SPRING 2010

即時控制系統設計
Design of Real-Time Control Systems

Lecture 25
Techniques for Enhancing the Performance of Discretized Controllers

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Feb10 – Jun10

Figures and images used in these lecture notes are adopted from

1: B.D.O. Anderson, "Controller Design: Moving from Theory to Practice," IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993

2: D. Raviv & E.W. Djaja, "Technique for Enhancing the Performance of Discretized Controllers," IEEE Control Systems Magazine, 19(3), pp. 52-57, June 1999

Digital Control Systems

- Study in Digital Control Systems
  - Controller Design of Digital Control Systems
    - Design Process
      - > Emulation:
        - » CT plant -> CT controller -> DT controller
      - > Discrete Design:
        - » CT plant -> DT plant -> DT controller
      - > Direct Design: (B.D.O. Anderson, 1992 Bode Prize Lecture)
        - » CT plant -> DT controller

Basic Design Concept

- Basic principles of low-order controller design

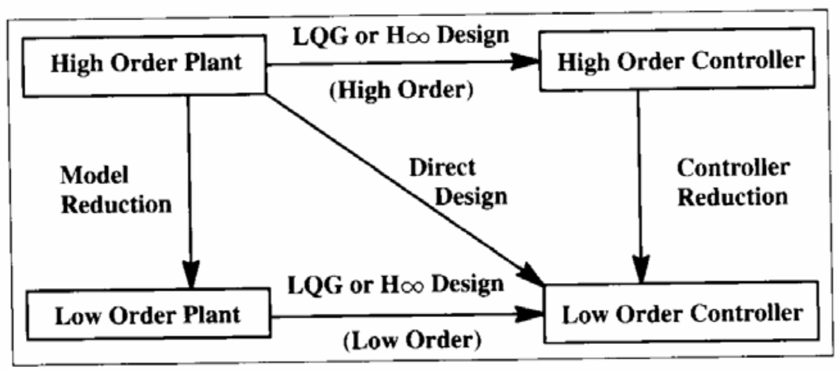


Fig. 1. Basic principles of low order controller design.

Anderson 1993

Basic Design Concept

- Textbook schemes for replacing a CT controller by a DT one

With $C(s)$ continuous time and $C_d(z)$ discrete time,

$C_d(z) = C\left(\frac{z-1}{T}\right)$	Euler or forward difference
$C_d(z) = C\left(\frac{z-1}{zT}\right)$	Balanced difference
$C_d(z) = C\left(\frac{2}{T}\frac{z-1}{z+1}\right)$	Tustin or bilinear
$C_d(z)=C\left(\frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}\frac{z-1}{Z+1}\right)$	Tustin with prewarping
	Poles and zeros of $C_d(z)$ are images under $z = e^{sT}$ of those of $C(s)$, with $C_d(1) = C(0)$.
$C_d(z)=\frac{(z-1)}{Tz}\frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\frac{e^{sT}}{z-e^{sT}}\frac{G(s)}{s}ds$	Step-invariance
$C_d(z)=\frac{(z-1)^2}{Tz}\frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\frac{e^{sT}}{z-e^{sT}}\frac{G(s)}{s^2}ds$	Ramp-invariance
	Zero-order hold equivalance.
	First-order hold equivalance.
	Triangular-hold equivalance.

Anderson 1993

- Digital control design through discretizing an analog controller

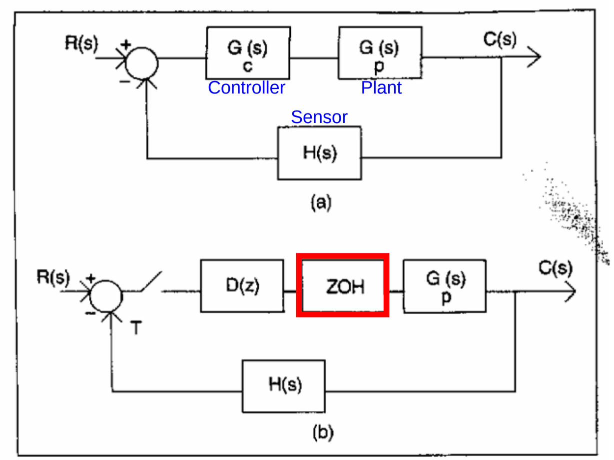


Fig. 1. (a) The analog closed-loop control system, (b) The digital closed-loop control system.

Raviv & Djaja 1999

- Given
  - A process $G_p(s)$
  - A sensor H(s)
  - A presumably well designed analog controller $G_c(s)$

- Find
  - A digital controller D(z)
    which produces closed-loop behavior
    similar to the analog system
    both in the time and frequency domains

Raviv & Djaja 1999

- Solutions:
  - Analog control design followed by controller discretization
    - More convenient
    - Deal with sampling time T at the final phase
  - Direct digital control design
  - To enhance the performance by the first method
    - Add a pole-zero pair in the z-plane
    - To compensate for the low-frequencies and mid-frequencies phase and gain response effects contributed by ZOH

Raviv & Djaja 1999

- Potential problem:
  - The ZOH causes a delay of approximately T/2

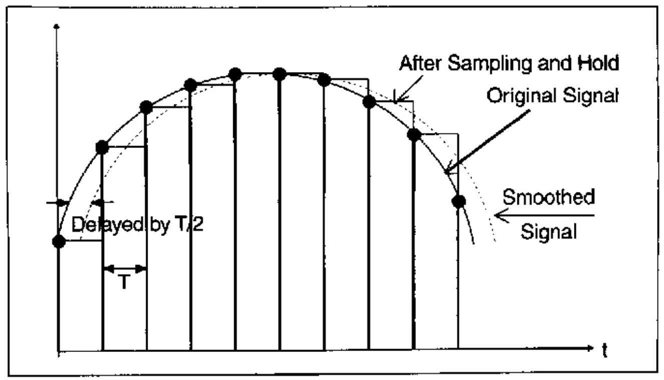


Fig. 2. A reconstructed signal using ZOH and its smoothed approximation.

Raviv & Djaja 1999

- A pole-zero compensation for delay:

$$C(z) = \frac{2z}{z+1}$$

– Provides a phase of (ωT/2)

– Which exactly cancels the frequency phase response of the ZOH obtained from

$$\frac{1-e^{-sT}}{s}$$

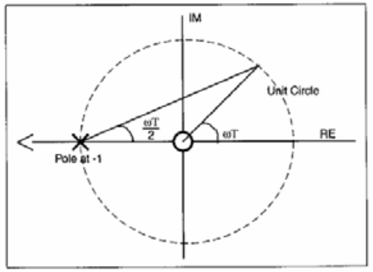


Fig. 3. The location of pole and zero of ZOH compensator in the Z-domain.

Raviv & Djaja 1999

- A pole-zero compensation for delay:
  - The ZOH transfer function:

$$\frac{1-e^{-sT}}{s} \approx \frac{T}{1+\frac{sT}{2}}$$ 1st-order Pade approximation

$$\Rightarrow \left.\frac{T}{1+\frac{sT}{2}}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{T}{2}\frac{z+1}{z}$$ Tustin transformation

  - The characteristic polynomial

$$1+\left(\frac{2z}{z+1}\right) D'(z)\,(1-z^{-1})\mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = 0$$

Raviv & Djaja 1999

- A pole-zero compensation for delay:
  - IF the proposed compensation causes instability a modified ZOH compensation

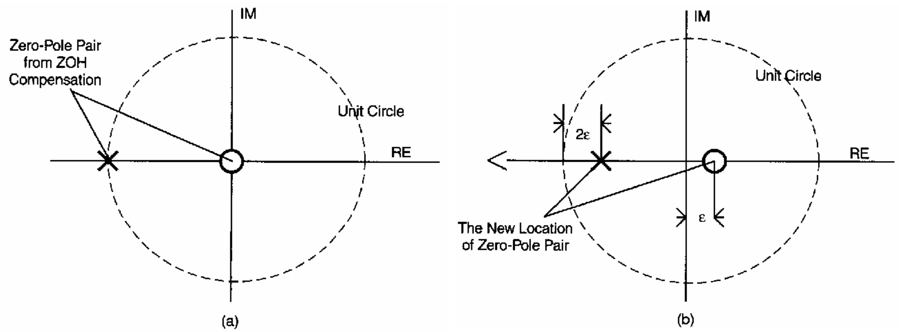
$$C'(z) = \frac{2(z-\varepsilon)}{z+1-2\varepsilon}$$

  - The characteristic polynomial

$$1+\left(\frac{2(z-\varepsilon)}{z+1-2\varepsilon}\right) D'(z)\,(1-z^{-1})\mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = 0$$

Raviv & Djaja 1999

- A pole-zero compensation for delay:



Raviv & Djaja 1999

- Lag Compensator:

$$G_p(s) = \frac{4\times 10^6}{s(s+20)(s+200)}$$

$$H(s) = 1$$

- Design specifications:
  1. Velocity error constant K_v at least 1000 s^{-1}
  2. Attenuation of all sinusoidal inputs of frequency above 400 rad/sec by at least 16
  3. Steady-state error of (up to) 1% for sinusoidal inputs for frequencies less than 1 rad/sec

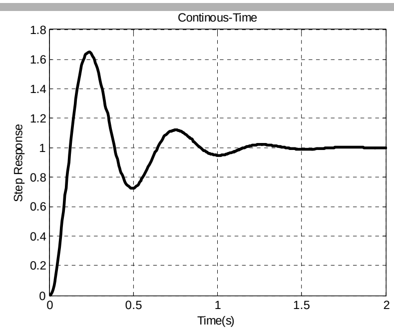
Raviv & Djaja 1999

$$\Rightarrow\ G_c(s) = \frac{1}{80}\frac{(s+8)}{(s+0.1)}$$

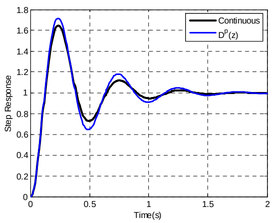
Table 1

	$D'(z)$	Multiplier	$D(z)$
T = 0.01 s	$\frac{0.0130z-0.0120}{z-0.9990}$	$\frac{2z}{z+1}$	$\frac{0.0260z^2-0.0240z}{z^2+0.0010z-0.9990}$
T = 0.05 s	$\frac{0.0150z-0.0100}{z-0.9950}$	$\frac{2z}{z+1}$	$\frac{0.0299z^2-0.0200z}{z^2+0.0050z-0.9950}$
T = 0.1 s	Unstable $\frac{0.0174z-0.0075}{z-0.9900}$	$\frac{2z}{z+1}$	Unstable $\frac{0.0348z^2-0.0150z}{z^2+0.0100z-0.9900}$
T = 0.1 s	Unstable $\frac{0.0174z-0.0075}{z-0.9900}$	$\frac{2(z-0.2)}{(z+0.6)}$	$\frac{0.0348z^2-0.0219z+0.0030}{z^2-0.3900z-0.5940}$

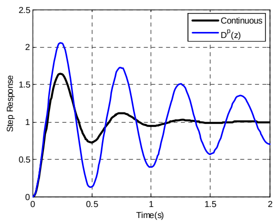
Raviv & Djaja 1999



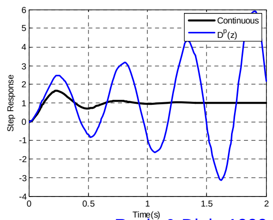
$T = 0.01s$



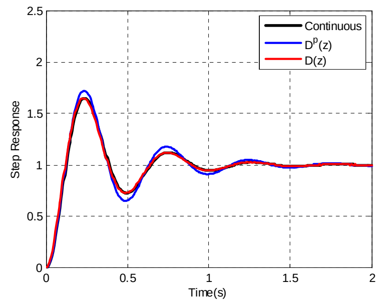
$T = 0.1s$



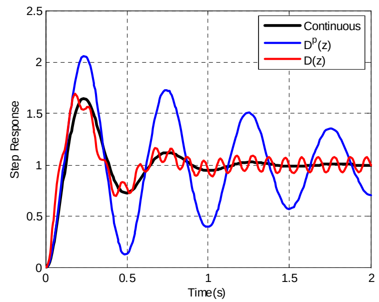
$T = 0.05s$



Raviv & Djaja 1999

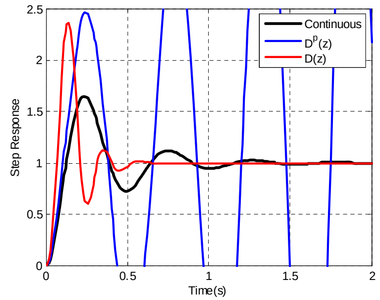


$T = 0.01s$



$T = 0.05s$

$T = 0.1s$



Raviv & Djaja 1999

Lead-Lag Compensator:

$$G_p(s) = \frac{1000}{s(1+\frac{s}{10})(1+\frac{s}{250})}$$

$$H(s) = 1$$

- Design specifications:
  1. Phase margin of at least 50°
  2. Velocity error constant K_v at least 1000 s^{-1}
  3. Attenuation of the input noise at 60 Hz and above by a factor of 100
  4. Steady-state error for frequencies less than 1 rad/sec less than 1%

Raviv & Djaja 1999

$$\Rightarrow G_c(s) = \frac{(1+\frac{s}{4.5})(1+\frac{s}{10})}{(1+\frac{s}{0.1})(1+\frac{s}{110})}$$

$$T = 0.01s$$

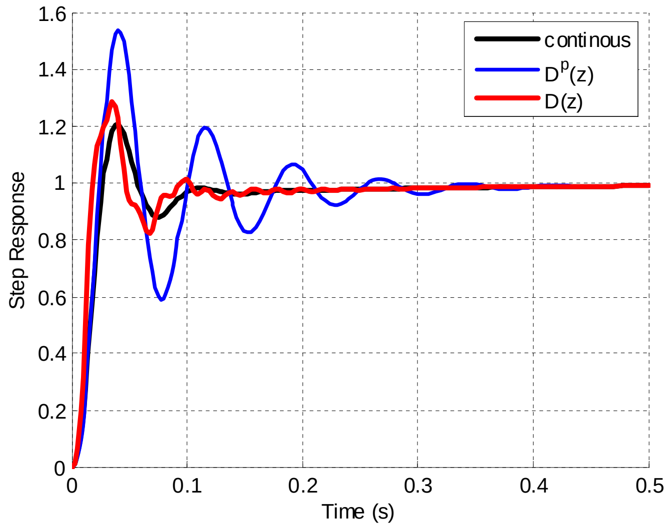
$$\Rightarrow D'(z) = \frac{0.6597z^2 - 1.2897z + 0.6300}{1.00z^2 - 1.2893z + 0.2900}$$

?

With the ZOH compensation of $\frac{2z}{z+1}$

$$\Rightarrow D(z) = \frac{1.3194z^3 - 2.5793z^2 + 1.26z}{z^3 - 0.2893z^2 - 0.9993z + 0.2900}$$

Raviv & Djaja 1999



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Raviv & Djaja 1999

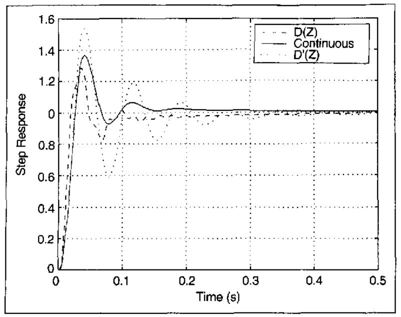
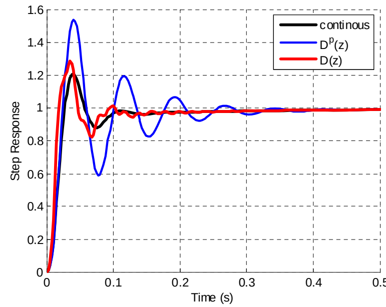
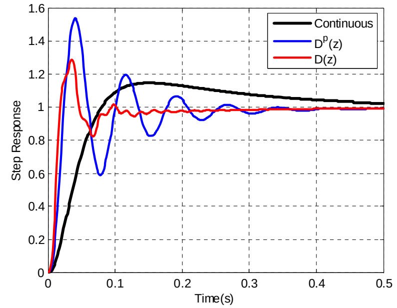


Fig. 6. Closed-loop step response of Example (b). T = 0.01 s.





Raviv & Djaja 1999

Examples

- **Katz's Example:**

$$G_p(s) = \frac{863.3}{s^2}$$

- Design specifications:
  1. Max phase lag at f = 3 Hz should not be more than 13°
  2. At any given frequency the CL gain should not exceed 5 dB beyond the CL dc gain
  3. Max tracking error due to an input disturbance moment of 0.028Nm should not be 0.01 rad

Raviv & Djaja 1999

Examples

$$G_c(s) = 2940\frac{(s+29.4)}{(s+294)^2}$$

$$T = 0.03s$$

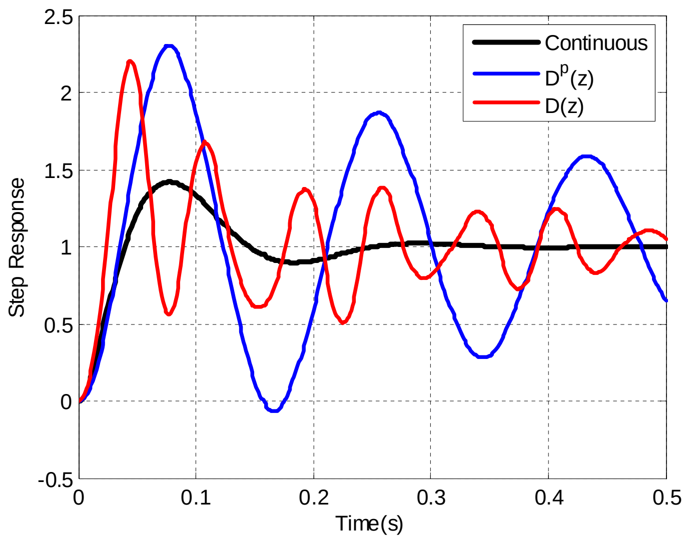
$$\Rightarrow \quad D'(z) = \frac{1.8958z^2 + 1.1685z - 0.7273}{z^2 + 1.1653z + 0.3395}$$

With the ZOH compensation of $\frac{2z}{z+1}$

$$\Rightarrow \quad D(z) = \frac{3.7916z^3 + 2.3369z^2 - 1.4546z}{z^3 + 2.1653z^2 + 1.5047z + 0.3395}$$

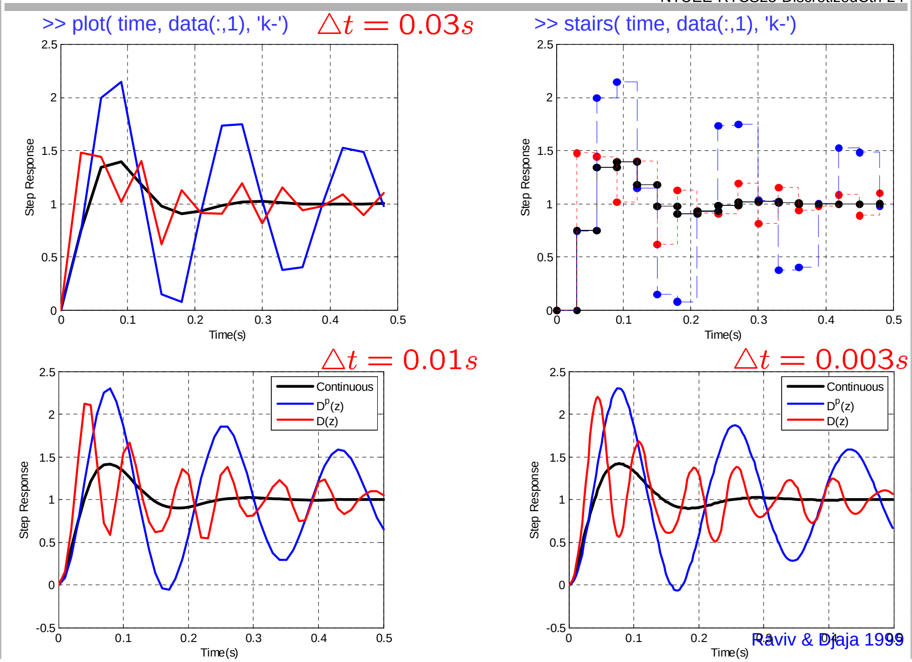
Raviv & Djaja 1999

Examples



Raviv & Djaja 1999

Examples $T = 0.03s$



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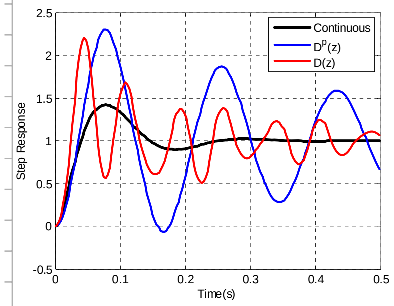
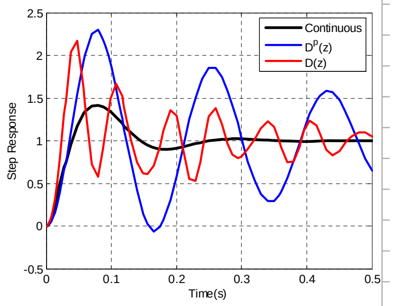
$\triangle t = 0.003s$



Sample time = -1 (inherited)



Raviv & Djaja 1999

- Rattan's Example:

$$G_p(s) = \frac{10}{s(s+1)}$$

$$\Rightarrow G_c(s) = \frac{1+0.416s}{1+0.319s}$$

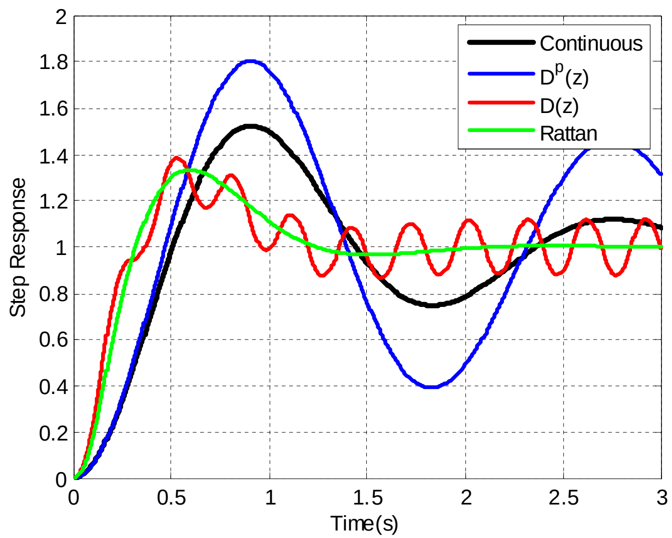
$T = 0.15s$

$$\Rightarrow D_{\text{Rattan}}(z) = \frac{3.436z - 2.191}{z + 0.2390}$$

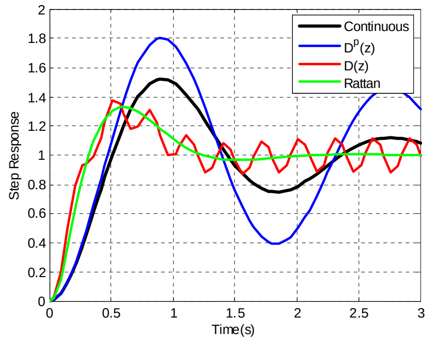
$$\Rightarrow D'(z) = \frac{2.294z - 1.5935}{z - 0.2991}$$ Tustin transformation

$$\Rightarrow D(z) = \frac{4.5888z^2 - 3.6459z + 0.3187}{z^2 + 0.5009z - 0.2393}$$

Raviv & Djaja 1999



Raviv & Djaja 1999



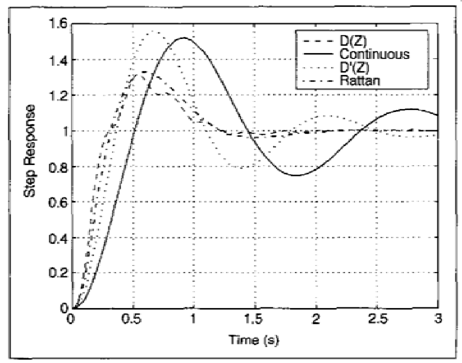


Fig. 8. Closed-loop step response of Example (d). T = 0.15 s.

Raviv & Djaja 1999

Homework 6

- By 11pm, 5/16/09 (Sunday) by e-mail to fengli@ntu.edu.tw
- Content:
  - Homework 6 (Discretized Controller)
  - Perform your simulation study of the four examples discussed in the paper by Raviv & Djaja, 1999
  - Submit R93921XXX.m of Matlab program
    - Name, Registration Number, Department, University, etc.
    - Date:
  - Submit R93921XXX.doc of Word file
    - Name, Registration Number, Department, University, etc.
    - Date:
    - From Matlab/Figure, use Edit/Copy Figure to copy every figure generated by the Matlab program
    - When copying figures, set up the following options:
      - Edit/Copy Options
        - Clipboard format -> Preserve information
        - Figure background color -> Transparent background
        - Size -> Match figure screen size
    - Discuss in detail how do you set up your simulation
    - Provide any possible description or explanation for each figure
    - Further discussions if possible