- Representation of Aperiodic Signals:
  the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by
  Linear Constant-Coefficient Difference Equations

Problem 5.6 (p.400) – Properties of FT [SS5:53]

5.6. Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties listed in Table 5.1.

(a) $x_1[n] = x[1-n] + x[-1-n]$

(b) $x_2[n] = \frac{x^*[-n]+x[n]}{2}$

(c) $x_3[n] = (n-1)^2 x[n]$

Problem 5.7 (p.401) – Properties of FT [SS5:53]

5.7. For each of the following Fourier transforms, use Fourier transform properties (Table 5.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) $X_1(e^{j\omega}) = e^{-j\omega}\sum_{k=1}^{10}(\sin k\omega)$

(b) $X_2(e^{j\omega}) = j\sin(\omega)\cos(5\omega)$

(c) $X_3(e^{j\omega}) = A(\omega) + e^{jB(\omega)}$ where

$$A(\omega) = \begin{cases} 1, & 0 \leq |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| \leq \pi \end{cases} \quad \text{and } B(\omega) = -\frac{3\omega}{2} + \pi.$$

Problem 5.8 (p.401) – Properties of FT [SS5:53, 54, 55]

5.8. Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\sin\frac{3}{2}\omega}{\sin\frac{\omega}{2}}\right) + 5\pi\delta(\omega), \qquad -\pi < \omega \leq \pi$$

Problem 5.9 (p.401) – Properties of FT [SS5:53, 54, 55]

5.9. The following four facts are given about a real signal $x[n]$ with Fourier trans $X(e^{jw})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] > 0$.
3. $\mathcal{Im}\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$.
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = 3$.

Determine $x[n]$.

Problem 5.10 (p.401) – Properties of FT [SS5:53, 54, 55]

5.10. Use Tables 5.1 and 5.2 in conjunction with the fact that

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$$

to determine the numerical value of

$$A = \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n.$$

Problem 5.11 (p.401) – Time Expansion [SS5:33-37]

5.11. Consider a signal $g[n]$ with Fourier transform $G(e^{j\omega})$. Suppose

$$g[n] = x_{(2)}[n],$$

where the signal $x[n]$ has a Fourier transform $X(e^{j\omega})$. Determine a real number α such that $0 < \alpha < 2\pi$ and $G(e^{j\omega}) = G(e^{j(\omega-\alpha)})$.