93學年 第一學期

非線性系統分析
Nonlinear Systems Analysis

Lecture 1

Course Information & Introduction

連豐力

台大電機系

Sep04 – Jan05

Syllabus

- Course Information:
  - No: 921 U3390
  - Time: Thu 1:30am-4:20pm
  - Room: EE-BLH 620
  - Website: http://cc.ee.ntu.edu.tw/~fengli/Teaching/NonlinearSystems
- Instructor:
  - 連豐力(Feng-Li Lian)
  - Office: EE-BLH 620
  - Email: fengli@ntu.edu.tw
  - Phone: 02-3366-3606
- Grading:
  - Homework (30%), or (25%)
  - Midterm (30%), or (25%)
  - Final (40%), or (30%)
  - Project (20 %)
- Textbook:
  - Nonlinear Systems by Hassan Khalil, 3rd Edition, 2002
- Reference:
  - Nonlinear Systems: Analysis, Stability and Control, by Shanker Sastry, 1999
  - Nonlinear Systems Analysis by Vidyasagar, 2nd Edition, 1993
  - Control in an Information Rich World, Report of the Panel on Future Directions in Control, Dynamics, and Systems. http://www.cds.caltech.edu/~murray/cdspanel/report/cdspanel-15aug02.pdf

Course Outline

- 18 weeks (9/16/02-1/13/03)
  - Exam: 2 weeks (11/25, 1/13),
  - Holiday: none
  - Total 16 week lectures,
  - Homework assignments: approximately 8-10;
    - No late homework accepted

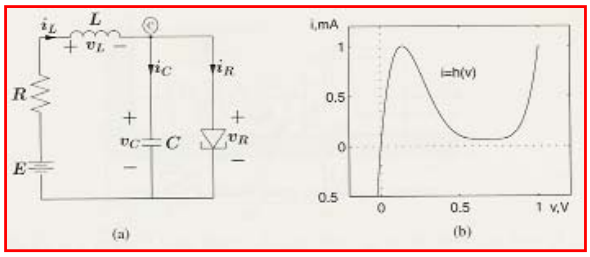
- Basic analysis
  1. Introduction (1 week)
  2. Math + Second-order systems (3)
  3. Fundamental properties (2~3)
  4. Lyapunov stability (4~5)
- Analysis of feedback systems
  5. Input-output stability (1)
  6. Passivity (1)
  7. Frequency domain analysis
- Advanced analysis
  8. Advanced stability analysis (2)
  9. Stability of perturbed systems
  10. Perturbation theory & averaging
  11. Singular perturbations
- Nonlinear control
  12. Feedback control
  13. Feedback linearization
  14. Nonlinear design tools

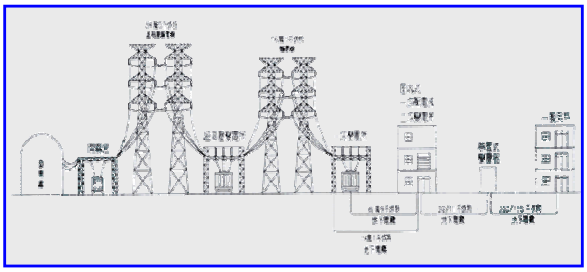
Today's Outline

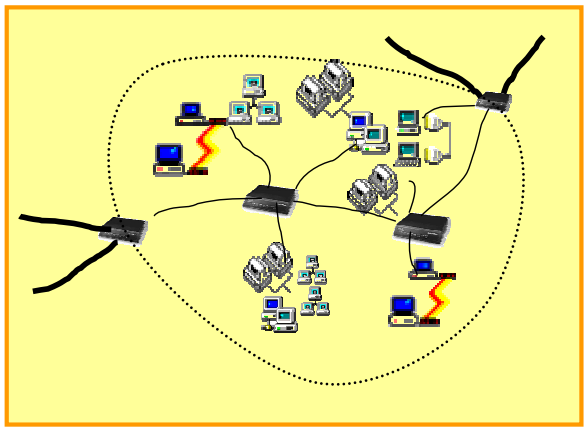
- Engineering Examples
  - Electrical engineering
  - Mechanical engineering
  - Chemical engineering
  - Transportation, etc.
- Analysis & Design Methodology
- Nonlinear Models & Phenomena
- Examples of Nonlinear Systems

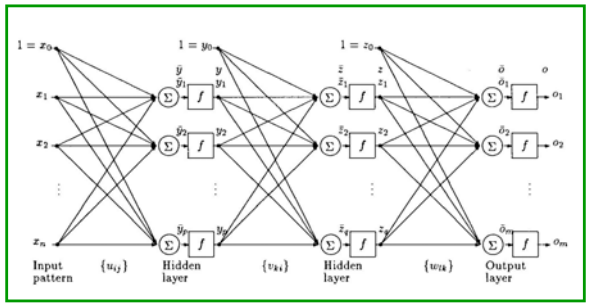
Engineering Examples - 1

- Electrical Engineering:
  - Tunnel-Diode Circuit
  - Electrical Power Transmission
  - Internet, Communication Network
  - Artificial Neural Network





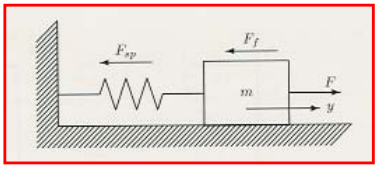


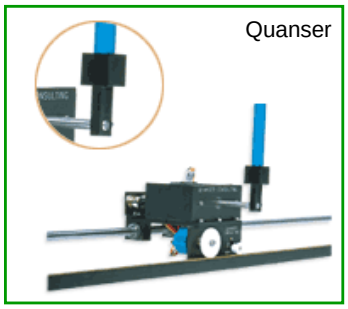


Engineering Examples - 2

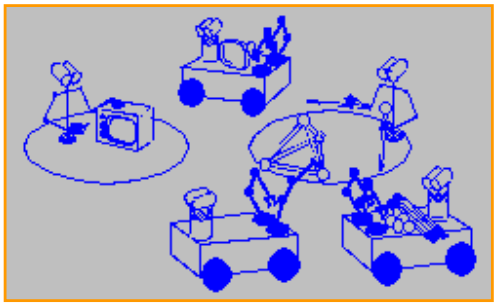
- Mechanical Engineering:
  - Mass-Spring System
  - (Inverted) Pendulum
  - Robot Arms, Walking robots
  - Vehicle Engine/Speed Control
  - Fluid Dynamics
  - Manufacturing/Machining Systems

SAE





Quanser

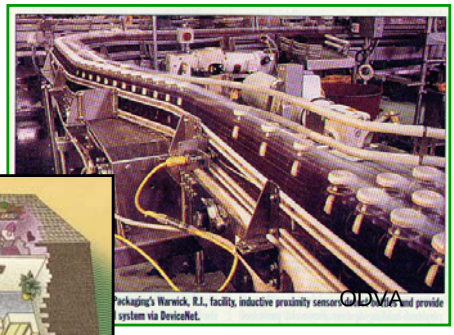


UMass Robotics

UM/ERC/ RMS

Engineering Examples - 3

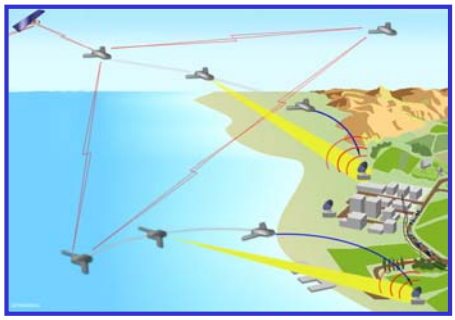
- Chemical Engineering:
  - Petroleum Engineering
  - Food Processing
  - Waster Material Handling
  - Heating, Ventilation & Air Conditioning
- Transportation:
  - Intelligent Highway System
  - Air Traffics Management
  - Unmanned Air/Ground Vehicle



ODVA

P&N Distribution

Berkeley PATH



DARPA MICA

Analysis & Design Philosophy

Problem → **Modeling** → **Analysis** → **Design (Control)**

- Mathematics, Statistics:
  - Differential Equations, Linear Algebra, Probability, Stochastic Processes
- Physics, Chemistry:
  - Electronics, Electrical Circuits, Electromagnetics, Dynamics, Thermodynamics, Heat Transfer
- Linear v.s. Nonlinear

- Stability
- Performance
- Robustness

- Hardware vs Software
- Optimization vs Control
  - Optimal Control, Robust Control, Adaptive Contrrol, Nonlinear Control

Analysis in Linear & Nonlinear Systems

	Linear Systems transfer functions	Nonlinear Systems no transfer functions
Analysis • Stability • Performance • Robustness	• Poles in LHP • Root Locus • Bode Plot	?
	• Steady-State Error < 5%	?
	• Phase/Gain Margin	?

Essentially Nonlinear Phenomena - 1

- Finite Escape Time
  - State goes to infinity in finite time
- Multiple Isolated Equilibria
  - Nonlinear systems has more than one isolated equilibrium points.
  - The state convergence depends on the initial conditions.
- Limit Cycles
  - Go into an oscillation of fixed magnitude and frequency, irrespective of the initial state.
- Subharmonic, harmonic, almost-periodic oscillations
  - Oscillation frequencies are submultiples or multiples of the input frequency.

Essentially Nonlinear Phenomena - 2

- Chaos
  - Complicated steady-state behavior
  - Sometime random
- Multiple modes of behavior
  - Unforced systems may have more than one limit cycle
  - Forced systems with periodic excitation may exhibit harmonic, sub-harmonic, or complicated steady-state behavior, depending upon the amplitude and frequency of the input.
  - Exhibit discontinuity jump even though under smoothly changed input.

Examples of Nonlinear Models

- Examples:
  - Pendulum Equation
  - Tunnel-Diode Circuit
  - Mass-Spring System
  - Negative-Resistance Oscillator
  - Artificial Neural Network
  - Adaptive Control
- Common Nonlinearities
  - Relay
  - Saturation
  - Dead zone
  - Quantization

Pendulum - 1

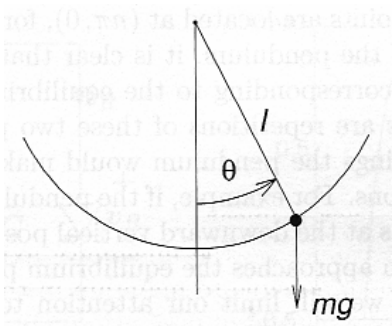


Figure 1.1: Pendulum.

Using Newton's Second Law,
Write the equation of motion
in the tangential direction:

$$ml\ddot{\theta} = -mg\sin\theta - kl\dot{\theta}$$

State model (let $x_1 = \theta, x_2 = \dot{\theta}$):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{l}\sin x_1 - \frac{k}{m}x_2 \end{aligned}$$

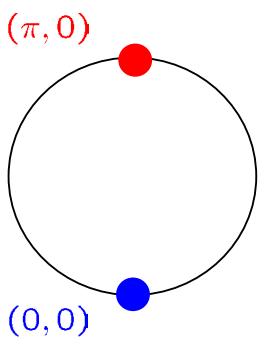
Pendulum - 2

Equilibrium points (let $\dot{x}_1 = \dot{x}_2 = 0$):

$$\begin{aligned} 0 &= x_2 \\ 0 &= -\frac{g}{l}\sin x_1 - \frac{k}{m}x_2 \end{aligned}$$

Equilibrium points are $(n\pi, 0), n = 0, \pm 1, \pm 2, \ldots,$
or, physically, $(0, 0)$ and $(\pi, 0)$.

Question? Which one is stable or unstable?



Tunnel-Diode Circuit - 1

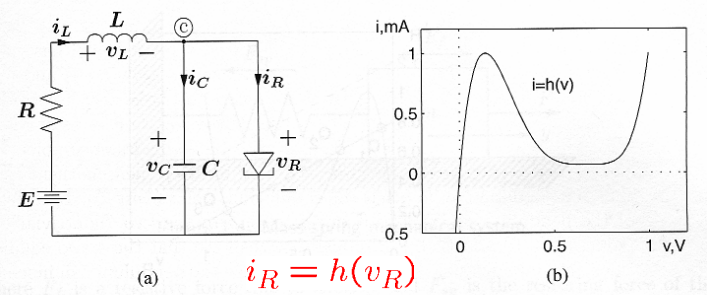


Figure 1.2: (a) Tunnel-diode circuit; (b) Tunnel-diode v_R–i_R characteristic.

Kirchhoff's current/voltage law:

$$\begin{aligned} i_C + i_R - i_L &= 0 \quad \text{(KCL)} \\ v_C - E + Ri_L + v_L &= 0 \quad \text{(KVL)} \end{aligned}$$

State model:

- state: $x_1 = v_C, x_2 = i_L$, and input: $u = E$,
- $i_C = C\frac{dv_C}{dt}$, $v_L = L\frac{di_L}{dt}$

$$\begin{aligned} \dot{x}_1 &= \frac{1}{C}[-h(x_1) + x_2] \\ \dot{x}_2 &= \frac{1}{L}[-x_1 - Rx_2 + u] \end{aligned}$$

Tunnel-Diode Circuit - 2

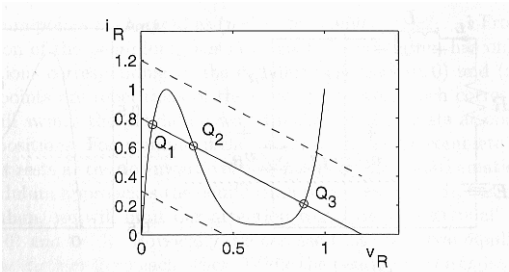


Figure 1.3: Equilibrium points of the tunnel-diode circuit.

Equilibrium points:

$$\begin{aligned} 0 &= -h(x_1) + x_2 \\ 0 &= -x_1 - Rx_2 + u \end{aligned}$$

That is, the roots of:

$$h(x_1) = \frac{E}{R} - \frac{1}{R}x_1$$

Mass-Spring System - 1

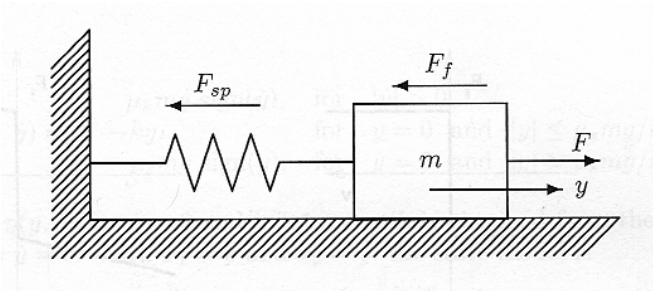


Figure 1.4: Mass-spring mechanical system.

Newton's Law of motion:

$$m\ddot{y} + F_f + F_{sp} = F$$

F_f due to friction:

$$\begin{aligned} \text{Static} &= \text{limit to } \pm\mu_s mg \\ \text{Coulomb} &= \begin{cases} -\mu_k mg, & \text{for} \quad v < 0 \\ \mu_k mg, & \text{for} \quad v > 0 \end{cases} \\ \text{Viscours} &= h(v) \approx cv \end{aligned}$$

Mass-Spring System - 2

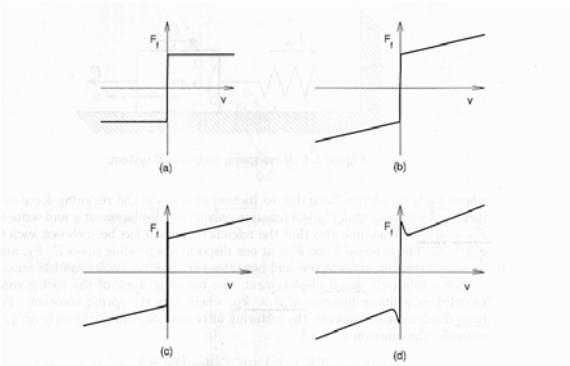


Figure 1.5: Examples of friction models. (a) Coulomb friction; (b) Coulomb plus linear viscous friction; (c) static, Coulomb, and linear viscous friction; (d) static, Coulomb, and linear viscous friction—Stribeck effect.

F_{sp} due to the spring $(= g(y))$:

$$\begin{aligned} g(y) &= ky, \text{(small y)} \\ &= k(1 - a^2y^2)y, |ay| < 1, \\ &\quad \text{(softening spring)} \\ &= k(1 + a^2y^2)y, \\ &\quad \text{(hardening spring)} \end{aligned}$$

Mass-Spring System - 3

Linear spring, static fricition, Coulumb friction, linear viscous friction, $F = 0$:

$$m\ddot{y} + ky + c\dot{y} + \eta(y, \dot{y}) = 0$$

where:

$$\eta(y, \dot{y}) = \begin{cases} \mu_k mg \text{ sign}(\dot{y}), & \text{for} \quad |\dot{y}| > 0 \\ -ky, & \text{for} \quad \dot{y} = 0 \text{ and } |y| \le \mu_s mg/k \\ -\mu_s mg \text{ sign}(y), & \text{for} \quad \dot{y} = 0 \text{ and } |y| > \mu_s mg/k \end{cases}$$

State model (let $x_1 = y, x_2 = \dot{y}$):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{c}{m}x_2 - \frac{1}{m}\eta(x_1, x_2) \end{aligned}$$

Remark:

1. An equilibrium set, rather than points;
2. A discontinuous function of the state.

Negative-Resistance Oscillator - 1

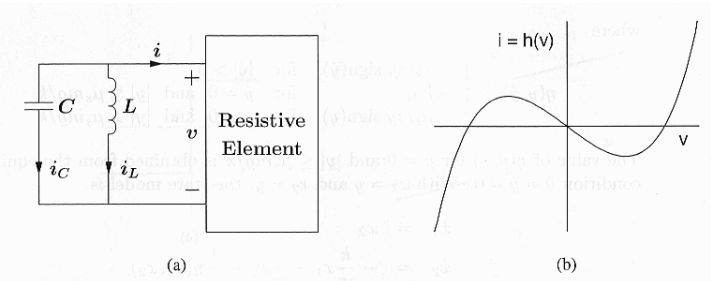


Figure 1.6: (a) Basic oscillator circuit; (b) Typical driving-point characteristic.

$$\begin{aligned} h(0) &= 0 \\ h'(0) &< 0 \\ h(v) \to \infty &\text{ as } v \to \infty \\ h(v) \to -\infty &\text{ as } v \to -\infty \end{aligned}$$

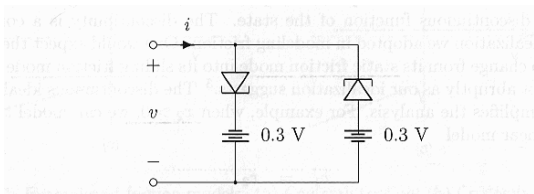


Figure 1.7: A negative-resistance twin-tunnel-diode circuit.

Kirchhoff's current law:

$$i_C + i_L + i = 0 \text{ or}$$

$$C\frac{dv}{dt} + \frac{1}{L}\int_{-\infty}^{t} v(s)ds + h(v) = 0$$

(dt) and $(\times L)$:

$$CL\frac{d^2v}{dt^2} + v + Lh'(v)\frac{dv}{dt} = 0$$

Negative-Resistance Oscillator - 2

Let $\tau = t/\sqrt{CL}, \epsilon = \sqrt{L/C}$:

$$\ddot{v} + \epsilon h'(v)\dot{v} + v = 0$$

The **Lienard's equation**:

$$\ddot{v} + f(v)\dot{v} + g(v) = 0$$

The **Van der Pol equation**:

$$\ddot{v} - \epsilon(1 - v^2)\dot{v} + v = 0$$

Artificial Neural Network (ANN) - 1

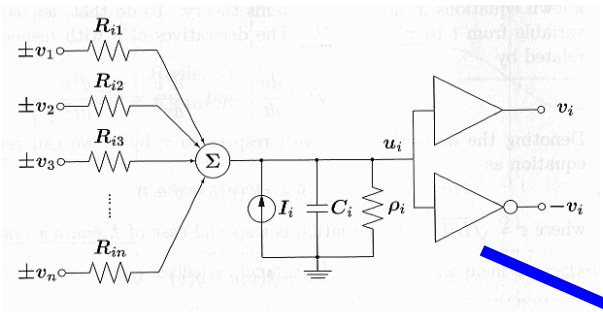
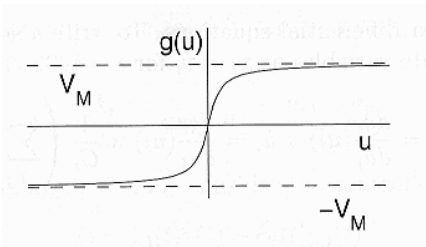


Figure 1.8: Hopfield neural network model.

$$v_i = g_i(u_i)$$



$g_i(\cdot)$: a sigmoid function:

$$g_i(u_i) = \frac{2V_M}{\pi}\tan^{-1}\left(\frac{\lambda\pi u_i}{2V_M}\right), \lambda > 0$$

$$g_i(u_i) = V_M\frac{e^{\lambda u_i} - e^{-\lambda u_i}}{e^{\lambda u_i} + e^{-\lambda u_i}} = V_M \tanh(\lambda u_i), \lambda > 0$$

Artificial Neural Network (ANN) - 2

Kirchhoff's current law:

$$
\begin{aligned}
C_i \frac{du_i}{dt} &= \sum_j \frac{1}{R_{ij}}(\pm v_j - u_i) - \frac{1}{\rho} u_i + I_i \\
&= \sum_j T_{ij} v_j - \frac{1}{R_i} u_i + I_i
\end{aligned}
$$

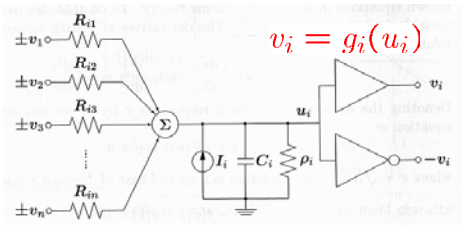


Figure 1.8: Hopfield neural network model.

State model (let $x_i = v_i, i = 1, 2, ..., n$):

$$
\begin{aligned}
\dot{x}_i &= \frac{dg_i}{du_i}(u_i) \times \dot{u}_i \\
&= \frac{dg_i}{du_i}(u_i) \times \frac{1}{C_i}\left[\sum_j T_{ij} x_j - \frac{1}{R_i} u_i + I_i\right]
\end{aligned}
$$

Let $h_i(x_i) = \frac{dg_i}{du_i}(u_i)|_{u_i = g_i^{-1}(x_i)}$,
and $h_i(x_i) > 0, \forall x_i \in (-V_M, V_M)$

Equilibrium points

$$
0 = \sum_j T_{ij} x_j - \frac{1}{R_i} g^{-1}(x_i) + I_i
$$

Direct Model Reference Adaptive Control (MRAC) - 1

Plant model: $\dot{y}_p = a_p y_p + k_p u$

Reference model: $\dot{y}_m = a_m y_m + k_m r$

Input: $u(t) = \theta_1^* r(t) + \theta_2^* y_p(t)$

$$
\theta_1^* = \frac{k_m}{k_p} \text{ and } \theta_2^* = \frac{a_m - a_p}{k_p}
$$

Gradient algorithm:

$$
\begin{aligned}
\dot{\theta}_1 &= -\gamma(y_p - y_m) r \\
\dot{\theta}_2 &= -\gamma(y_p - y_m) y_p
\end{aligned}
$$

Error state:

$$
\begin{aligned}
e_0 &= y_p - y_m, \\
\phi_1 &= \theta_1 - \theta_1^*, \\
\phi_2 &= \theta_2 - \theta_2^*
\end{aligned}
$$

Direct Model Reference Adaptive Control (MRAC) - 2

Error dynamics:

$$\begin{aligned}\dot{e}_0 &= a_m e_0 + k_p\phi_1 r(t) + k_p\phi_2[e_0 + y_m(t)]\\ \dot{\phi}_1 &= -\gamma e_0 r(t)\\ \dot{\phi}_2 &= -\gamma e_0[e_0 + y_m(t)]\end{aligned}$$

Equilibrium points:

$$\begin{aligned}0 &= a_m e_0 + k_p\phi_1 r(t) + k_p\phi_2[e_0 + y_m(t)]\\ 0 &= -\gamma e_0 r(t)\\ 0 &= -\gamma e_0[e_0 + y_m(t)]\end{aligned}$$

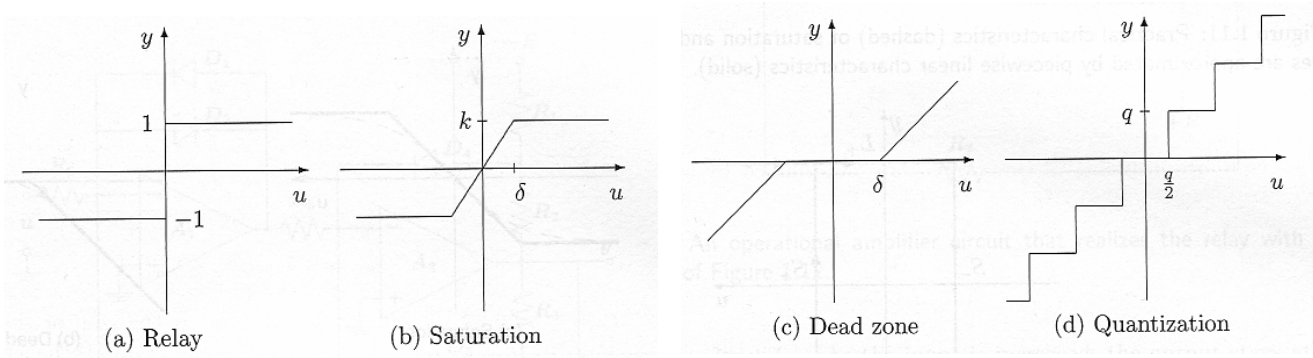
If $r(t) = 0, i.e., y_m(t) = 0$

$$\begin{aligned}0 &= [a_m + k_p\phi_2]e_0\\ 0 &= -\gamma e_0^2\end{aligned}$$

At steady state:

$$\begin{aligned}e_0 &\rightarrow 0\\ \phi_i &\rightarrow 0 \quad (?)\end{aligned}$$

Common Nonlinearities



(a) Relay (b) Saturation (c) Dead zone (d) Quantization

Homework Assignment 1

- Due 9/30 in class:
  1) 1.11 (page 26)
  2) 1.15 (page 28)
  3) 1.21 (page 33)

- Outline of a technical paper/report:
  - Introduction
  - Background/Literature Survey
  - Problem Formulation
  - Analysis/Design/Proposed Solutions
  - Validation & Verification by Simulation/Experiment
  - Conclusions & Future Work
  - References

- 每一個作業：
  - 請註明：作業次別，姓名，學號，系級，日期
  - 請說明清楚該次作業之撰寫為：
    1) 自行獨立完成作業，且未參考過去之作業或解答或與其他同學討論。
    2) 自行獨立完成作業，曾與同學或學長討論之後，但未參考過去之作業或解答。
    3) 曾參考其他同學或過去之作業或解答之後，完成作業。
    4) 完全抄襲其他相關資料。

1.11 A phase-locked loop [64] can be represented by the block diagram of Figure 1.20. Let $\{A, B, C\}$ be a minimal realization of the scalar, strictly proper transfer function $G(s)$. Assume that all eigenvalues of A have negative real parts, $G(0) \neq 0$, and $\theta_i =$ constant. Let z be the state of the realization $\{A, B, C\}$.

(a) Show that the closed-loop system can be represented by the state equations

$$\dot{z} = Az + B\sin e, \qquad \dot{e} = -Cz$$

(b) Find all equilibrium points of the system.

(c) Show that when $G(s) = 1/(\tau s + 1)$, the closed-loop model coincides with the model of a pendulum equation.

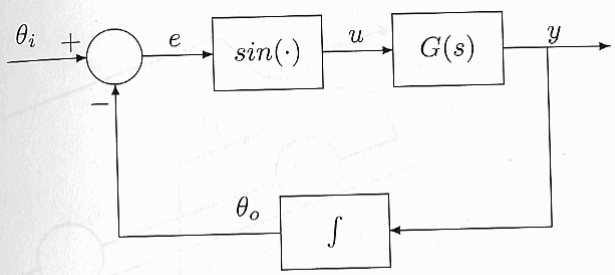


Figure 1.20: Exercise 1.11.

1.15 Consider the inverted pendulum of Figure 1.24 [110]. The pivot of the pendulum is mounted on a cart that can move in a horizontal direction. The cart is driven by a motor that exerts a horizontal force F on the cart. The figure shows also the forces acting on the pendulum, which are the force mg at the center of gravity, a horizontal reaction force H, and a vertical reaction force V at the pivot. Writing horizontal and vertical Newton's laws at the center of gravity of the pendulum yields

$$m\frac{d^2}{dt^2}(y + L\sin\theta) = H \quad \text{and} \quad m\frac{d^2}{dt^2}(L\cos\theta) = V - mg$$

Taking moments about the center of gravity yields the torque equation

$$I\ddot{\theta} = VL\sin\theta - HL\cos\theta$$

while a horizontal Newton's law for the cart yields

$$M\ddot{y} = F - H - k\dot{y}$$

Here m is the mass of the pendulum, M is the mass of the cart, L is the distance from the center of gravity to the pivot, I is the moment of inertia of the pendulum with respect to the center of gravity, k is a friction coefficient, y is the displacement of the pivot, θ is the angular rotation of the pendulum (measured clockwise), and g is the acceleration due to gravity.

(a) Carrying out the indicated differentiation and eliminating V and H, show that the equations of motion redúce to

$$I\ddot{\theta} = mgL\sin\theta - mL^2\ddot{\theta} - mL\ddot{y}\cos\theta$$

$$M\ddot{y} = F - m\left(\ddot{y} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta\right) - k\dot{y}$$

(b) Solving the foregoing equations for $\ddot{\theta}$ and $\ddot{y}$, show that

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{y} \end{bmatrix} = \frac{1}{\Delta(\theta)} \begin{bmatrix} m+M & -mL\cos\theta \\ -mL\cos\theta & I+mL^2 \end{bmatrix} \begin{bmatrix} mgL\sin\theta \\ F + mL\dot{\theta}^2\sin\theta - k\dot{y} \end{bmatrix}$$

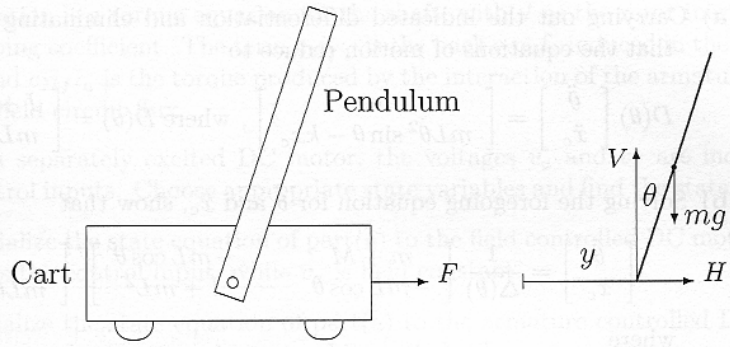


Figure 1.24: Inverted pendulum of Exercise 1.15.

where

$$\Delta(\theta) = (I + mL^2)(m + M) - m^2L^2\cos^2\theta \geq (I + mL^2)M + mI > 0$$

(c) Using $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = y$, and $x_4 = \dot{y}$ as the state variables and $u = F$ as the control input, write down the state equation.

1.21 The valves in the hydraulic system of Figure 1.30 obey the flow relationships $w_1 = k_1\sqrt{p_1 - p_2}$ and $w_2 = k_2\sqrt{p_2 - p_a}$. The pump has the characteristic shown in Figure 1.29 for $(p_1 - p_a)$ versus w_p. The various components and variables are defined in the previous two exercises.

(a) Using $(p_1 - p_a)$ and $(p_2 - p_a)$ as the state variables, find the state equation.

(b) Find all equilibrium points of the system.

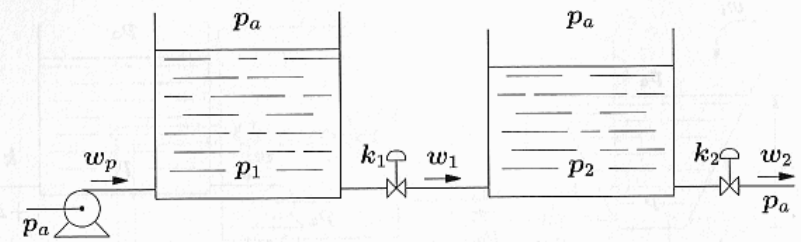


Figure 1.30: The hydraulic system of Exercise 1.21.