- Representation of Aperiodic Signals:
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Problem 4.10 (p.336) – Convolution, Multiplication [SS4:70-71]

4.10. **(a)** Use Tables 4.1 and 4.2 to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

(b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{+\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt$$

Problem 4.11 (p.336) – Convolution, Multiplication [SS4:70-71]

4.11. Given the relationships

$$y(t) = x(t) * h(t)$$

and

$$g(t) = x(3t) * h(3t),$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form

$$g(t) = Ay(Bt).$$

Determine the values of A and B.

Problem 4.12 (p.336) – Convolution, Multiplication [SS4:70-71]

4.12. Consider the Fourier transform pair

$$e^{-|t|} \overset{\mathfrak{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

(a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.

(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

Hint: See Example 4.13.

Problem 4.13 (p.336) – Convolution, Multiplication [SS4:70-71]

4.13. Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5),$$

and let

$$h(t) = u(t) - u(t - 2).$$

(a) Is $x(t)$ periodic?
(b) Is $x(t) * h(t)$ periodic?
(c) Can the convolution of two aperiodic signals be periodic?

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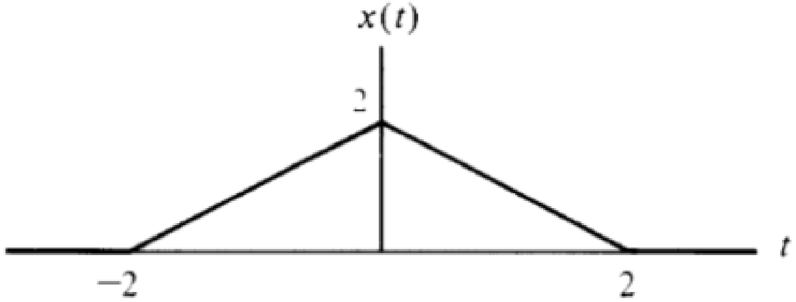
7. (10%) Given the following triangular signal $x(t)$.



a) (4%) Express $x(t)$ as the convolution of a rectangular pulse with itself, i.e., find a rectangular pulse $r(t)$, such that $x(t) = r(t) * r(t)$. Justify your answer.

b) (4%) In addition to using two identical rectangular pulses, express $x(t)$ as the convolution of two different functions, i.e., find $p(t)$ and $q(t)$ ($p(t) \neq q(t)$) such that $x(t) = p(t) * q(t)$. Justify your answer.

c) (2%) Determine the Fourier transform $X(jw)$ of the signal $x(t)$.

Problem 4.14 (p.337) – Facts [SS4:70-71]

4.14. Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

1. $x(t)$ is real and nonnegative.
2. $\mathfrak{F}^{-1}\{(1 + j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t.
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$.

Determine a closed-form expression for $x(t)$.

Problem 4.15 (p.337) – Facts [SS4:70-71]

4.15. Let $x(t)$ be a signal with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

1. $x(t)$ is real.

2. $x(t) = 0$ for $t \leq 0$.

3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \Re e\{X(j\omega)\}e^{j\omega t}\,d\omega = |t|e^{-|t|}$.

Determine a closed-form expression for $x(t)$.

Problem 4.16 (p.337) – Convolution, Multiplication [SS4:70-71]

4.16. Consider the signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\frac{\pi}{4})}{(k\frac{\pi}{4})} \delta(t - k\frac{\pi}{4}).$$

(a) Determine $g(t)$ such that

$$x(t) = \left(\frac{\sin t}{\pi t}\right) g(t).$$

(b) Use the multiplication property of the Fourier transform to argue that $X(j\omega)$ is periodic. Specify $X(j\omega)$ over one period.

Problem 4.17 (p.337) – Rea, Imaginary, Even, Odd [SS4:35]

4.17. Determine whether each of the following statements is true or false. Justify your answers.

(a) An odd and imaginary signal always has an odd and imaginary Fourier transform.

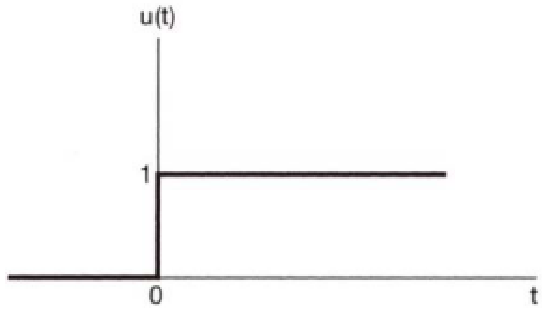
(b) The convolution of an odd Fourier transform with an even Fourier transform is always odd.

Problem 4.18 (p.337) – Convolution, Multiplication [SS4:70-71]

4.18. Find the impulse response of a system with the frequency response

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

10. (10 %) Consider the following unit step function.

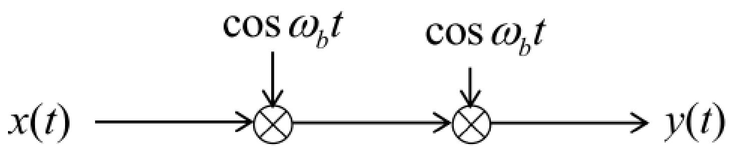


a) (4%) Find and sketch the even part $u_e(t)$ and the odd part $u_o(t)$ of $u(t)$. Justify your answer.

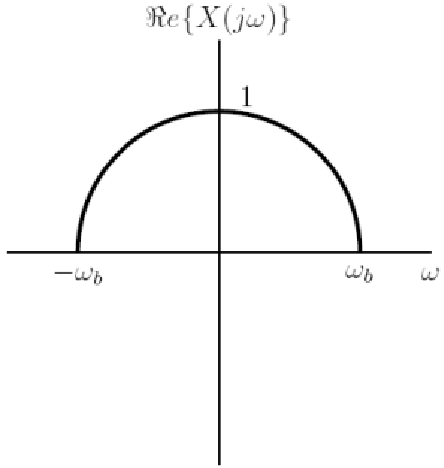
b) (4%) Find the Fourier transforms $U_e(jw)$ and $U_o(jw)$ of $u_e(t)$ and $u_o(t)$, respectively. Justify your answer.

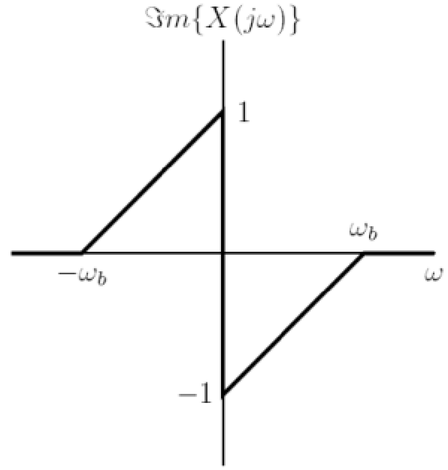
c) (2%) Find the Fourier transform $U(jw)$ of $u(t)$. Justify your answer.

6. (8 %) Suppose the Fourier transform $X(j\omega)$ of the input $x(t)$ to the following system



has real and imaginary parts given below:





Provide a sketch of the real and imaginary parts of $Y(j\omega)$.

10. (8 %) Consider an impulse train $p(t)$ with period T and an aperiodic signal $x(t)$ with zero value outside the interval $-T/2 \le t < T/2$, that is, $x(t) = 0$ when $t < -T/2$ or when $t \ge T/2$. Also consider a periodic signal $\tilde{x}(t)$ with period T and $\tilde{x}(t) = x(t), \ -T/2 \le t < T/2$.

a) Suppose $p(t)$ can be expressed as $\sum_{K=-\infty}^{\infty} \delta(t-kT)$. Show that $\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j2\pi kt/T}$ is also a valid expression of $p(t)$. That is, prove that

$$\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j2\pi kt/T}.$$

b) Take the Fourier transform of each side of the above equation and use the two relations

i. $\tilde{x}(t)$ can be considered the convolution of $x(t)$ with $p(t)$

ii. $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-T/2}^{T/2} x(t)e^{-j\omega t}dt$

to show that $\tilde{X}(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$, where a_k is the Fourier series coefficient of $\tilde{x}(t)$.

8. [16] Use some properties or derivations to answer the following questions.

(a) $x(t) = te^{-2|t-1|}$, what is $X(j\omega)$? [4]

(b) $x[n] = \left(\sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4})\right) * \left(\frac{\sin(\frac{\pi n}{6})}{\pi n} + \frac{\sin(\frac{\pi n}{2})}{\pi n}\right)$, what is $X(e^{j\omega})$? [6]

Note that * denotes convolution.

(c) $x(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)\left(\frac{\sin(2\pi t)}{2\pi t}\right)$, what is $X(j\omega)$? [6]

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Problem 4.19 (p.337) – Frequency Response [SS4:75]

4.19. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}.$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

Determine $x(t)$.

Problem 4.20 (p.337) – RLC [SS4:75]

3.20. Consider a causal LTI system implemented as the *RLC* circuit shown in Figure P3.20. In this circuit, $x(t)$ is the input voltage. The voltage $y(t)$ across the capacitor is considered the system output.

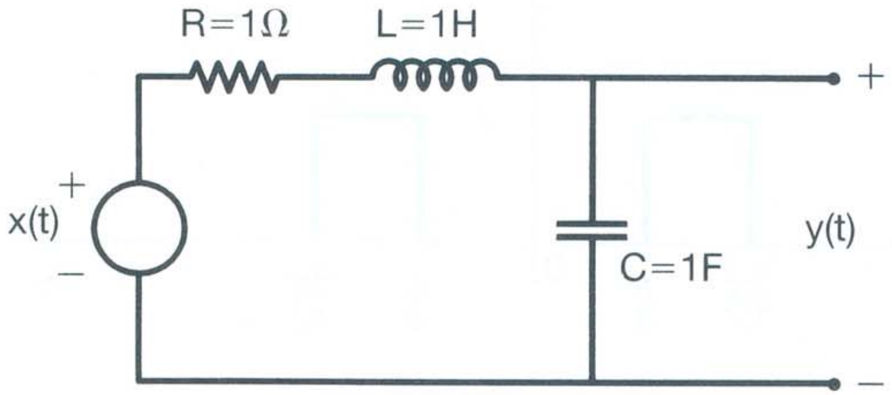

Figure P3.20

(a) Find the differential equation relating $x(t)$ and $y(t)$.

(b) Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.

(c) Determine the output $y(t)$ if $x(t) = \sin(t)$.

4.20. Find the impulse response of the causal LTI system represented by the *RLC* circuit considered in Problem 3.20. Do this by taking the inverse Fourier transform of the circuit's frequency response. You may use Tables 4.1 and 4.2 to help evaluate the inverse Fourier transform.

8. (10%) Consider that the input and output of an LTI system are related by the following linear constant coefficient differential equation (LCCDE):

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

a) (2%) Show that the left-hand side of the equation has a Fourier transform that can be expressed as: $A(jw)Y(jw)$, where $Y(jw)$ is the Fourier Transform of $y(t)$. Find $A(jw)$.

b) (2%) Show that the right-hand side of the equation has a Fourier transform that can be expressed as: $B(jw)X(jw)$, where $X(jw)$ is the Fourier Transform of $x(t)$. Find $B(jw)$.

c) (2%) Show that $Y(jw)$ can be expressed as $Y(jw) = H(jw)X(jw)$, and find $H(jw)$.

d) (4%) Find the impulse response $h(t)$ of the LTI system.

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7. (12%) Consider that the input and output of an LTI system are related by the following linear constant coefficient differential equation (LCCDE):

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

a) Find the impulse response $h(t)$ of the LTI system. (3%)

b) Is this system causal ? Justify your answer. (3%)

c) Is this system stable ? Justify your answer. (3%)

d) Find the output $y(t)$ of the LTI system when the input signal $x(t) = e^{-t}u(t)$, where $u(t)$ is the unit step function. (3%)

7. [12] Consider a system characterized by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 9y(t) = \frac{dx(t)}{dt} + 5x(t).$$

(a) Find the impulse response $h(t)$ of the system. [6]

(b) Find the output of the system with the input signal: $x(t) = e^{-3t}u(t)$. [6]