Outline

- A Historical Perspective
- The Response of LTI Systems to Complex Exponentials
- Fourier Series Representation of Continuous-Time Periodic Signals
- Convergence of the Fourier Series
- Properties of Continuous-Time Fourier Series
- Fourier Series Representation of Discrete-Time Periodic Signals
- Properties of Discrete-Time Fourier Series
- Fourier Series & LTI Systems
- Filtering & Examples of CT & DT Filters

Problem 3.1 (pp.250) – FS coefficients [SS3:21]

3.1. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are

$$a_1 = a_{-1} = 2, a_3 = a^*_{-3} = 4j.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

Problem 3.3 (p.251) – FS coefficients [SS3:21]

3.3. For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

Problem 3.4 (p.251) – FS coefficients [SS3:21]

3.4. Use the Fourier series analysis equation (3.39) to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

Problem 3.22 (p.255) – FS coefficients [SS3:21]

3.22. Determine the Fourier series representations for the following signals:

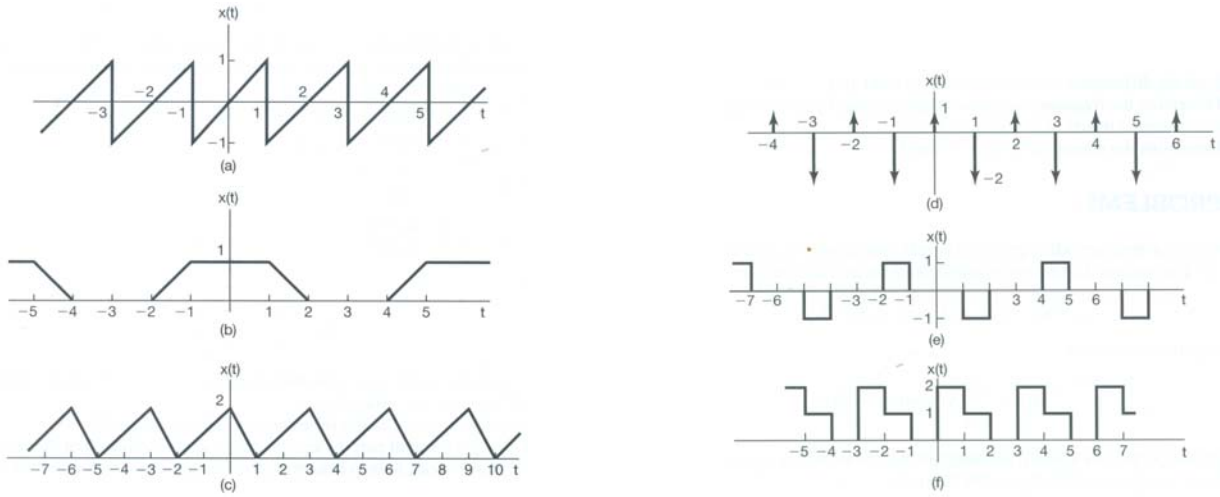
(a) Each $x(t)$ illustrated in Figure P3.22(a)–(f).

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1$$

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$



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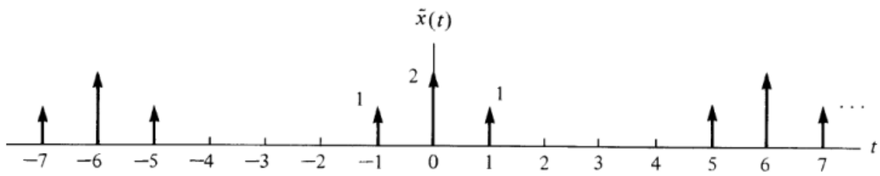
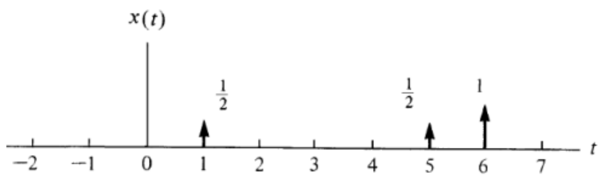
6. (10%) Consider the periodic signal $\tilde{x}(t)$ shown in the following picture, which is composed solely of impulses.



a) (3%) Find the Fourier series a_k of $\tilde{x}(t)$.

b) (3%) Find the Fourier transform $X(jw)$ of the following signal $x(t)$, which is composed of only three impulses.



c) (2%) $\tilde{x}(t)$ can be expressed as $x(t)$ periodically repeated, i.e.,

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k x(t - kT)$$

Determine c_k and T and demonstrate graphically that the above equation is valid.

d) (2%) Verify that the Fourier series of $\tilde{x}(t)$ is composed of scaled samples of $X(jw)$.

8. (8%) Consider the signal $x(t) = \cos 2\pi t$. Since $x(t)$ is periodic with a fundamental period of 1, it is also periodic with a period of N, where N is any positive integer. What are the Fourier series coefficients of $x(t)$ if we regard it as a periodic signal with period 3?

3. [16] Consider the unit impulse response $h(t) = \cos(\pi t)\sin(10\pi t)$ of a linear-time-invariant system.
   (a) Determine its fundamental period. [4]
   (b) Determine the Fourier series coefficients of $h(t)$. [4]
   (c) Determine the Fourier transform of the even part of $h(t)$. [4]
   (d) Determine the response of the system to the input $x(t) = e^{-at}u(t)$, $a > 0$.[4]

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Problem 3.5 (p.251) – Shifting, Reversal [SS3:46-47]

3.5. Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that

$$x_2(t) = x_1(1-t) + x_1(t-1),$$

how is the fundamental frequency ω_2 of $x_2(t)$ related to ω_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k. You may use the properties listed in Table 3.1.

Problem 3.6 (p.251) – Real-Valued, Even [SS3:52-53]

3.6. Consider three continuous-time periodic signals whose Fourier series representations are as follows:

$$x_1(t) = \sum_{k=0}^{100} \left(\frac{1}{2}\right)^k e^{jk\frac{2\pi}{50}t},$$

$$x_2(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}t},$$

$$x_3(t) = \sum_{k=-100}^{100} j \sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}t}.$$

Use Fourier series properties to help answer the following questions:

(a) Which of the three signals is/are real valued?

(b) Which of the three signals is/are even?

Problem 3.7 (p.251) – Differentiation [SS3:50]

3.7. Suppose the periodic signal $x(t)$ has fundamental period T and Fourier coefficients a_k. In a variety of situations, it is easier to calculate the Fourier series coefficients b_k for $g(t) = dx(t)/dt$, as opposed to calculating a_k directly. Given that

$$\int_T^{2T} x(t)\,dt = 2,$$

find an expression for a_k in terms of b_k and T. You may use any of the properties listed in Table 3.1 to help find the expression.

Problem 3.8 (p.252) – Table 3.1 [SS3:55]

3.8. Suppose we are given the following information about a signal $x(t)$:

1. $x(t)$ is real and odd.
2. $x(t)$ is periodic with period $T = 2$ and has Fourier coefficients a_k.
3. $a_k = 0$ for $|k| > 1$.
4. $\frac{1}{2}\int_0^2 |x(t)|^2\,dt = 1$.

Specify two different signals that satisfy these conditions.