Outline

- Discrete-Time Linear Time-Invariant Systems
  - The convolution sum
- Continuous-Time Linear Time-Invariant Systems
  - The convolution integral
- Properties of Linear Time-Invariant Systems
- Causal Linear Time-Invariant Systems Described by Differential & Difference Equations
- Singularity Functions

Problem 2.25 (p.144) – Distributive [SS2:46]

2.25. Let the signal

$$y[n] = x[n] * h[n],$$

where

$$x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$$

and

$$h[n] = \left(\frac{1}{4}\right)^n u[n+3].$$

(a) Determine $y[n]$ *without* utilizing the distributive property of convolution.
(b) Determine $y[n]$ *utilizing* the distributive property of convolution.

Problem 2.26 (p.144) – Associative [SS2:49]

2.26. Consider the evaluation of

$$y[n] = x_1[n] * x_2[n] * x_3[n],$$

where $x_1[n] = (0.5)^n u[n]$, $x_2[n] = u[n+3]$, and $x_3[n] = \delta[n] - \delta[n-1]$.

(a) Evaluate the convolution $x_1[n] * x_2[n]$.

(b) Convolve the result of part (a) with $x_3[n]$ in order to evaluate $y[n]$.

(c) Evaluate the convolution $x_2[n] * x_3[n]$.

(d) Convolve the result of part (c) with $x_1[n]$ in order to evaluate $y[n]$.

Problem 2.14 & 2.15 (p.140) – Stable LTI [SS2:60-61]

2.14. Which of the following impulse responses correspond(s) to stable LTI systems?

(a) $h_1(t) = e^{-(1-2j)t}u(t)$ **(b)** $h_2(t) = e^{-t}\cos(2t)u(t)$

2.15. Which of the following impulse responses correspond(s) to stable LTI systems?

(a) $h_1[n] = n\cos(\frac{\pi}{4}n)u[n]$ **(b)** $h_2[n] = 3^n u[-n+10]$

Problem 2.28 & 2.29 (p.144) – Causal, Stable [SS2:57,60,61]

2.28. The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(a) $h[n] = (\frac{1}{5})^n u[n]$
(b) $h[n] = (0.8)^n u[n+2]$
(c) $h[n] = (\frac{1}{2})^n u[-n]$
(d) $h[n] = (5)^n u[3-n]$
(e) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$
(f) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$
(g) $h[n] = n(\frac{1}{3})^n u[n-1]$

2.29. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(a) $h(t) = e^{-4t} u(t-2)$
(b) $h(t) = e^{-6t} u(3-t)$
(c) $h(t) = e^{-2t} u(t+50)$
(d) $h(t) = e^{2t} u(-1-t)$
(e) $h(t) = e^{-6|t|}$
(f) $h(t) = te^{-t} u(t)$
(g) $h(t) = (2e^{-t} - e^{(t-100)/100})u(t)$

Problem 2.33 (p.146) – Linear [SS2:82]

2.33. Consider a system whose input $x(t)$ and output $y(t)$ satisfy the first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t). \qquad \text{(P2.33–1)}$$

The system also satisfies the condition of initial rest.

(a) (i) Determine the system output $y_1(t)$ when the input is $x_1(t) = e^{3t}u(t)$.
(ii) Determine the system output $y_2(t)$ when the input is $x_2(t) = e^{2t}u(t)$.
(iii) Determine the system output $y_3(t)$ when the input is $x_3(t) = \alpha e^{3t}u(t) + \beta e^{2t}u(t)$, where α and β are real numbers. Show that $y_3(t) = \alpha y_1(t) + \beta y_2(t)$.
(iv) Now let $x_1(t)$ and $x_2(t)$ be arbitrary signals such that

$$x_1(t) = 0, \text{ for } t < t_1,$$
$$x_2(t) = 0, \text{ for } t < t_2.$$

Letting $y_1(t)$ be the system output for input $x_1(t)$, $y_2(t)$ be the system output for input $x_2(t)$, and $y_3(t)$ be the system output for $x_3(t) = \alpha x_1(t) + \beta x_2(t)$, show that

$$y_3(t) = \alpha y_1(t) + \beta y_2(t).$$

We may therefore conclude that the system under consideration is linear.

Problem 2.33 (p.146) – Linear [SS2:82]

(b) (i) Determine the system output $y_1(t)$ when the input is $x_1(t) = Ke^{2t}u(t)$.

(ii) Determine the system output $y_2(t)$ when the input is $x_2(t) = Ke^{2(t-T)}u(t-T)$. Show that $y_2(t) = y_1(t-T)$.

(iii) Now let $x_1(t)$ be an arbitrary signal such that $x_1(t) = 0$ for $t < t_0$. Letting $y_1(t)$ be the system output for input $x_1(t)$ and $y_2(t)$ be the system output for $x_2(t) = x_1(t-T)$, show that

$$y_2(t) = y_1(t-T).$$

We may therefore conclude that the system under consideration is time invariant. In conjunction with the result derived in part (a), we conclude that the given system is LTI. Since this system satisfies the condition of initial rest, it is causal as well.