- Representation of Aperiodic Signals:
  the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by
  Linear Constant-Coefficient Difference Equations

Problem 5.12 (p.401) – Convolution, Multiplication [SS5:41]

5.12. Let

$$y[n] = \left(\frac{\sin \frac{\pi}{4} n}{\pi n}\right)^2 * \left(\frac{\sin \omega_c n}{\pi n}\right),$$

where $*$ denotes convolution and $|\omega_c| \leq \pi$. Determine a stricter constraint on ω_c which ensures that

$$y[n] = \left(\frac{\sin \frac{\pi}{4} n}{\pi n}\right)^2.$$

Problem 5.15 (p.402) – Convolution, Multiplication [SS5:41]

5.15. Let the inverse Fourier transform of $Y(e^{j\omega})$ be

$$y[n] = \left(\frac{\sin \omega_c n}{\pi n}\right)^2,$$

where $0 < \omega_c < \pi$. Determine the value of ω_c which ensures that

$$Y(e^{j\pi}) = \frac{1}{2}.$$

5.16. The Fourier transform of a particular signal is

$$X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(1/2)^k}{1 - \frac{1}{4}e^{-j(\omega-\pi/2k)}}.$$

It can be shown that

$$x[n] = g[n]q[n],$$

where $g[n]$ is of the form $\alpha^n u[n]$ and $q[n]$ is a periodic signal with period N.

(a) Determine the value of α.

(b) Determine the value of N.

(c) Is $x[n]$ real?

Problem 5.17 (p.402) – Duality [SS5:57-60]

5.17. The signal $x[n] = (-1)^n$ has a fundamental period of 2 and corresponding Fourier series coefficients a_k. Use duality to determine the Fourier series coefficients b_k of the signal $g[n] = a_n$ with a fundamental period of 2.

Problem 5.18 (p.402) – Duality [SS5:57-60]

5.18. Given the fact that

$$a^{|n|} \stackrel{\mathfrak{F}}{\longleftrightarrow} \frac{1-a^2}{1-2a\cos\omega + a^2}, \quad |a| < 1,$$

use duality to determine the Fourier series coefficients of the following continuous-time signal with period $T = 1$:

$$x(t) = \frac{1}{5-4\cos(2\pi t)}.$$