

- Introduction
- Continuous-Time & Discrete-Time Signals
- Transformations of the Independent Variable
- Exponential & Sinusoidal Signals
- The Unit Impulse & Unit Step Functions
- Continuous-Time & Discrete-Time Systems
- Basic System Properties

Problem 1.16, 17 (pp. 58-59) – memory, invertible, causal, linear [SS1-78]

1.16. Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n - 2].$$

- (a) Is the system memoryless?
- (b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.
- (c) Is the system invertible?

1.17. Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

- (a) Is this system causal?
- (b) Is this system linear?

1.18. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.

- (a) Is this system linear?
- (a) Is this system time-invariant?
- (c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C . We conclude that the given system is stable. Express C in terms of B and n_0 .

(a) $y(t) = x(t - 2) + x(2 - t)$

(b) $y(t) = [\cos(3t)]x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t - 2), & t \geq 0 \end{cases}$

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t - 2), & x(t) \geq 0 \end{cases}$

(f) $y(t) = x(t/3)$

(g) $y(t) = \frac{dx(t)}{dt}$

(a) $y[n] = x[-n]$

(b) $y[n] = x[n - 2] - 2x[n - 8]$

(c) $y[n] = nx[n]$

(d) $y[n] = \mathcal{E}\{x[n - 1]\}$

(e) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n + 1], & n \leq -1 \end{cases}$

(f) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

(g) $y[n] = x[4n + 1]$

- (a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P1.31(a) is the signal $y_1(t)$ illustrated in Figure P1.31(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P1.31(c).
- (b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P1.31(d).

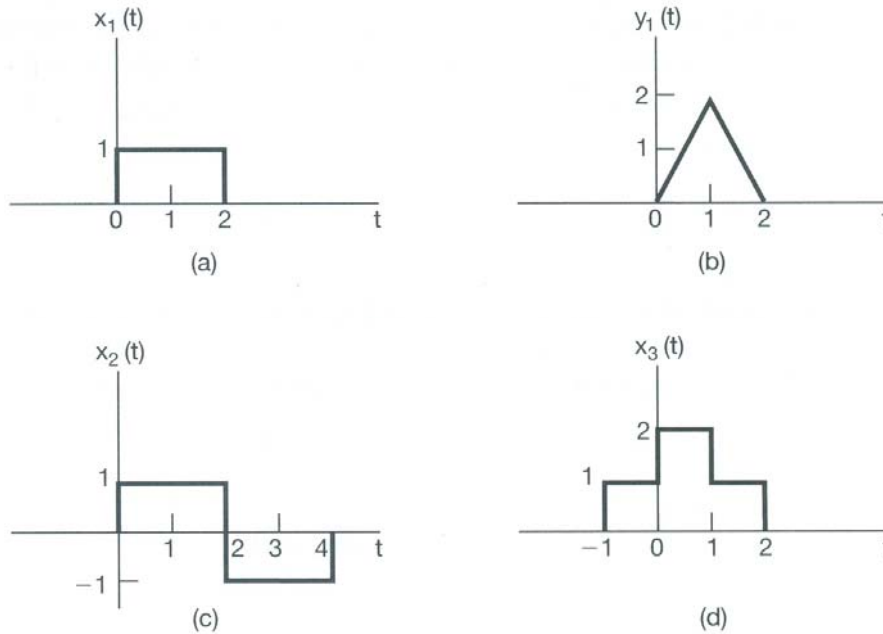


Figure P1.31

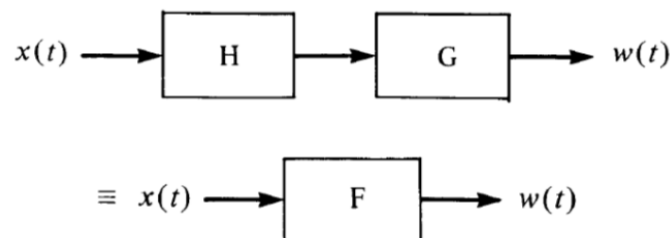
Midterm: 2014-3

3. (10%) Consider the following systems:

$$H : y(t) = x(t+1)$$

$$G : y(t) = x(2t)$$

- a) (4%) What is H^{-1} , the inverse of H ? What is G^{-1} , the inverse of G ? Justify your answer.
- b) (6%) Consider the system in the following figure. That is, F is equivalent to the interconnection of H and G . Find F^{-1} , the inverse of F and draw it in block diagram form in terms of H^{-1} and G^{-1} . Justify your answer.



1. (24 %) The response $y[n]$ of a discrete-time system is related to the input $x[n]$ by

$$y[n] = \frac{1}{4}(x[n-1] + 2x[n] + x[n+1]).$$

- Is it a linear time-invariant system? Justify your answer. (3%)
- Is the system causal? Justify your answer. (3%)
- Is the system stable? Justify your answer. (3%)
- Is the system a recursive filter? Justify your answer. (3%)
- Determine the impulse response $h[n]$. (3%)
- Determine the step response of the system. (3%)
- Determine the frequency response $H(e^{j\omega})$ of the system by the eigenfunction approach and by Fourier transform. (3%)
- Is it possible to find another LTI system with impulse response $\tilde{h}[n] \neq h[n]$ such that $\tilde{H}(e^{j\omega}) = H(e^{j\omega})$? Justify your answer. (3%)

1. [20] Suppose the output $y[n]$ of an LTI system is related to the input $x[n]$ by the following equation:

$$y[n] = \sum_{k=n}^{n+b} x[k-a], \text{ where } a \text{ and } b \text{ are two finite positive integers.}$$

- Is it possible for the system to be causal? Justify your answer. [4]
 - Is this system time-invariant? Justify your answer. [4]
 - Is this system linear? Justify your answer. [4]
 - Determine the impulse response of the system. [4]
 - If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C . We concluded that the given system is stable. Express C in terms of B , a , and b . [4]
1. [28] Suppose the output $y[n]$ of an LTI system is related to the input $x[n]$ by the following equation:

$$y[n] = \sum_{k=n-2}^{n+2} x[k].$$

- Is the system causal? Justify your answer. [4]
- Is this system time-invariant? Justify your answer. [4]
- Is this system linear? Justify your answer. [4]
- Determine the impulse response of the system. [4]
- Does the system introduce a delay to the input? Justify your answer. [4]
- Draw a diagram of the frequency response and show the frequencies at which the frequency response has value zero. [4]
- If the system equation becomes

$$y[n] = x[n-4] + x[n-2] + x[n] + x[n+2] + x[n+4],$$

determine the frequency response of this system by the property of time expansion. [4]