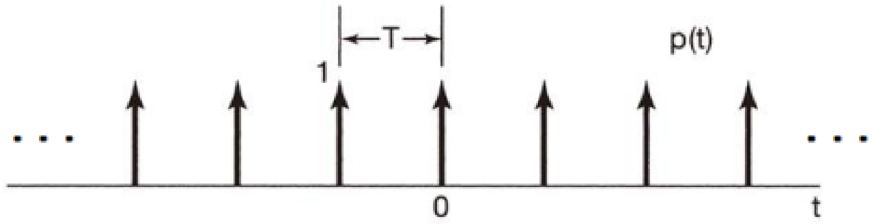
- Representation of Aperiodic Signals: the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations

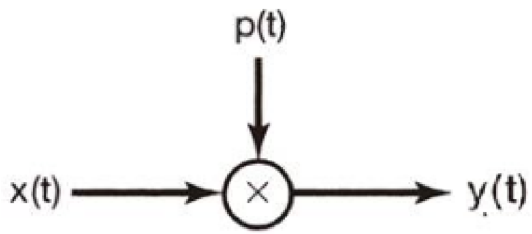
9. (10 %) Answer the following questions.

a) (2%) Consider the following periodic signal $p(t)$.

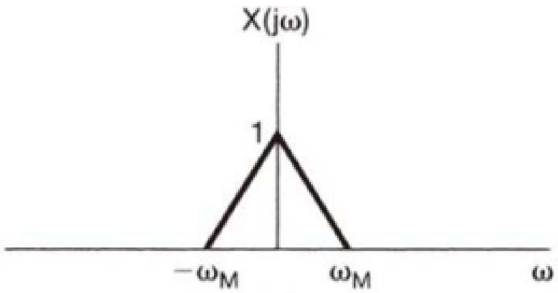


Find the Fourier transform $P(jw)$ of $p(t)$.

b) (3%) Consider the following operation.



Assume that the Fourier transform $X(jw)$ of one signal $x(t)$ is as follows.



Assume that $w_M < \frac{\pi}{T}$. Sketch the Fourier transform $Y(jw)$ of $y(t)$ and express $Y(jw)$ in the form: $Y(jw) = A\sum^{\infty} X(jw+kB)$. What are the value of A and B?

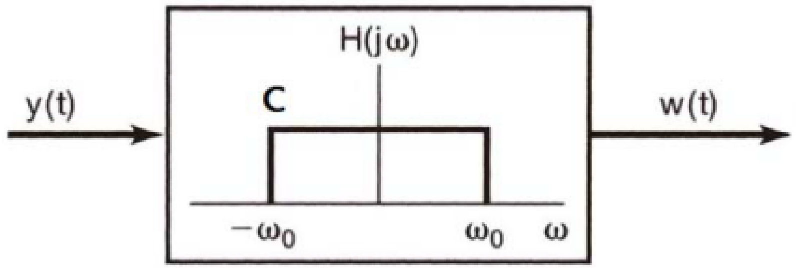
c) (3%) Let $r[n] = y(nT)$ and assume that:

$$Y(jw) = \sum_{n=-\infty}^{\infty} y(nT)e^{-jwnT}, \text{ and}$$

$$R(e^{jw}) = \sum_{n=-\infty}^{\infty} r[n]e^{-jwn}.$$

Show that $R(e^{jw}) = Y(\frac{jw}{T})$ and sketch the graph of $R(e^{jw})$.

d) (2%) Consider the following operation:



where $H(jw)$ an ideal lowpass filter with a magnitude C in the passband between $-w_o$ and w_o. Find the value of C and the range of w_o, such that $w(t) = x(t)$. Justify your answer.

Midterm 2013-1

1. (24 %) The response $y[n]$ of a discrete-time system is related to the input $x[n]$ by

$$y[n] = \tfrac{1}{4}(x[n-1] + 2x[n] + x[n+1]).$$

a) Is it a linear time-invariant system? Justify your answer. (3%)
b) Is the system causal? Justify your answer. (3%)
c) Is the system stable? Justify your answer. (3%)
d) Is the system a recursive filter? Justify your answer. (3%)
e) Determine the impulse response $h[n]$. (3%)
f) Determine the step response of the system. (3%)
g) Determine the frequency response $H(e^{j\omega})$ of the system by the eigenfunction approach and by Fourier transform. (3%)
h) Is it possible to find another LTI system with impulse response $\tilde{h}[n] \neq h[n]$ such that $\tilde{H}(e^{j\omega}) = H(e^{j\omega})$? Justify your answer. (3%)

Midterm 2012-5, 2012-6

5. [12] Consider a discrete-time signal $x[n]$ given by $x[n] = a^{|n|}$ with $|a| < 1$ and a continuous-time signal $y(t)$ given by $y(t) = \dfrac{1}{5 - 4\cos(2\pi t)}$ with period $T = 1$.

   (a) Derive the discrete-time Fourier transform (DT-FT) of $x[n]$ using the DT-FT analysis equation. [6]

   (b) Use the concept of duality to determine the Fourier series coefficients of $y(t)$. [6]

6. [8] Consider the discrete-time Fourier transform pair: $x[n] \xleftrightarrow{DT-FT} X(e^{j\omega})$. If another signal $y[n]$ is defined by the following relationship:

$$y[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases},$$

Show that $Y(e^{j\omega}) = X(e^{jm\omega})$, where $Y(e^{j\omega})$ is the Fourier transform of $y[n]$.

8. [16] Use some properties or derivations to answer the following questions.

(a) $x(t) = te^{-2|t-1|}$, what is $X(j\omega)$? [4]

(b) $x[n] = \left(\sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4})\right) * \left(\frac{\sin(\frac{\pi n}{6})}{\pi n} + \frac{\sin(\frac{\pi n}{2})}{\pi n}\right)$, what is $X(e^{j\omega})$? [6]

Note that * denotes convolution.

(c) $x(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)\left(\frac{\sin(2\pi t)}{2\pi t}\right)$, what is $X(j\omega)$? [6]