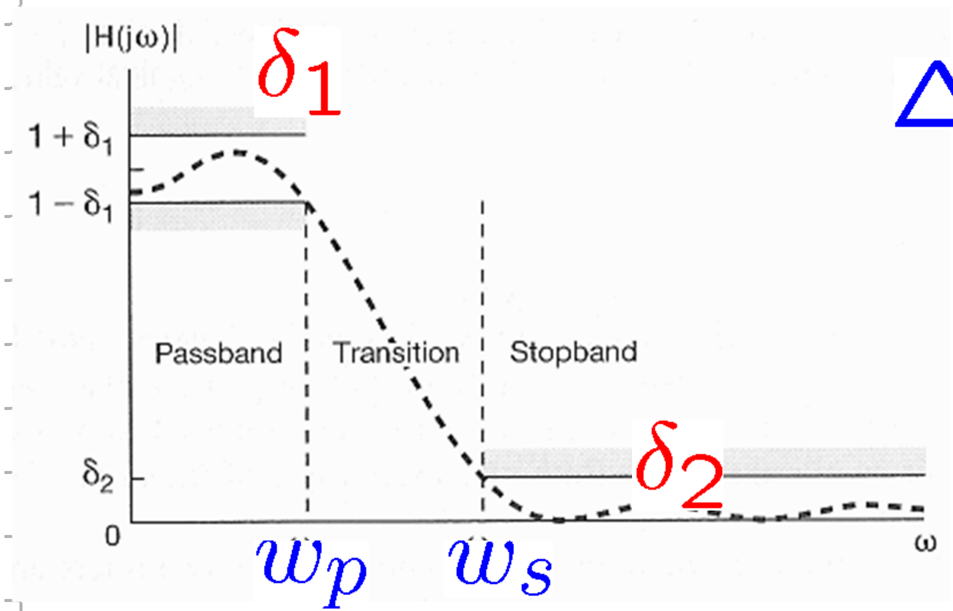
$$\tau(w) = -\frac{d}{dw}\left\{ \measuredangle H(jw) \right\}$$

$$y(t)$$

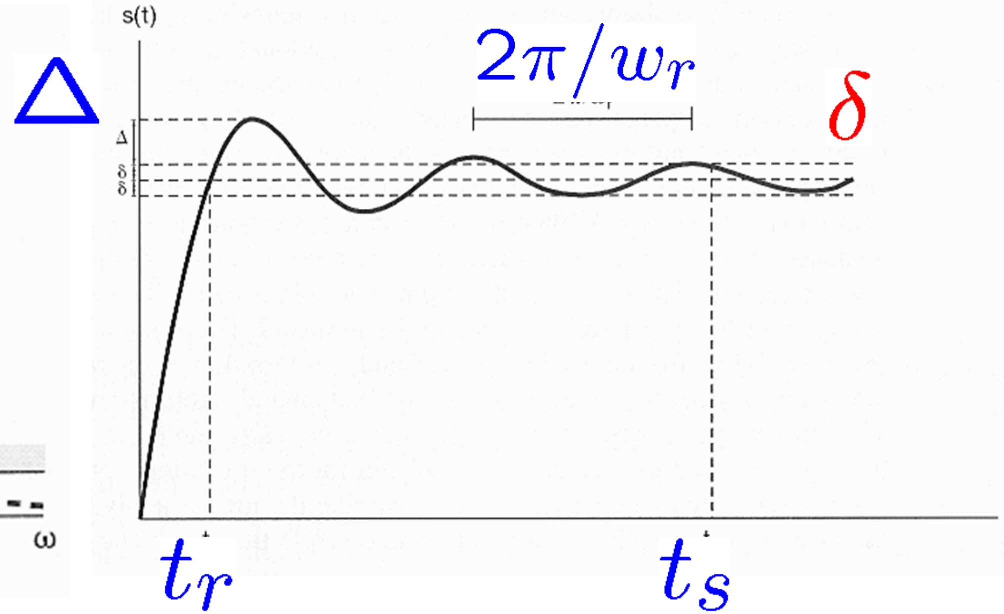




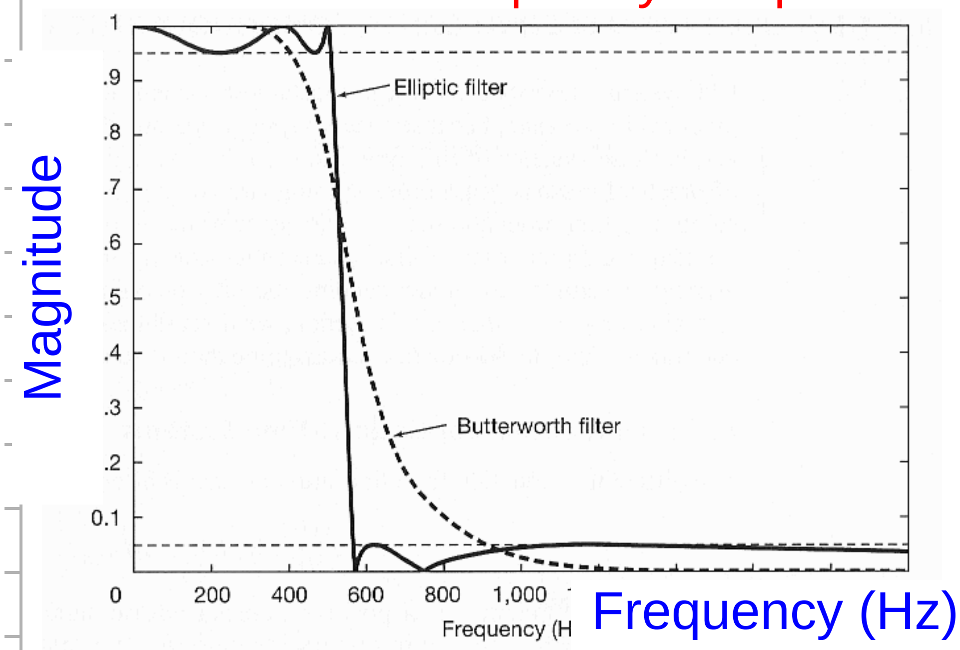


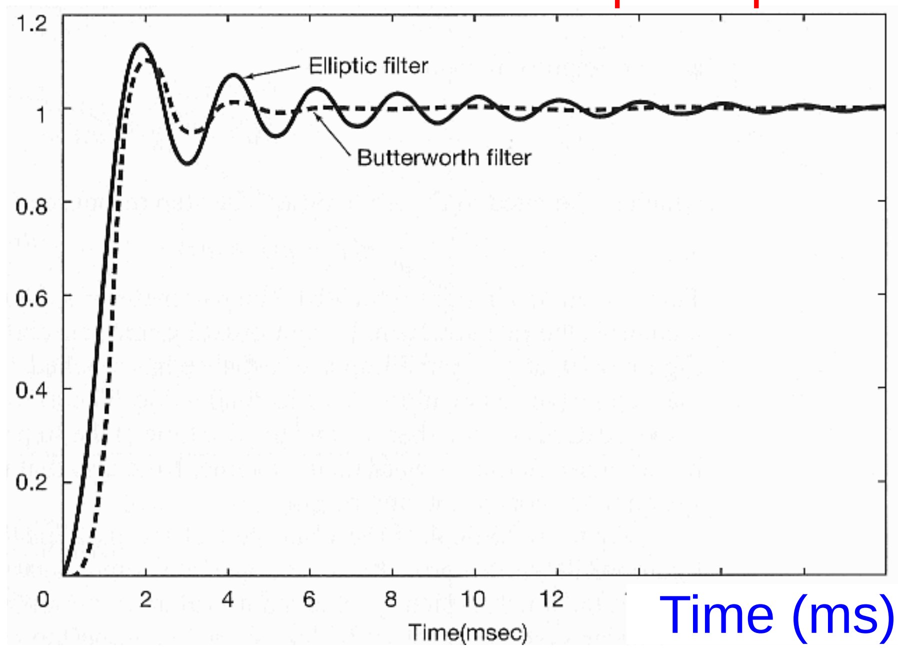


Frequency Response



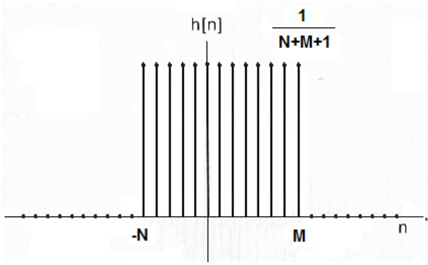
Step Response

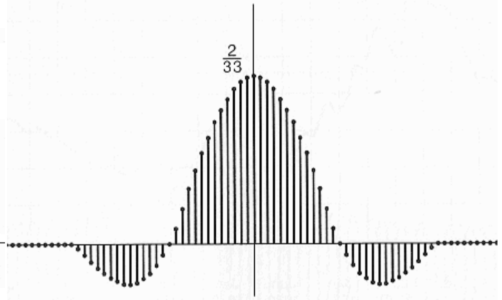


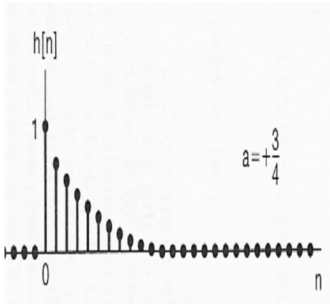


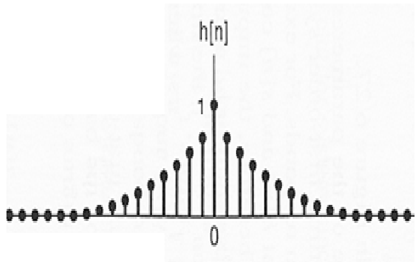
- Lowpass Filters:

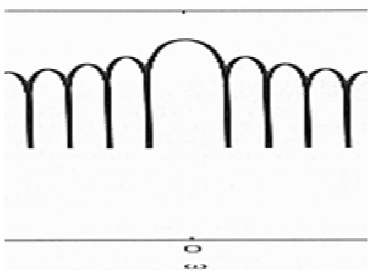
$$\frac{1}{N+M+1} \qquad \frac{\sin(2\pi n/33)}{\pi n} \qquad h[n] = a^n\, u[n] \qquad x[n] = a^{|n|}$$

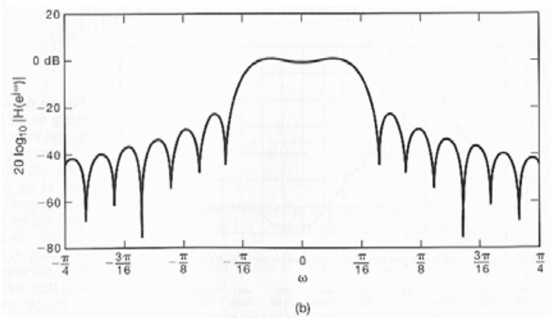


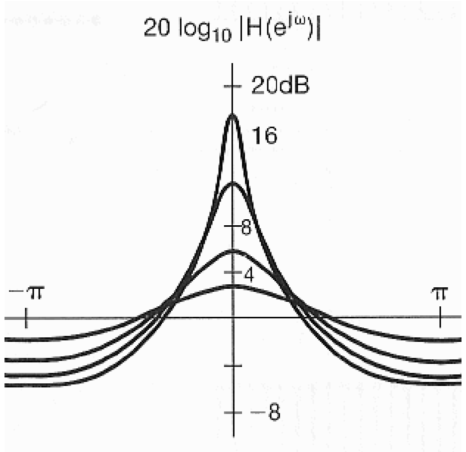




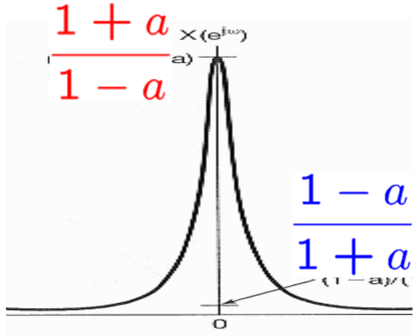








$$H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$



$$X(e^{jw}) = \frac{1-a^2}{1 - 2a\cos w + a^2}$$

Fig 6.35 | Fig 6.37 | Fig 6.28 | Ex 5.2

- The Magnitude-Phase Representation of the FT
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order CT Systems
- 1st-Order & 2nd-Order DT Systems
- Time- & Frequency-Domain Analysis of Systems