Nonlinear Systems Analysis

Lecture 6

Appendix B: Contraction Mapping

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Outline

- Appendix B: Contraction Mapping
  - Vector space
  - Normed linear space
  - Banach space
  - Contraction mapping theorem

B: Linear Vector Spaces – 1 (Appendix B; page 653)

- A linear vector space χ over the field R
  is a set of elements $x, y, z, \ldots,$ called vectors,
  such that for any two vectors $x, y \in \chi$

- the sum $x + y$ is defined, and
  -
  -
  -

- and there is zero vector $0 \in \chi$
  - such that

B: Linear Vector Spaces – 2

- For any numbers $\alpha, \beta \in R$,
  the scalar multiplication $\alpha\, x$ is defined, and
  -
  -
  -
  -
  -
  - and

- A linear space χ is a normed linear space
  if, to each vector $x \in \chi$,
  there is a real-valued norm $||x||$
  that satisfies:
-
-
-

- Convergence:
- Assume that χ is a normed linear space.
- A sequence $\{x_k\} \in \chi$ converges to $x \in \chi$
  if
- Closed Set:
- A set $S \subset \chi$ is closed
  iff

- Cauchy Sequence:

- A sequence $\{x_k\} \in \chi$ is said to be
  a Cauchy sequence
  if

- Banach Space:

- A normed linear space χ is complete
  if

- A complete normed linear space is
  a space.

- Theorem B.1 (Contraction Mapping):

- Let S be a closed subset of
  a Banach space χ and
  let T be a mapping that maps S into S.

- Suppose that

- THEN
  - there exists a unique vector $x^* \in S$ satisfying
  - x^* can be obtained by the method of successive approximation, starting from any arbitrary initial vector in S.

- Proof:
- Select an arbitrary $x_1 \in S$ and define the sequence $\{x_k\}$
- Since T maps S into S,
- Show that $\{x_k\}$ is Cauchy:
- Show that $x^* = T(x^*)$:
- Show that x^* is the unique fixed point of T in S.

- Show that $\{x_k\}$ is Cauchy:

B: Contraction Mapping Theorem – 6

- Show that $x^* = T(x^*)$:

B: Contraction Mapping Theorem – 7

- Show that x^* is the unique fixed point of T in S.

- T maps S into S.
- T is a contraction mapping over S.