Nonlinear Systems Analysis

Lecture 6

Appendix B: Contraction Mapping

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Outline

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- Appendix B: Contraction Mapping
 - Vector space
 - Normed linear space
 - Banach space
 - Contraction mapping theorem

B: Linear Vector Spaces – 1 (Appendix B; page 653)	Feng-Li Lian © 2005 NTUEE-NSA-AppB-3
$ullet$ A linear vector space χ over the field R	-
is a set of elements $x, y, z,$, called vectors,	
such that for any two vectors $x,y\in \mathbf{\chi}$	-
ullet the sum $x+y$ is defined, and	
-	-
	-

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- ullet and there is zero vector $0 \in \chi$
 - such that

B: Linear Vector Spaces – 2

• For any numbers $\alpha, \beta \in R$, the scalar multiplication α x is defined, and

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- and

- A linear space χ is a normed linear space if, to each vector $x \in \chi$, there is a real-valued norm ||x|| that satisfies:
- •
- •
- •

B: Banach Space - 1

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- Convergence:
- ullet Assume that χ is a normed linear space.
- ullet A sequence $\{x_k\}\in\chi$ converges to $x\in\chi$ if
- Closed Set:
- A set $S \subset \chi$ is closed iff

B: Contraction Mapping Theorem – 1 (Appendix B; page 655)

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• Theorem B.1 (Contraction Mapping):

space.

• Let S be a closed subset of a Banach space χ and let T be a mapping that maps S into S.

Suppose that

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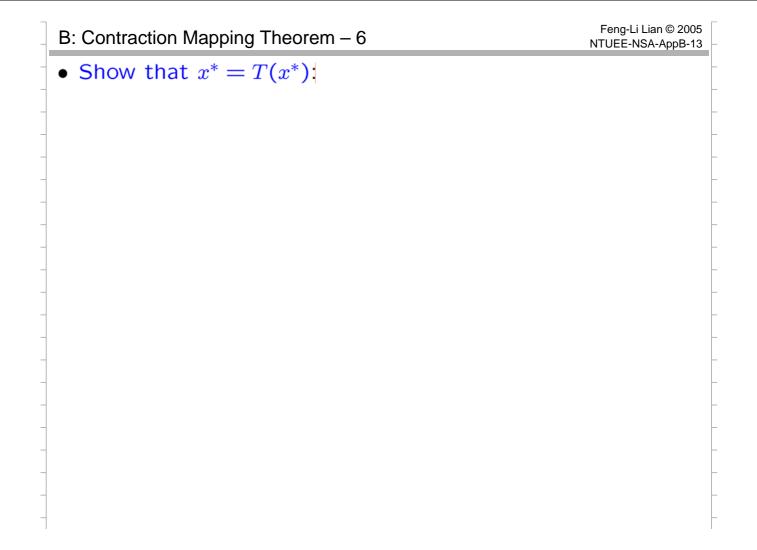
- THEN
 - there exists a unique vector $x^* \in S$ satisfying
 - $-x^*$ can be obtained by the method of successive approximation, starting from any arbitrary initial vector in S.

B: Contraction Mapping Theorem – 3

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- Proof:
- Select an arbitrary $x_1 \in S$ and define the sequence $\{x_k\}$
- Since T maps S into S,
- Show that $\{x_k\}$ is Cauchy:
- Show that $x^* = T(x^*)$:
- Show that x^* is the unique fixed point of T in S.

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B: Contraction Mapping Theorem – 7

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• Show that x^* is the unique fixed point of T in S.

 \bullet T maps S into S.

ullet T is a contraction mapping over S.