SPRING 2010

即時控制系統設計
Design of Real-Time Control Systems

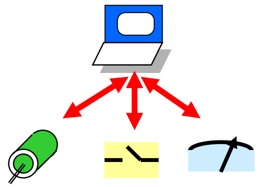
Lecture 33
Networked Control Methodology

Feng-Li Lian

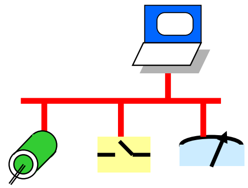
NTU-EE

Feb10 – Jun10

- Real-Time Control Systems
  - Controlled by one Computer Processor
    - Centralized control systems
    - Real-time operating systems
  - Controlled by one Communication Medium
    - Distributed control systems
    - Real-time communications



Centralized Control System



Distributed Control System

- Yodyium Tipsuwan, Mo-Yuen Chow
- "Control methodologies in networked control systems,"
- Control Engineering Practice 11 (2003) 1099–1111
- Abstract:
  - The use of a data network in a control loop has gained increasing attentions in recent years due to its cost effective and flexible applications.
  - One of the major challenges in this so-called networked control system (NCS) is the network-induced delay effect in the control loop.
  - Network delays degrade the NCS control performance and destabilize the system.
  - A significant emphasis has been on developing control methodologies to handle the network delay effect in NCS.
  - This survey paper presents recent NCS control methodologies.
  - The overview on NCS structures and description of network delays including characteristics and effects are also covered.

- Networked Control Methodology:
  - Assumptions
  1. Augmented Deterministic Discrete-Time Model Methodology
  2. Queuing Methodology
  3. Optimal Stochastic Control Methodology
  4. Perturbation Methodology
  5. Sampling Time Scheduling Methodology
  6. Robust Control Methodology
  7. Fuzzy Logic Modulation Methodology
  8. Event-Based Methodology
  9. End-User Control Adaptation Methodology

Assumptions:

- Network transmissions are error-free
- Every frame or packet always has the same constant length
- The difference between the sampling times of the controller and of the sensor, called time skew Δ_k is constant
- The computational delay τ^c is constant and is much smaller than the sampling period T
- The network traffic cannot be overloaded
- Every dimension of the output measurement or the control signal can be packed into one single frame or packet.

State-Space Model:

- Continuous-time plant

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}$$

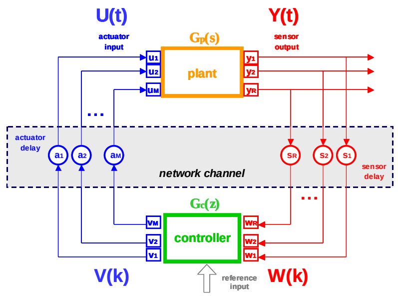
- Discrete-time plant

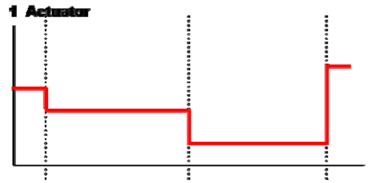
$$\begin{aligned}\mathbf{x}[k+1] &= \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k]\\ \mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k]\end{aligned}$$

$$\begin{cases}\mathbf{\Phi} = e^{\mathbf{A}T}\\ \mathbf{\Gamma} = \left(\int_0^T e^{\mathbf{A}\eta}d\eta\right)\mathbf{B}\end{cases}$$

- Discrete-time controller

$$\begin{aligned}\mathbf{z}[k+1] &= \mathbf{F}\mathbf{z}[k] + \mathbf{G}\mathbf{w}[k]\\ \mathbf{v}[k] &= \mathbf{H}\mathbf{z}[k] + \mathbf{J}\mathbf{w}[k]\end{aligned}$$

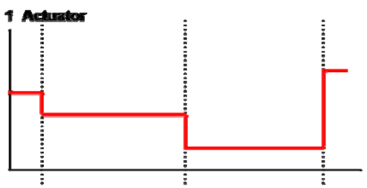


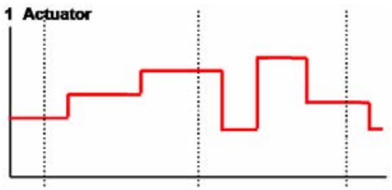


$$\begin{aligned}\mathbf{w}[k] &= \mathbf{y}[k-i],\ i=\{1,\cdots,N\}\\ \mathbf{u}[k] &= \mathbf{v}[k-j],\ j=\{1,\cdots,M\}\end{aligned}$$

State-Space Model:

$$\begin{aligned}\mathbf{w}[k] &= \mathbf{y}[k-i],\ i=\{1,\cdots,N\}\\ \mathbf{u}[k] &= \mathbf{v}[k-j],\ j=\{1,\cdots,M\}\end{aligned}$$





$$\begin{aligned}\mathbf{x}[k+1] &= \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}_0^k\,\mathbf{v}[k] + \mathbf{\Gamma}_1^k\,\mathbf{v}[k-1] + \cdots + \mathbf{\Gamma}_M^k\,\mathbf{v}[k-M]\\ \mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k]\end{aligned}$$

$$\begin{aligned}\mathbf{z}[k+1] &= \mathbf{F}\mathbf{z}[k] + \mathbf{G}\mathbf{w}[k] = \mathbf{F}\mathbf{z}[k] + \mathbf{G}\mathbf{y}[k-i]\\ \mathbf{v}[k] &= \mathbf{H}\mathbf{z}[k] + \mathbf{J}\mathbf{w}[k] = \mathbf{H}\mathbf{z}[k] + \mathbf{J}\mathbf{y}[k-i]\end{aligned}$$

Augmented (Closed-Loop) State-Space Model:

$$\begin{bmatrix}\mathbf{x}_{k+1}\\ \hline \mathbf{y}_k\\ \vdots\\ \mathbf{y}_{k-i+1}\\ \vdots\\ \mathbf{y}_{k-N+1}\\ \hline \mathbf{z}_{k+1}\\ \hline \mathbf{v}_k\\ \vdots\\ \mathbf{v}_{k-M+1}\end{bmatrix} = \left[\begin{array}{c|cccccc|c|cccc}\mathbf{\Phi} & 0 & 0 & \mathbf{\Gamma}_0^k\mathbf{J} & 0 & 0 & & \mathbf{\Gamma}_0^k\mathbf{H} & \mathbf{\Gamma}_1^k & \cdots & \cdots & \mathbf{\Gamma}_M^k\\ \mathbf{C} & & & & & & & & & & & \\ & \mathbf{I} & & & & & & & & & & \\ & & \ddots & & & & & & & & & \\ & & & & \mathbf{I} & & & & & & & \\ \hline & & & \mathbf{G} & & & & \mathbf{F} & & & & \\ \hline & & & \mathbf{J} & & & & \mathbf{H} & & & & \\ & & & & & & & & \mathbf{I} & & & \\ & & & & & & & & & \ddots & & \\ & & & & & & & & & & \mathbf{I} & \end{array}\right] \begin{bmatrix}\mathbf{x}_k\\ \hline \mathbf{y}_{k-1}\\ \vdots\\ \mathbf{y}_{k-i}\\ \vdots\\ \mathbf{y}_{k-N}\\ \hline \mathbf{z}_k\\ \hline \mathbf{v}_{k-1}\\ \vdots\\ \mathbf{v}_{k-M}\end{bmatrix}$$

$$\mathbf{X}_{k+1} = \mathbf{\Xi}_k\,\mathbf{X}_k$$

▪ Augmented (Closed-Loop) State-Space Model:

$$\Xi_k = \left[\begin{array}{c|ccc|c|cccc} \Phi & 0 & \cdots & 0 & \Gamma_0^k \mathbf{H} & \Gamma_1^k & \cdots & \cdots & \Gamma_M^k \\ \hline \mathbf{C} & & & & & & & & \\ & \mathbf{I} & & & & & & & \\ & & \ddots & & & & & & \\ & & & \mathbf{I} & & & & & \\ \hline & & & & \mathbf{F} & & & & \\ \hline & & & & \mathbf{H} & & & & \\ & & & & & \mathbf{I} & & & \\ & & & & & & \ddots & & \\ & & & & & & & \mathbf{I} & \end{array}\right] + \left[\begin{array}{c} \Gamma_0^k \mathbf{J} \\ \hline 0 \\ \vdots \\ 0 \\ \hline \mathbf{G} \\ \hline \mathbf{J} \\ 0 \\ \vdots \\ 0 \end{array}\right] \left[\begin{array}{c|ccc|c|ccc} 0 & \Upsilon_1^k & \cdots & \Upsilon_N^k & 0 & 0 & \cdots & 0 \end{array}\right]$$

▪ Augmented (Closed-Loop) State-Space Model:

– For periodic delays: $\tau^*_{k+K} = \tau^*_k$

$$\mathbf{X}_{k+1} = \Xi_k \cdot \mathbf{X}_k$$

$$\mathbf{X}_{k+1+1} = \Xi_{k+1} \cdot \mathbf{X}_{k+1} = \Xi_{k+1} \cdot \left(\Xi_k \cdot \mathbf{X}_k \right)$$

$$\vdots$$

$$\mathbf{X}_{k+1+K} = \left(\Xi_{k+K} \cdots \Xi_k \right) \mathbf{X}_k$$

$$= \Omega \ \mathbf{X}_k$$

– The closed-loop system is asymptotically stable if all the eigenvalues of Ω are contained within the unit circle

▪ Deterministic Predictor-Based Delay Compensation:

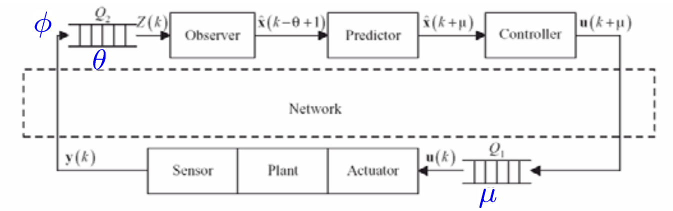


Fig. 7. Configuration of NCS in the deterministic predictor-based delay compensation methodology.

– FIFO queues with shift registers Q_1 & Q_2 of sizes μ & θ

- Past measurement: $Z_k = \left\{ y_{k-\phi}, y_{k-\phi+1}, \cdots \right\}$
- Oberver estimates the plant state $\hat{\mathbf{x}}_{k-\theta+1}$
- Predictor uses $\hat{\mathbf{x}}_{k-\theta+1}$ to predict $\hat{\mathbf{x}}_{k+\mu}$
- Controller computes predictive control $\mathbf{u}_{k+\mu}$ from $\hat{\mathbf{x}}_{k+\mu}$

▪ Probabilistic Predictor-Based Delay Compensation:

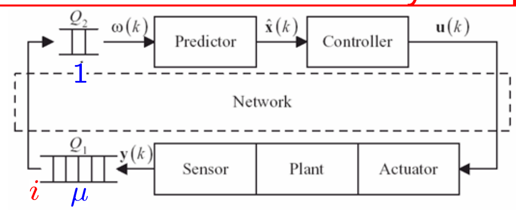


Fig. 8. Configuration of NCS in the probabilistic predictor-based delay compensation methodology.

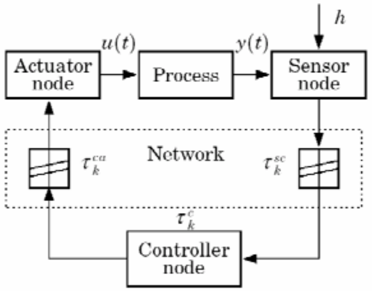
– FIFO queues with shift registers Q_1 & Q_2 of sizes μ & 1

- $\mathbf{w}_k = \mathbf{w}_{k-1}$, or any value in $\left\{ y_k, y_{k-1}, \cdots, y_{k-\mu} \right\}$
- Predictor estimates $\hat{\mathbf{x}}_k$ by

$$\hat{\mathbf{x}}_k = \mathbf{P}_0(\Phi^{i-1}\mathbf{w}_k + \mathbf{W}_i) + \mathbf{P}_1(\Phi^i \mathbf{w}_k + \mathbf{W}_{i+1})$$

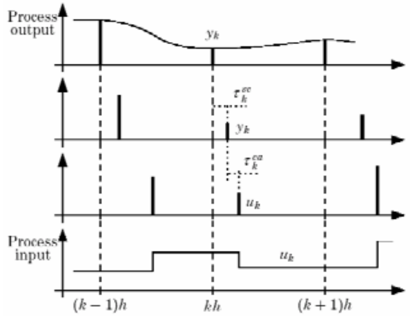
$$\mathbf{W}_i = \begin{cases} 0, & i = 1 \\ \left[\Gamma, \Phi\Gamma, \cdots, \Phi^{i-2}\Gamma\right] \cdot \left[\mathbf{u}_{k-1}^T, \mathbf{u}_{k-2}^T, \cdots, \mathbf{u}_{k-i+1}^T\right]^T, & i \neq 1 \end{cases}$$

Assumptions:



- h: sampling time
- τ^{sc}, τ^{ca}: communication delays
  independently randomly varying
  known probability distribution
- $\tau^{sc} + \tau^{ca} < h \qquad \tau_k = \begin{bmatrix} \tau_k^{sc} \\ \tau_k^{ca} \end{bmatrix}$
- τ^{c}: controller compuation time
- $\mathbf{v}(t), \mathbf{w}(t)$: actuation & sensing noise
  with zero mean, covariance matrices $\mathbf{R_1}, \mathbf{R_2}$

CT & DT Models:



$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{v}$$

$$\Rightarrow \begin{cases} \mathbf{x}_{k+1} &= \mathbf{\Phi x}_k + \mathbf{\Gamma}_0(\tau_k)\mathbf{u}_k + \mathbf{\Gamma}_1(\tau_k)\mathbf{u}_{k-1} + \mathbf{v}_k \\ \mathbf{y}_k &= \mathbf{Cx}_k + \mathbf{w}_k \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{\Phi} &= e^{\mathbf{A}h} \\ \mathbf{\Gamma}_0(\tau_k) &= \left(\int_0^{h-\tau_k^{sc}-\tau_k^{ca}} e^{\mathbf{A}\eta} d\eta\right)\mathbf{B} \\ \mathbf{\Gamma}_1(\tau_k) &= \left(\int_{h-\tau_k^{sc}-\tau_k^{ca}}^{h} e^{\mathbf{A}\eta} d\eta\right)\mathbf{B} \end{cases}$$

Information for Controller Design:

- at time k:

$$\mathcal{Y}_k = \left\{\mathbf{y}_k, \mathbf{y}_{k-1}, \ldots, \mathbf{u}_{k-1}, \mathbf{u}_{k-2}, \ldots, \tau_k^{sc}, \tau_{k-1}^{sc}, \ldots, \tau_{k-1}^{ca}, \tau_{k-2}^{ca}, \ldots\right\}$$

$$\Rightarrow \mathbf{u}_k = f(\mathcal{Y}_k)$$

- Choose the following cost function:

$$\mathbf{J_N} = E\left\{\mathbf{x}_N^T \mathbf{Q}_N \mathbf{x}_N\right\} + E\left\{\sum_{k=0}^{N-1} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}^T \mathbf{Q} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}\right\}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix}, \qquad \mathbf{Q} \geq 0,\ \mathbf{Q}_{22} > 0$$

Optimal State Feedback Law:

- If $\mathbf{y}_k = \mathbf{x}_k$, i.e., noise free measurement:

$$\Rightarrow \mathbf{u}_k^* = -\mathbf{L}(\tau_k^{sc}) \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_{k-1}^* \end{bmatrix}$$

$$\mathbf{L}(\tau_k^{sc}) = \left(\mathbf{Q}_{22} + \tilde{\mathbf{S}}_{k+1}^{22}\right)^{-1} \begin{bmatrix} \mathbf{Q}_{12}^T + \tilde{\mathbf{S}}_{k+1}^{21} & \tilde{\mathbf{S}}_{k+1}^{23} \end{bmatrix}$$

$$\tilde{\mathbf{S}}_{k+1}(\tau_k^{sc}) = E_{\tau_k^{ca}}\left\{\mathbf{G}^T(\tau_k^{sc}, \tau_k^{ca})\mathbf{S}_{k+1}\mathbf{G}(\tau_k^{sc}, \tau_k^{ca}) \;\middle|\; \tau_k^{sc}\right\}$$

$$\mathbf{G}(\tau_k^{sc}, \tau_k^{ca}) = \begin{bmatrix} \mathbf{\Phi} & \mathbf{\Gamma}_0(\tau_k^{sc}, \tau_k^{ca}) & \mathbf{\Gamma}_1(\tau_k^{sc}, \tau_k^{ca}) \\ 0 & \mathbf{I} & 0 \end{bmatrix}$$

$$\mathbf{S}_k(\tau_k^{sc}) = E_{\tau_k^{sc}}\left\{\mathbf{F}_1^T(\tau_k^{sc})\mathbf{Q}\mathbf{F}_1(\tau_k^{sc}) + \mathbf{F}_2^T(\tau_k^{sc})\tilde{S}_{k+1}(\tau_k^{sc})\mathbf{F}_2(\tau_k^{sc})\right\}$$

$$\mathbf{F}_1(\tau_k^{sc}) = \begin{bmatrix} \mathbf{I} & 0 \\ \multicolumn{2}{c}{-\mathbf{L}(\tau_k^{sc})} \end{bmatrix} \qquad \mathbf{F}_2(\tau_k^{sc}) = \begin{bmatrix} \mathbf{I} & 0 \\ \multicolumn{2}{c}{-\mathbf{L}(\tau_k^{sc})} \\ 0 & \mathbf{I} \end{bmatrix} \qquad \mathbf{S}_N = \begin{bmatrix} \mathbf{Q}_N & 0 \\ 0 & 0 \end{bmatrix}$$

Derivation of Optimal State Feedback Controller:

$$\Rightarrow \mathbf{z}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_{k-1} \end{bmatrix}$$

$$\mathbf{z}_k^T \mathbf{S}_k \mathbf{z}_k + \alpha_k = \min_{\mathbf{u}_k} E_{\tau_k^{sc},\tau_k^{ca},\mathbf{v}_k} \left\{ \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}^T \mathbf{Q} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} + \mathbf{z}_{k+1}^T \mathbf{S}_{k+1} \mathbf{z}_{k+1} \right\} + \alpha_{k+1}$$

$$= E_{\tau_k^{sc}} \min_{\mathbf{u}_k} E_{\tau_k^{ca},\mathbf{v}_k} \left\{ \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}^T \mathbf{Q} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} + \mathbf{z}_{k+1}^T \mathbf{S}_{k+1} \mathbf{z}_{k+1} \mid \tau_k^{sc} \right\} + \alpha_{k+1}$$

$$= E_{\tau_k^{sc}} \min_{\mathbf{u}_k} \left\{ \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}^T \mathbf{Q} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} + \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \\ \mathbf{u}_{k-1} \end{bmatrix}^T \tilde{\mathbf{S}}_{k+1}(\tau_k^{sc}) \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \\ \mathbf{u}_{k-1} \end{bmatrix} \right\}$$

$$+ \alpha_{k+1} + tr\left\{ \mathbf{S}_{k+1}^{11} \mathbf{R}_1 \right\}$$

Optimal State Estimation:

$\rightarrow$ To minimize $E\left\{ [\mathbf{x}_k - \hat{\mathbf{x}}_k]^T [\mathbf{x}_k - \hat{\mathbf{x}}_k] \mid \mathcal{Y}_k \right\}$

$$\Rightarrow \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \bar{\mathbf{K}}_k \left(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1} \right)$$

$$\hat{\mathbf{x}}_{k+1|k} = \mathbf{\Phi}\, \hat{\mathbf{x}}_{k|k-1} + \mathbf{\Gamma}_0(\tau_k)\mathbf{u}_k + \mathbf{\Gamma}_1(\tau_k)\mathbf{u}_{k-1} + \bar{\mathbf{K}}_k \left(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1} \right)$$

$$\hat{\mathbf{x}}_{0|-1} = E(\mathbf{x}_0)$$

$$\mathbf{P}_{k+1} = \mathbf{\Phi}\,\mathbf{P}_k\,\mathbf{\Phi}^T + \mathbf{R}_1 - \mathbf{\Phi}\,\mathbf{P}_k\,\mathbf{C}^T \left[\mathbf{C}\,\mathbf{P}_k\,\mathbf{C}^T + \mathbf{R}_2 \right]^{-1} \mathbf{C}\,\mathbf{P}_k\,\mathbf{\Phi}$$

$$\mathbf{P}_0 = \mathbf{R}_0 = E\left(\mathbf{x}_0 \mathbf{x}_0^T \right)$$

$$\mathbf{K}_k = \mathbf{\Phi}\,\mathbf{P}_k\,\mathbf{C}^T \left[\mathbf{C}\,\mathbf{P}_k\,\mathbf{C}^T + \mathbf{R}_2 \right]^{-1}$$

$$\bar{\mathbf{K}}_k = \mathbf{P}_k\,\mathbf{C}^T \left[\mathbf{C}\,\mathbf{P}_k\,\mathbf{C}^T + \mathbf{R}_2 \right]^{-1}$$

Separation Property:

$$\Rightarrow \begin{cases} \mathbf{x}_{k+1} = \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}_0(\tau_k)\mathbf{u}_k + \mathbf{\Gamma}_1(\tau_k)\mathbf{u}_{k-1} + \mathbf{v}_k \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{w}_k \end{cases}$$

$$\rightarrow \left\{ \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \bar{\mathbf{K}}_k \left(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1} \right) \right.$$

$$\Rightarrow \mathbf{u}_k^* = -\mathbf{L}(\tau_k^{sc}) \begin{bmatrix} \hat{\mathbf{x}}_{k|k} \\ \mathbf{u}_{k-1}^* \end{bmatrix}$$

$$\Rightarrow \mathbf{L}(\tau_k^{sc}) = \left(\mathbf{Q}_{22} + \tilde{\mathbf{S}}_{k+1}^{22} \right)^{-1} \left[\mathbf{Q}_{12}^T + \tilde{\mathbf{S}}_{k+1}^{21} \quad \tilde{\mathbf{S}}_{k+1}^{23} \right]$$

Example:

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 35 \\ -61 \end{bmatrix} \xi$$

$$y = [2 \quad 1]\, x + \eta,$$

$E[\xi(t)] = E[\eta(t)] = 0$

$E[\xi(t_1)\xi(t_2)] =$

$E[\eta(t_1)\eta(t_2)] = \delta(t_1 - t_2).$

$\xi(t)$ and $\eta(t)$ have mean zero and unit incremental variance.

$$J = E \lim_{T\to\infty} \frac{1}{T} \int_0^T (x^T H^T H x + u^2)dt,$$

$$H = 4\sqrt{5}\,[\sqrt{35} \quad 1]$$

$$h = 0.05$$

τ_k^{sc} and τ_k^{ca} uniformly distributed on the interval $[0, \alpha h/2]$

$0 \le \alpha \le 1.$

- Example:
  - Comparison:
    - An LQG-controller neglecting the time delays

$$\mathrm{sp}(\Phi - \Gamma L) = \{0.700 \pm 0.0702i\}$$
$$\mathrm{sp}(\Phi - KC) = \{0.743, 0.173\}.$$
$$L = \begin{bmatrix} 38.911 \\ 8.094 \end{bmatrix}^T, K = \begin{bmatrix} 2.690 \\ -4.484 \end{bmatrix}, \overline{K} = \begin{bmatrix} 2.927 \\ -5.012 \end{bmatrix}$$

a small phase margin, $\phi_m = 10.9°$

the stability limit $\alpha_{crit} = 0.425$

    - An LQG-controller designed for the mean delays
    - The scheme with buffers proposed in Luck & Ray 90
    - The optimal stochastic control

- Example:

Monte Carlo Simulation of $2 \cdot 10^4$ samples

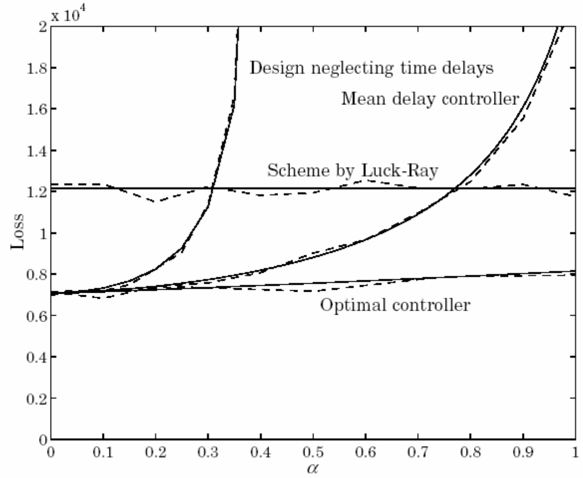
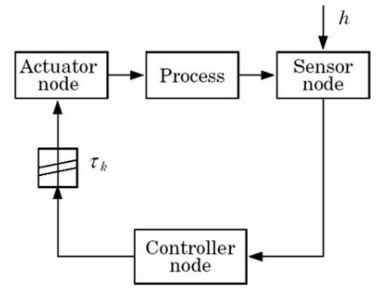
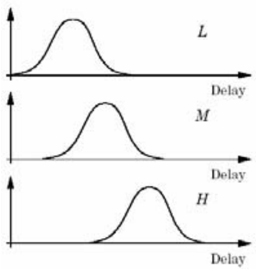
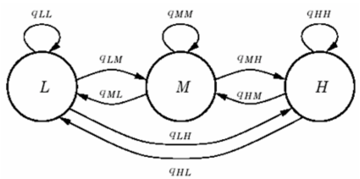


Figure 5.4 Exact calculated performance (solid lines) of the four schemes, and simulated performance (dashed lines) of system (5.43) as a function of the amount of stochastics in the time-delays. The time-delays are uniformly distributed on $[0, \alpha h/2]$. For $\alpha > 0.425$ the controller neglecting the time delays fails to stabilize the process.



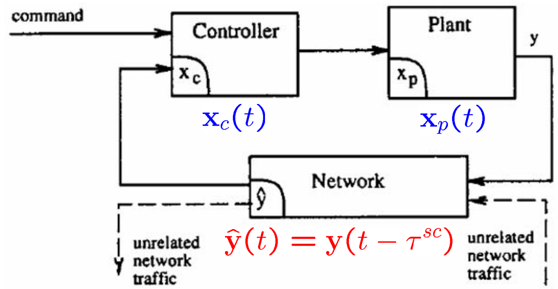




$$\mathbf{x}_{k+1} = \mathbf{A}(r_k)\mathbf{x}_k + \mathbf{B}(r_k)\mathbf{u}_k$$
$$\Rightarrow \mathbf{u}_k^* = -\mathbf{L}(\tau_k^{sc}, r_k)\begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_{k-1}^* \end{bmatrix}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \bar{\mathbf{K}}_k\left(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1}\right) \quad \Rightarrow \mathbf{u}_k^* = -\mathbf{L}(\tau_k^{sc}, r_k)\begin{bmatrix} \hat{\mathbf{x}}_{k|k} \\ \mathbf{u}_{k-1}^* \end{bmatrix}$$

- Nonlinear CT Approach:
  - For obtaining Maximal Allowable Transfer Interval:



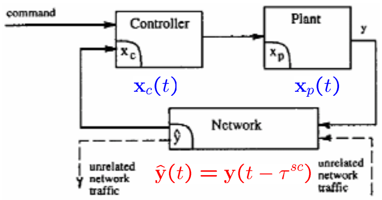
$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_p(t) \\ \mathbf{x}_c(t) \end{bmatrix}$$

$$\mathbf{e}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{e}(t))$$
$$\dot{\mathbf{e}}(t) = \mathbf{g}(t, \mathbf{x}(t), \mathbf{e}(t))$$

$\Rightarrow$ Find a bound ρ, s.t. $\tau^{sc} < \rho$

Problem Formulation:



$$\begin{cases} \dot{\mathbf{x}}_p(t) = \mathbf{f}_p\left(t, \mathbf{x}_p(t), \mathbf{u}_p(t)\right) \\ \mathbf{y}(t) = \mathbf{g}_p\left(t, \mathbf{x}_p(t)\right) \end{cases}$$

$$\begin{cases} \dot{\mathbf{x}}_c(t) = \mathbf{f}_c\left(t, \mathbf{x}_c(t), \hat{\mathbf{y}}(t)\right) \\ \mathbf{u}_p(t) = \mathbf{g}_c\left(t, \mathbf{x}_c(t)\right) \end{cases}$$

$$\mathbf{e}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t)$$

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_p(t) \\ \mathbf{x}_c(t) \end{bmatrix}$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(t, \mathbf{x}(t), \mathbf{e}(t)\right) \\ \dot{\mathbf{e}}(t) = \mathbf{g}\left(t, \mathbf{x}(t), \mathbf{e}(t)\right) \end{cases}$$

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix}$$

$$\left\{ \dot{\mathbf{z}}(t) = \mathbf{h}\left(t, \mathbf{z}(t)\right) \right.$$

$\mathbf{f}, \mathbf{g}, \mathbf{h}$ are globally Lipschitz with constants k_f, k_g, k_h

Perturbation Approach:

- Consdier $\mathbf{e}(t)$ as a perturbation
- If $\mathbf{e}(t) \equiv \mathbf{0}$, i.e., $\hat{\mathbf{y}}(t) = \mathbf{y}(t)$
- Exist a Lyapunov function $V(t, \mathbf{x})$ and $c_i > 0$

$$c_1 ||\mathbf{x}||^2 \leq V(t, \mathbf{x}) \leq c_2 ||\mathbf{x}||^2$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}, \mathbf{0}) \leq -c_3 ||\mathbf{x}||^2$$

$$\left| \frac{\partial V}{\partial x} \right| \leq c_4 ||\mathbf{x}||$$

Lemma:

- Consider a static or dynamic network scheduler starting at time t_0, visiting each node at least once, with integer periodicity p, maximum allowable transfer interval τ, maximum growth in error e(t) in τ seconds strictly bounded by $\beta \in (0, \infty)$
- Then, for any time $t > t_0 + p\,\tau$, the error is bounded as $||e(t)|| < \beta\, p\,(p-1)/2$

i.e., $||\dot{\mathbf{e}}(t)|| < \beta \Rightarrow ||\mathbf{e}(t)|| < \beta\, p(p-1)/2 < \beta \left(\sum_{j=1}^{p} j \right)$

Theorem:

- Consider a NCS whose continuous dynamics are described before with the control law designed so that the set of Lyapunov function properties is satisfied, with p nodes of sensors operating under static or dynamic scheduling of periodic visiting,
- A maximum allowable transfer interval that satisfies

$$\tau < \min \left\{ (\ln(2)/k_h p),\ (S/8),\ (S c_3 / 32 k_f c_4)(c_1/c_2)^{3/2} \right\}$$

- Where $S = \left((\sqrt{c_2/c_1} + 1)\, k_h \, (\sum_{i=1}^{p} j) \right)^{-1}$
- Then, the NCS is exponentially stable

Sketch of Theorem Proof:

$$\tau < \min\left\{ \left(\frac{\ln(2)}{k_h p}\right), \left(\frac{S}{8}\right), \left(\frac{S c_3}{32 k_f c_4}\right)\left(\frac{c_1}{c_2}\right)^{3/2} \right\}$$

$$\tau < \frac{\ln(2)}{k_h p} \Rightarrow e^{k_h p \tau} < 2$$

$$\tau < \frac{S}{8} \Rightarrow \tau < S$$

$$\Rightarrow \frac{2\tau}{S} < \frac{1}{4}$$

$$\tau < \frac{S c_3}{32 k_f c_4}\left(\frac{c_1}{c_2}\right)^{3/2} \Rightarrow k_f c_4 \frac{2\tau}{S} < c_3$$

$$\Rightarrow \sqrt{\frac{\frac{4 k_f c_4 \tau}{S}\sqrt{\frac{c_2}{c_1}}}{c_1}} \le \frac{1}{2}$$

Sketch of Theorem Proof:

- For any $t \in [t_0, t_0 + p\tau]$, by Bellman-Gronwall,

$$\Rightarrow ||\mathbf{z}(t)|| \le e^{k_h p \tau} ||\mathbf{z}(t_0)|| \le 2||\mathbf{z}(t_0)||$$

$$\Rightarrow ||\dot{\mathbf{e}}(t)|| \le ||\dot{\mathbf{z}}(t)||$$

$$\le ||\mathbf{h}(t, \mathbf{z}(t))|| \le k_h ||\mathbf{z}(t)|| \le 2\, k_h\, ||\mathbf{z}(t_0)||$$

$$2\,\tau\, k_h\, ||\mathbf{z}(t_0)|| \le 2\,\tau\, k_h \left(1 + \sqrt{c_2/c_1}\right) ||\mathbf{z}(t_0)|| = \beta$$

$$\Rightarrow ||\mathbf{e}(t_0 + p\tau)|| \le \beta \left(\sum_{j=1}^{p} j\right) ||\mathbf{z}(t_0)|| = \frac{2\tau}{S} ||\mathbf{z}(t_0)||$$

Sketch of Theorem Proof:

$$\Rightarrow \begin{cases} ||\mathbf{e}(t)|| < \frac{2\tau}{S}||\mathbf{z}(t_0)||, & \forall t \ge t_0 + p\tau \\ V(t, \mathbf{x}(t)) < 4\, c_2\, ||\mathbf{z}(t_0)||^2, & \forall t \ge t_0 \\ ||\mathbf{x}(t)|| < 2\sqrt{\frac{c_2}{c_1}}\, ||\mathbf{z}(t_0)||, & \forall t \ge t_0 \end{cases}$$

$$\dot{V}(\mathbf{x}) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}, \mathbf{e})$$

$$= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \mathbf{x}} \left(\mathbf{f}(t, \mathbf{x}, 0) + \mathbf{f}(t, \mathbf{x}, \mathbf{e}) - \mathbf{f}(t, \mathbf{x}, 0)\right)$$

$$= -c_3\, ||\mathbf{x}||^2 + c_4\, ||\mathbf{x}||\, k_f\, ||\mathbf{e}||$$

Sketch of Theorem Proof:

- At a time $t \ge t_0 + p\tau$

$$\dot{V}(t, \mathbf{x}(t)) \le -\frac{c_3}{c_2} V(t, \mathbf{x}(t)) + k_f\, c_4 \frac{4\tau}{S}\sqrt{\frac{c_2}{c_1}}\, ||\mathbf{z}(t_0)||^2$$

- By Comparison Lemma and

  let $\alpha = \frac{c_3}{c_2}$, $\alpha\delta = 4\, k_f\, c_4 \frac{4\tau}{S}\sqrt{\frac{c_2}{c_1}}$

  - $\forall t > p\tau$,

$$V(t_0 + t, \mathbf{x}(t_0 + t)) \le ||\mathbf{z}(t_0)||^2 \left[e^{-\alpha(t - \tau p)}(4c_2 - \delta) + \delta\right]$$

  - Fix $t_1 > p\tau$, s.t. $e^{-\alpha(t_1 - \tau p)} < \delta/(4c_2 - \delta)$

$$V(t_0 + t, \mathbf{x}(t_0 + t)) \le 2\,\delta ||\mathbf{z}(t_0)||^2$$

Sketch of Theorem Proof:

- Based on the Lyapunov bound & the bound on τ,

$$||\mathbf{x}(t_0+t_1))|| \leq \sqrt{\frac{2\delta}{c_1}}||\mathbf{z}(t_0)|| \leq \frac{1}{2}||\mathbf{z}(t_0)||$$

- Because

$$||\mathbf{z}(t_0+t_1))|| \leq ||\mathbf{x}(t_0+t_1))|| + ||\mathbf{e}(t_0+t_1))||$$
$$\leq \frac{1}{2}||\mathbf{z}(t_0))|| + \frac{1}{4}||\mathbf{z}(t_0))|| = \frac{3}{4}||\mathbf{z}(t_0))||$$

$$\Rightarrow ||\mathbf{z}(t_0+kt_1))|| \leq \left(\frac{3}{4}\right)^k ||\mathbf{z}(t_0))||$$

$\Rightarrow$ exponentially stable

Example:

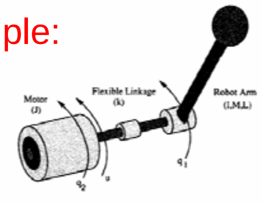


Fig. 2. Schematic of the single-link flexible-joint robot.

- μ: mean of Poisson distribution
  - Theoretical value: 0.001
  - Experimental value: 0.015

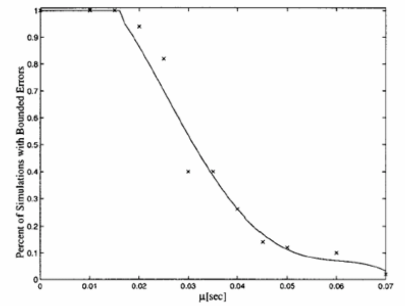


Fig. 3. Single link: the percent of stable cases as function of τ. The system loses stability when μ is around 0.015

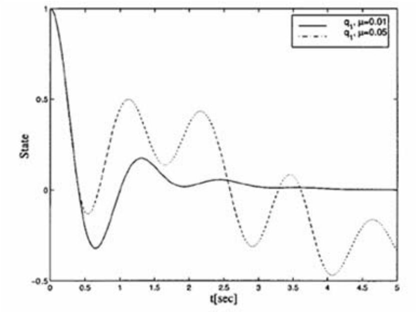


Fig. 4. Single link: stable (μ = 0.01) (solid line) and unstable (μ = 0.05) (dashed line) trajectories.

Robust Control:

– Network delay formulation:

τ_k^{sc}, τ_k^{ca} are bounded and $\in [\tau_{min}, \tau_{max}]$

$$\tau^n = \frac{1}{2}(\tau_{max}+\tau_{min}) + \frac{1}{2}(\tau_{max}-\tau_{min})\,\delta, \quad -1 \leq \delta \leq 1$$
$$= \text{constant delay} + \text{uncertain delay}$$
$$= (1-\alpha)\ \tau_{max} + \alpha\ \tau_{max}\delta, \quad 0 \leq \alpha \leq \frac{1}{2}$$

– Delay approximated by the first-order Pade approximation:

$$e^{-\tau^n s} = e^{-s(1-\alpha)\tau_{max}}\ e^{-s\alpha\tau_{max}\delta}$$

Robust Control:

– By the first-order Pade approximation:

$$e^{-\tau^n s} = e^{-s(1-\alpha)\tau_{max}}\ e^{-s\alpha\tau_{max}\delta}$$
$$\approx \frac{1-s\tau^n/2}{1+s\tau^n/2} \approx \frac{1-s(1-\alpha)\tau_{max}/2}{1+s(1-\alpha)\tau_{max}/2} \times \frac{1-s\alpha\tau_{max}\delta/2}{1+s\alpha\tau_{max}\delta/2}$$

$$\Rightarrow \frac{1-s\alpha\tau_{max}\delta/2}{1+s\alpha\tau_{max}\delta/2} = 1 + \left(\frac{\alpha\tau_{max}s}{1+\alpha\tau_{max}s/3.465}\right)\Delta$$
$$= 1 + W_m(s)\Delta$$

Robust Control:

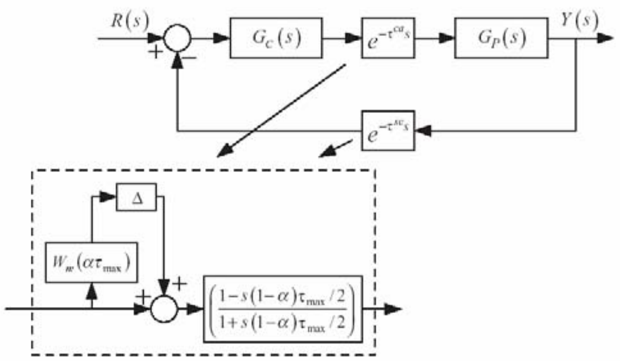


Fig. 11. Configuration of NCS in the robust control methodology.

> H_∞ design and μ-synthesis can be applied to design a CT controller

Fuzzy Compensation of DC Motor Control over Network:

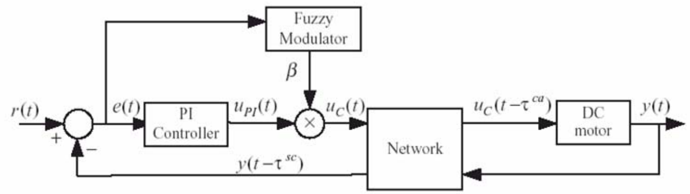


Fig. 12. Configuration of NCS in the fuzzy logic modulation methodology.

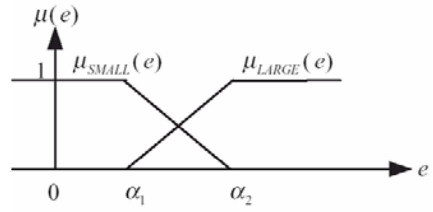


Fig. 13. Membership functions of $e(t)$.

Networked Control of a Robotic Manipulator over the Internet:

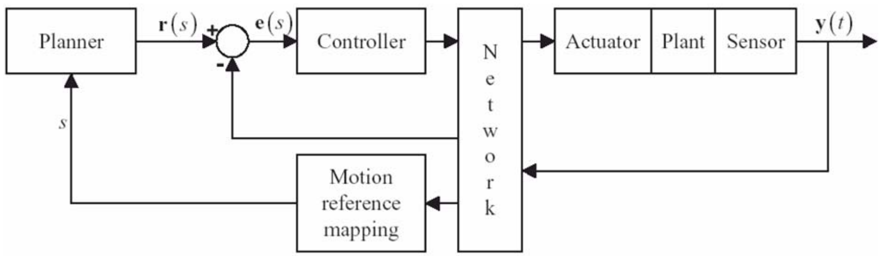


Fig. 14. Configuration of NCS in the event-based methodology.

End-User Control Adaptation:

- Adapt Controller Parameters with respect to
  - Current network traffic condition or
  - Current given network quality of service (QoS)
- Example: PI control for a DC motor speed control

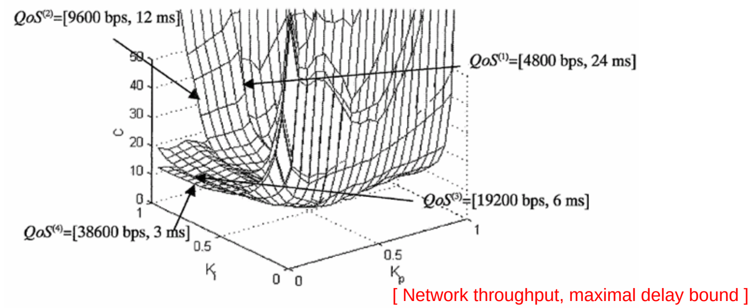


Fig. 15. Cost surface with respect to controller gains under different QoS conditions.

- End-User Control Adaptation:

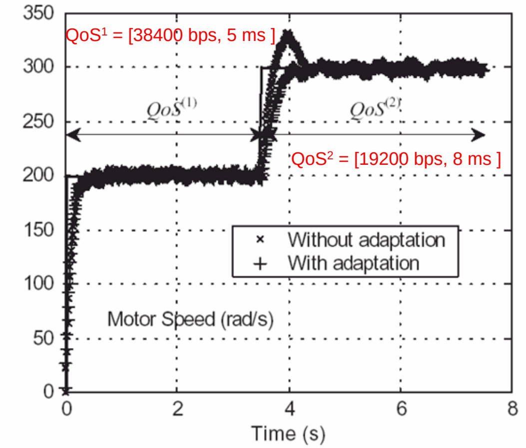


Fig. 16. Step responses of an actual networked DC motor speed control system in the end-user control adaptation methodology; ×: without adaptation, +: with adaptation.