- Representation of Aperiodic Signals:
  the Continuous-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Continuous-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Systems Characterized by
  Linear Constant-Coefficient Differential Equations

Problem 4.6 (p.335) – Properties of FT [SS4:27]

4.6. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in Table 4.1.

(a) $x_1(t) = x(1-t) + x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2}x(t-1)$

Problem 4.7 (p.335) – Rea, Imaginary, Even, Odd [SS4:35]

4.7. For each of the following Fourier transforms, use Fourier transform properties (Table 4.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) $X_1(j\omega) = u(\omega) - u(\omega - 2)$

(b) $X_2(j\omega) = \cos(2\omega)\sin(\frac{\omega}{2})$

(c) $X_3(j\omega) = A(\omega)e^{jB(\omega)}$, where $A(\omega) = (\sin 2\omega)/\omega$ and $B(\omega) = 2\omega + \frac{\pi}{2}$

(d) $X(j\omega) = \sum_{k=-\infty}^{\infty}(\frac{1}{2})^{|k|}\,\delta(\omega - \frac{k\pi}{4})$

Problem 4.8 (p.335) – Differentiation, Integration [SS4:38]

4.8. Consider the signal

$$x(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ t + \frac{1}{2}, & -\frac{1}{2} \le t \le \frac{1}{2}. \\ 1, & t > \frac{1}{2} \end{cases}$$

(a) Use the differentiation and integration properties in Table 4.1 and the Fourier transform pair for the rectangular pulse in Table 4.2 to find a closed-form expression for $X(j\omega)$.

(b) What is the Fourier transform of $g(t) = x(t) - \frac{1}{2}$?

Problem 4.9 (p.335) – Differentiation, Integration [SS4:70-71]

4.9. Consider the signal

$$x(t) = \begin{cases} 0, & |t| > 1 \\ (t+1)/2, & -1 \le t \le 1 \end{cases}.$$

(a) With the help of Tables 4.1 and 4.2, determine the closed-form expression for $X(j\omega)$.

(b) Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$?

3. (8 %) Suppose a continuous-time signal $x(t)$ is an odd signal. Show that the Fourier transform $X(j\omega)$ of $x(t)$ is given by

$$X(j\omega) = -2j\int_0^\infty x(t)\sin\omega t\ dt\ .$$

7. [12] Let $X(j\omega)$ be the Fourier transform of a signal $x(t)$. Assume that another signal $g(t)$ has the same shape as that of $X(j\omega)$, i.e., $g(t) = X(jt)$.
   (a) Show that the Fourier transform $G(j\omega)$ of $g(t)$ has the same shape as **$2\pi x(-t)$. [6]**
   (b) Using the result of Part (a), find the Fourier transform of a signal $x(t) = e^{jQt}$, where Q is a real number. Justify your answer. **[6]**