## Outline



- **Discrete-Time Linear Time-Invariant Systems**
  - The convolution sum
- Continuous-Time Linear Time-Invariant Systems
  - The convolution integral
- Properties of Linear Time-Invariant Systems
- Causal Linear Time-Invariant Systems Described by Differential & Difference Equations
- Singularity Functions

$x[n] \rightarrow$ h[n] $\rightarrow y[n]$ $\qquad$ $x(t) \rightarrow$ h(t) $\rightarrow y(t)$

## Problem 2.1 (p.137) – Convolution Sum [SS2:12]



**2.1.** Let

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3] \quad \text{and} \quad h[n] = 2\delta[n+1] + 2\delta[n-1].$$

Compute and plot each of the following convolutions:

**(a)** $y_1[n] = x[n] * h[n]$ **(b)** $y_2[n] = x[n+2] * h[n]$

**(c)** $y_3[n] = x[n] * h[n+2]$

# Problem 2.4 (p.138) – Convolution Sum [SS2:12]



**2.4.** Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases},$$

$$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{otherwise} \end{cases}.$$

# Problem 2.21 (p.141) – Convolution Sum [SS2:12]



**2.21.** Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

**(a)** $\left.\begin{aligned} x[n] &= \alpha^n u[n], \\ h[n] &= \beta^n u[n], \end{aligned}\right\} \quad \alpha \neq \beta$

**(b)** $x[n] = h[n] = \alpha^n u[n]$

**(c)** $x[n] = (-\frac{1}{2})^n u[n-4]$
$h[n] = 4^n u[2 - n]$

**(d)** $x[n]$ and $h[n]$ are as in Figure P2.21.

## Problem 2.7 (p.138) – Convolution Sum [SS2:12]



**2.7.** A linear system $S$ has the relationship

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n-4]$.

**(a)** Determine $y[n]$ when $x[n] = \delta[n-1]$.
**(b)** Determine $y[n]$ when $x[n] = \delta[n-2]$.
**(c)** Is $S$ LTI?
**(d)** Determine $y[n]$ when $x[n] = u[n]$.

## Midterm 2014-4, 2013-5



4. (10%) Consider a discrete-time, linear, time-invariant system that has unit sample response $h[n] = (\frac{1}{2})^n u[n]$ and input $x[n]$.

   a) (3%) Evaluate and sketch the response $y_1[n]$ of the system if $x[n] = x_1[n] = \delta[n-d]$, for some integer $d > 0$. Justify your answer.
   b) (3%) Evaluate and sketch the response $y_2[n]$ of the system if $x[n] = x_2[n] = u[n]$. Justify your answer.
   c) (4%) Identify the relationship between $y_1[n]$ and $y_2[n]$. Justify your answer.

5. (8 %) Compute the convolution of the two sequences

$$x[n] = \begin{cases} a^n, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} 1, & 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}.$$

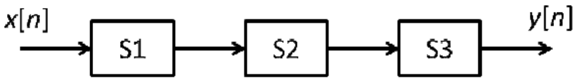
4. [10] Consider three systems with the following input $i[n]$ – output $o[n]$ relationships:

$$S1: o[n] = \begin{cases} i[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$S2: o[n] = i[n] + \frac{1}{2} i[n-1] + \frac{1}{4} i[n-2].$$

$$S3: o[n] = i[2n]$$

Suppose that these systems are interconnected in series as follows.




(a) Find the input-output relationship between $x[n]$ and $y[n]$. [6]

(b) Find the response of the overall system to the input $x[n] = \delta[n] - \delta[n-1]$. [4]