- Representation of Aperiodic Signals:
  the Continuous-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Continuous-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Systems Characterized by
  Linear Constant-Coefficient Differential Equations

Problem 4.1-4.2 (p.334) – FT of Aperiodic Signals [SS4:11]

4.1. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $e^{-2(t-1)}u(t-1)$ **(b)** $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1)+\delta(t-1)$ **(b)** $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

Problem 4.4-4.5 (p.334) – IFT [SS4:11]

4.4. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transforms of:

(a) $X_1(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)$

(b) $X_2(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2 \\ -2, & -2 \leq \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$

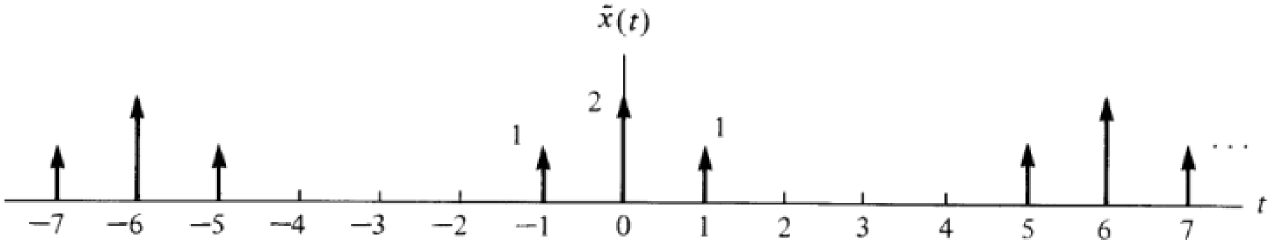
4.5. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transform of $X(j\omega) = |X(j\omega)|e^{j\sphericalangle X(j\omega)}$, where

$$|X(j\omega)| = 2\{u(\omega + 3) - u(\omega - 3)\},$$

$$\sphericalangle X(j\omega) = -\frac{3}{2}\omega + \pi.$$

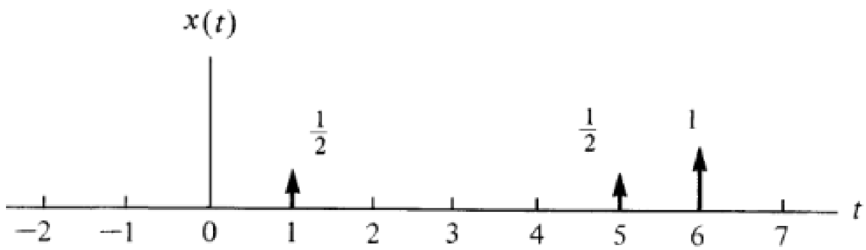
Use your answer to determine the values of t for which $x(t) = 0$.

6. (10%) Consider the periodic signal $\tilde{x}(t)$ shown in the following picture, which is composed solely of impulses.



a) (3%) Find the Fourier series a_k of $\tilde{x}(t)$.

b) (3%) Find the Fourier transform $X(jw)$ of the following signal $x(t)$, which is composed of only three impulses.



c) (2%) $\tilde{x}(t)$ can be expressed as $x(t)$ periodically repeated, i.e.,

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k x(t - kT)$$

Determine c_k and T and demonstrate graphically that the above equation is valid.

d) (2%) Verify that the Fourier series of $\tilde{x}(t)$ is composed of scaled samples of $X(jw)$.

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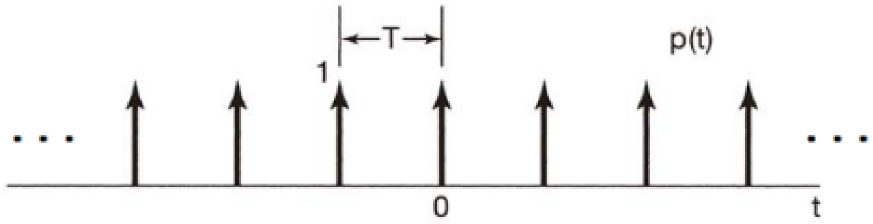
Problem 4.3 (p.334) – FT of Periodic Signals [SS4:22]

4.3. Determine the Fourier transform of each of the following periodic signals:

(a) $\sin(2\pi t + \frac{\pi}{4})$ **(b)** $1 + \cos(6\pi t + \frac{\pi}{8})$

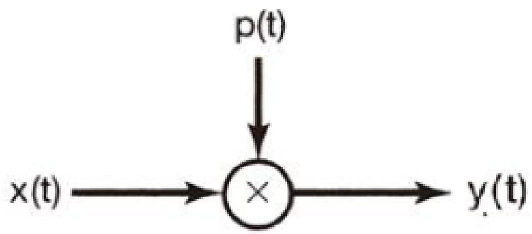
9. (10 %) Answer the following questions.

a) (2%) Consider the following periodic signal $p(t)$.

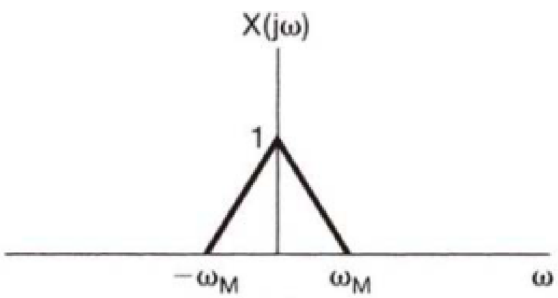


Find the Fourier transform $P(jw)$ of $p(t)$.

b) (3%) Consider the following operation.



Assume that the Fourier transform $X(jw)$ of one signal $x(t)$ is as follows.



Assume that $w_M < \frac{\pi}{T}$. Sketch the Fourier transform $Y(jw)$ of $y(t)$ and express $Y(jw)$ in the form: $Y(jw) = A\sum^{\infty} X(jw+kB)$. What are the value of A and B?

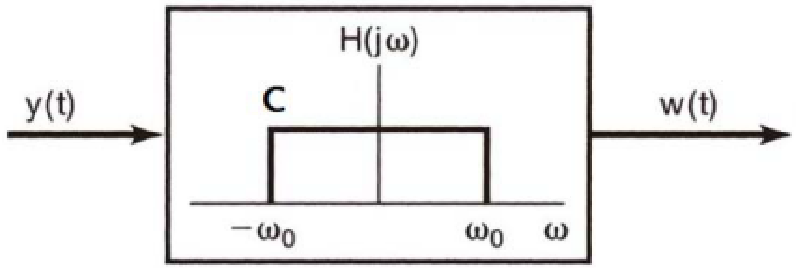
c) (3%) Let $r[n] = y(nT)$ and assume that:

$$Y(jw) = \sum_{n=-\infty}^{\infty} y(nT)e^{-jwnT}, \text{ and}$$

$$R(e^{jw}) = \sum_{n=-\infty}^{\infty} r[n]e^{-jwn}.$$

Show that $R(e^{jw}) = Y(\frac{jw}{T})$ and sketch the graph of $R(e^{jw})$.

d) (2%) Consider the following operation:



where $H(jw)$ an ideal lowpass filter with a magnitude C in the passband between $-w_o$ and w_o. Find the value of C and the range of w_o, such that $w(t) = x(t)$. Justify your answer.

3. [16] Consider the unit impulse response $h(t) = \cos(\pi t)\sin(10\pi t)$ of a linear-time-invariant system.

(a) Determine its fundamental period. [4]

(b) Determine the Fourier series coefficients of $h(t)$. [4]

(c) Determine the Fourier transform of the even part of $h(t)$. [4]

(d) Determine the response of the system to the input $x(t) = e^{-at}u(t),\ a > 0$.[4]