Outline

- Introduction
- Continuous-Time & Discrete-Time Signals
- Transformations of the Independent Variable
- Exponential & Sinusoidal Signals
- The Unit Impulse & Unit Step Functions
- Continuous-Time & Discrete-Time Systems
- Basic System Properties

Problem 1.1, 1.2 (p. 57) – x+jy [SS1:28]

1.1. Express each of the following complex numbers in Cartesian form $(x + jy)$: $\frac{1}{2}e^{j\pi}$, $\frac{1}{2}e^{-j\pi}$, $e^{j\pi/2}$, $e^{-j\pi/2}$, $e^{j5\pi/2}$, $\sqrt{2}e^{j\pi/4}$, $\sqrt{2}e^{j9\pi/4}$, $\sqrt{2}e^{-j9\pi/4}$, $\sqrt{2}e^{-j\pi/4}$.

1.2. Express each of the following complex numbers in polar form $(re^{j\theta}$, with $-\pi < \theta \leq \pi)$: $5, -2, -3j, \frac{1}{2} - j\frac{\sqrt{3}}{2}, 1+j, (1-j)^2, j(1-j), (1+j)/(1-j), (\sqrt{2}+j\sqrt{2})/(1+j\sqrt{3})$.

Problem 1.9, 1.10, 1.11 (pp. 57-58) – Fundamental Period [SS1:21]

1.9. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.

(a) $x_1(t) = je^{j10t}$ **(b)** $x_2(t) = e^{(-1+j)t}$ **(c)** $x_3[n] = e^{j7\pi n}$

(d) $x_4[n] = 3e^{j3\pi(n+1/2)/5}$ **(e)** $x_5[n] = 3e^{j3/5(n+1/2)}$

1.10. Determine the fundamental period of the signal $x(t) = 2\cos(10t+1) - \sin(4t-1)$.

1.11. Determine the fundamental period of the signal $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

Problem 1.25, 1.26 (p. 61) – Fundamental Period [SS1:21]

1.25. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$ **(b)** $x(t) = e^{j(\pi t - 1)}$

(c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$ **(d)** $x(t) = \mathcal{E}v\{\cos(4\pi t)u(t)\}$

(e) $x(t) = \mathcal{E}v\{\sin(4\pi t)u(t)\}$ **(f)** $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$ **(b)** $x[n] = \cos(\frac{n}{8} - \pi)$ **(c)** $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$ **(e)** $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

Problem 1.12 (p. 58) – δ[n] & u[n] [SS1:57]

1.12. Consider the discrete-time signal

$$x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k].$$

Determine the values of the integers M and n_0 so that $x[n]$ may be expressed as

$$x[n] = u[Mn - n_0].$$

Problem 1.13 (p. 58) – δ(t) & integral [SS1:57]

1.13. Consider the continuous-time signal

$$x(t) = \delta(t+2) - \delta(t-2).$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Problem 1.14 (p. 58) – impulse train & derivative [SS1:57]

1.14. Consider a periodic signal

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ -2, & 1 < t < 2 \end{cases}$$

with period $T = 2$. The derivative of this signal is related to the "impulse train"

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

with period $T = 2$. It can be shown that

$$\frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2).$$

Determine the values of A_1, t_1, A_2, and t_2.

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2. (10%) Consider the signals:

$$x(t) = \cos(\frac{2\pi t}{3}) + 2\sin(\frac{16\pi t}{3})$$

$$y(t) = \sin(\pi t)$$

a) (5%) Show that $z(t) = x(t)y(t)$ is periodic, and find the fundamental period of $z(t)$.

b) (5%) Write $z(t)$ as a linear combination of harmonically related complex exponentials. That is, find a number T and complex numbers c_k such that:

$$z(t) = \sum_k c_k e^{jk(2\pi/T)t} .$$

4. (8 %) Let V be the set of all periodic discrete-time signals with fundamental period N. Consider the functions $\phi_k[n] = e^{jk\frac{2\pi}{N}n}$, $k = 0, \pm 1, \pm 2, \cdots$.

a) Show that

$$\sum_{n=<N>} \phi_k[n] = \begin{cases} N, & k = 0, \pm N, \pm 2N, \cdots \\ 0, & \text{otherwise} \end{cases} . \text{ (4\%)}$$

b) For any element $x[n]$ in V, is it true that $x[n]$ can be generated by the linear combinations of the functions $\phi_k[n]$, i.e., $x[n] = \sum_{k=<N>} d_k \phi_k[n]$ with some coefficients d_k? If your answer is yes, determine the coefficients d_k. If your answer is no, explain why. (4%)