

Do
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P. 339
§ 2.4.3 A is diagonal

$$\bar{A} = P(A)P^{-1}$$

diagonal?

diagonalization

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$H(s) = \frac{6s^2 + 58s + 138}{s^2 + 15s + 174s + 120}$$

dim(A) = 3
eig(A): distinct

$$H(s) = \frac{6s^2 + 58s + 138}{(s+4)(s+5)(s+6)}$$

$$H(s) = \frac{2}{s+4} + \frac{2}{s+5} + \frac{3}{s+6}$$

$$\hat{y}(s) = H(s)\hat{u}(s)$$

$$= (H_1(s) + H_2(s) + H_3(s))\hat{u}(s)$$

$$= H_1(s)\hat{u}_1(s) + H_2(s)\hat{u}_2(s) + H_3(s)\hat{u}_3(s)$$

$$\begin{cases} \dot{x}_1 = -5x_1 + 2u \\ \dot{x}_2 = -6x_2 + 3u \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u$$

$$C = \begin{bmatrix} * & * & * \\ * & 0 & * \\ * & * & * \end{bmatrix}$$

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$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

equivalent

$$By \quad \bar{A} = (PA)P^{-1}, \quad \bar{B} = PB, \quad \bar{C} = CP^{-1}, \quad \bar{D} = D$$

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \\ y = \bar{C}\bar{x} + \bar{D}u \end{cases}$$

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Balanced Realization

L. O

$$W_r = \begin{matrix} n \times n \\ n \times n \end{matrix} \quad W_o = \begin{matrix} n \times n \\ n \times n \end{matrix}$$

$$\dot{x} = \begin{bmatrix} -1 & -\frac{a}{2} \\ 4a & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2a \end{bmatrix} u$$

$$W_r = \begin{bmatrix} 0.5 & 0 \\ 0 & a^2 \end{bmatrix}$$

$$W_o = \begin{bmatrix} 0.5 & 0 \\ 0 & \frac{1}{a^2} \end{bmatrix}$$

$$\Rightarrow H(s) = \frac{3s + 18}{s^2 + 3s + 18}$$

$$\text{rank}(C) = \dots = 2$$

$$\text{rank}(B) = \dots = 2 \quad a \neq 0$$

$$\exists P \quad PP^T = P^T P = I_{nn}$$

$$\bar{W}_r = P W_r P^T$$

$$\bar{W}_o = (P^T)^T W_o P^{-1}$$

$$(\bar{W}_r \bar{W}_o) = (P W_r P^T) ((P^T)^T W_o P^{-1})$$

$$= P (W_r W_o) P^{-1}$$

Similar

$$\text{eig}(\bar{W}_r \bar{W}_o) = \text{eig}(W_r W_o)$$

$$W_r W_o = \sum \lambda_i = P (W_r W_o) P^{-1}$$

$$\sum \lambda_i = \sum \lambda_i = \sum \begin{bmatrix} \lambda_i & 0 \\ 0 & \lambda_i \end{bmatrix}$$

$$(A, B, C) \rightarrow (\bar{A}, \bar{B}, \bar{C})$$

$$\begin{matrix} W_r \\ W_o \end{matrix} \quad \begin{matrix} \bar{W}_r \\ \bar{W}_o \end{matrix}$$

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$$u(t) = B^T e^{A^T(t-\tau)} n_1 - B^T e^{A^T(t-\tau)} n_2$$

$$W_r(0, T) n_1 = x_1$$

$$x_0 = W_o^{-1}(0, T) \left[\int_0^T e^{A^T(t-\tau)} C^T y(\tau) d\tau \right]$$

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$$W_r = Q^T \Lambda_r Q$$

diagonal

$$\det(\sigma^2 I - W_r W_o)$$

$$= \det(\sigma^2 I - R^T R W_o)$$

$$= \det(\sigma^2 I - R^T W_o R)$$

$$= Q^T \Lambda_r^{-1} Q$$

$$= \Theta^T (\Lambda_r^{-1})^{\frac{1}{2}} \Lambda_r^{\frac{1}{2}} \Theta$$

$$= \begin{bmatrix} R^T R \\ R \end{bmatrix}$$

$$R W_o R^T = U \Sigma^2 U^T$$

$$R, U, \Sigma$$

$$U^T U = I$$

$$P \equiv \Sigma^{\frac{1}{2}} U^T (R^T)^{-1}$$

$$P^T \equiv R^T U \Sigma^{\frac{1}{2}}$$

$$\bar{W}_o = (P^T)^T W_o P^{-1}$$

$$= (\Sigma^{\frac{1}{2}} U^T R) W_o (R^T U \Sigma^{\frac{1}{2}})$$

$$(A, B, C) \Rightarrow W_r, W_o$$

$$\Rightarrow \begin{cases} \bar{A} = PAP^{-1} \\ \bar{B} = PB \\ \bar{C} = CP^{-1} \\ \bar{D} = D \end{cases}$$

$$\bar{W}_r = P W_r P^T$$

$$= \Sigma^{\frac{1}{2}} U^T (R^T)^{-1} W_r R^T U \Sigma^{\frac{1}{2}}$$

$$= \Sigma^{\frac{1}{2}} U^T \underbrace{R^T R}_{I} U \Sigma^{\frac{1}{2}}$$

$$= \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}}$$

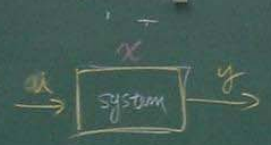
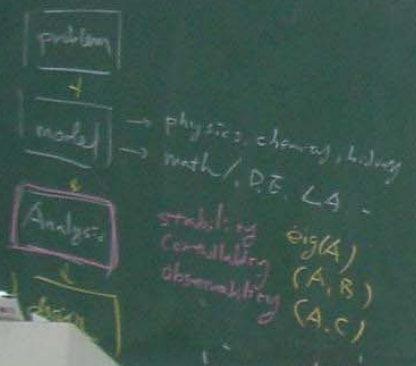
$$= \Sigma$$

$$\Rightarrow \bar{W}_o = \bar{W}_r = \Sigma$$

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Chap 9 state feedback and state observer



$$\hat{q}(s) = H(s) \hat{q}(s)$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

① unstable
weakly stable

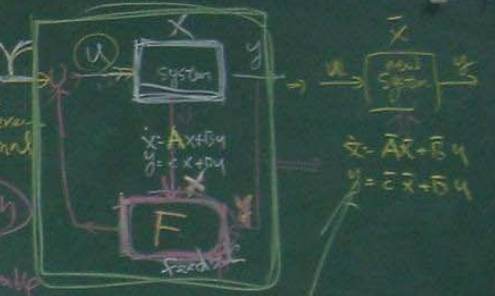
$$\text{eig}(A) \rightarrow \text{eig}(\bar{A})$$

$$\bar{A} = PAP^{-1}$$

⇒ need (A, B) controllable
⇒ How, $U(t)$? (state feedback)

② $y \in \mathbb{R}^p$
 $x \in \mathbb{R}^n$
 $n > p$
 $y \rightarrow x$
 $\rightarrow \rightarrow 10$

⇒ need (A, C) observable
⇒ How, state observer



$$\begin{cases} \dot{\hat{x}} = \bar{A}\hat{x} + \bar{B}u \\ y = \bar{C}\hat{x} + \bar{D}u \end{cases}$$

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