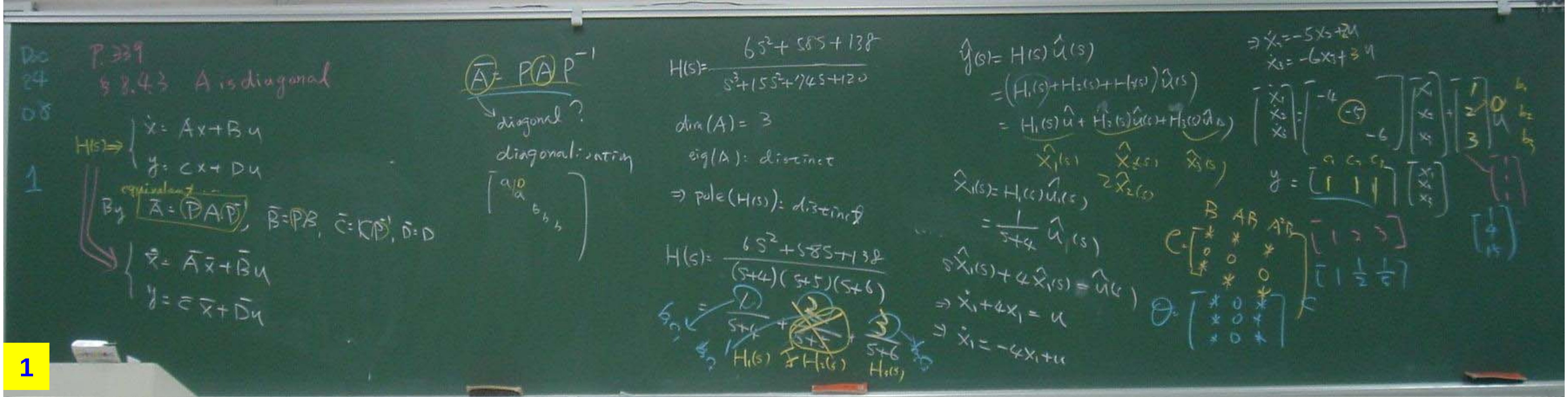
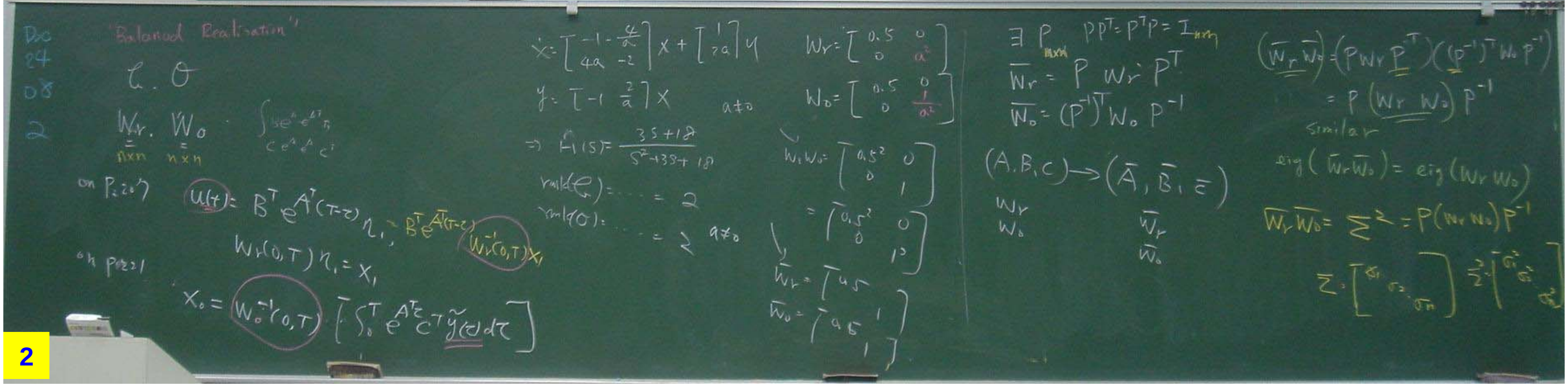
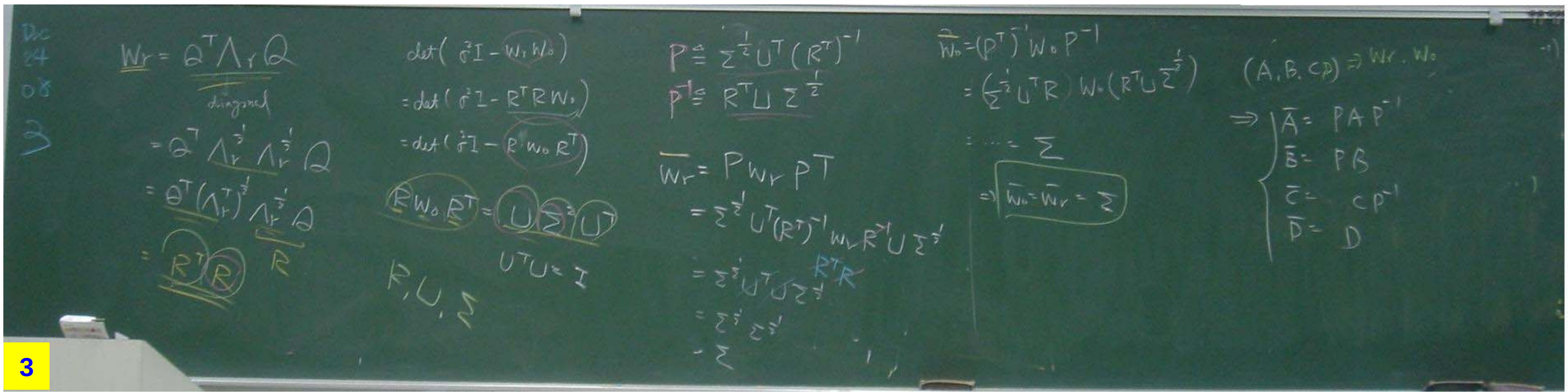
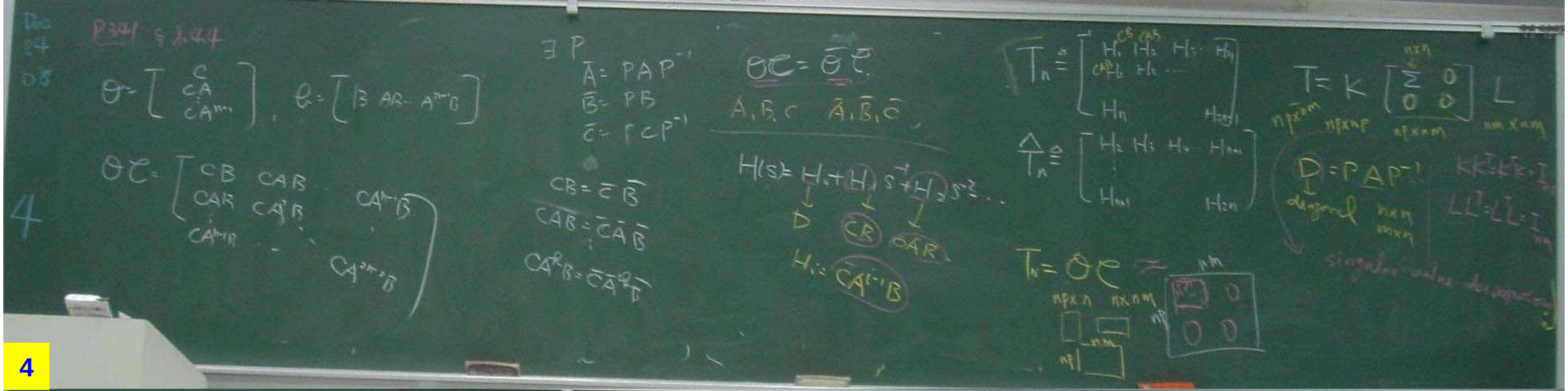
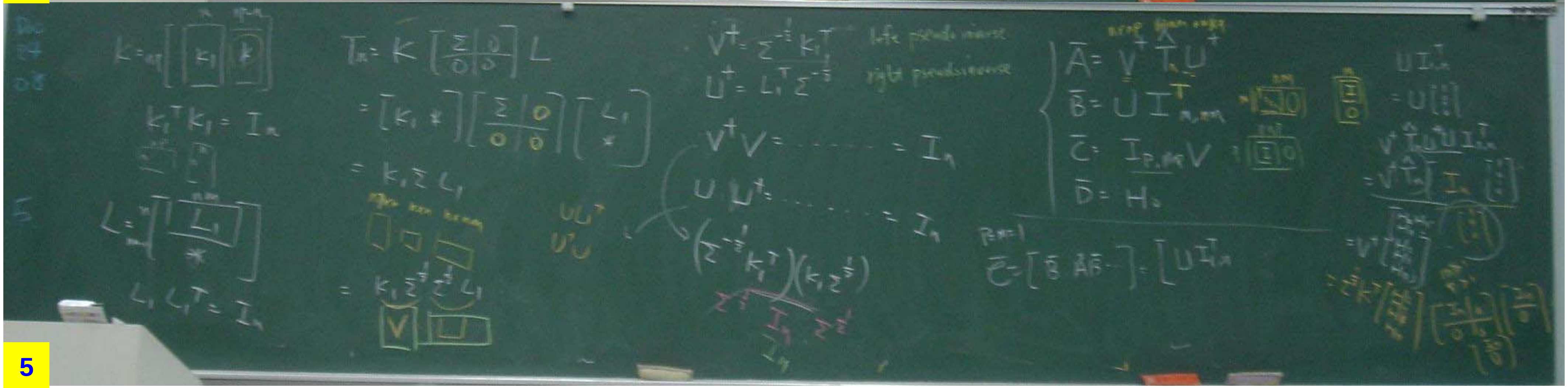

Dec 24 08
1
P.339
§8.4.3 A is diagonal
ẋ = Ax + Bu
y = Cx + Du
By Ā = PAP⁻¹, B̄ = PB, C̄ = CP⁻¹, D̄ = D
Ā = PAP⁻¹
diagonal?
diagonalization
dim(A) = 3
eig(A): distinct
⇒ pole(H(s)): distinct
ŷ(s) = H(s) û(s)
y = [1 1 1]
1


Dec 24 08
2
"Balanced Realization"
Wr. Wo
n×n n×n
Wr(0,T) η₁ = x₁
ẋ = [−1 −4/a; 4a −2] x + [1; 2a] u
y = [−1 2/a] x
a≠0
Wr = [0.5 0; 0 a²]
∃ P PPᵀ = PᵀP = I
W̄r = P Wr Pᵀ
W̄o = (P⁻¹)ᵀ Wo P⁻¹
(A, B, C) → (Ā, B̄, C̄)
similar
eig(W̄rW̄o) = eig(WrWo)
W̄rW̄o = Σ² = P(WrWo)P⁻¹
2

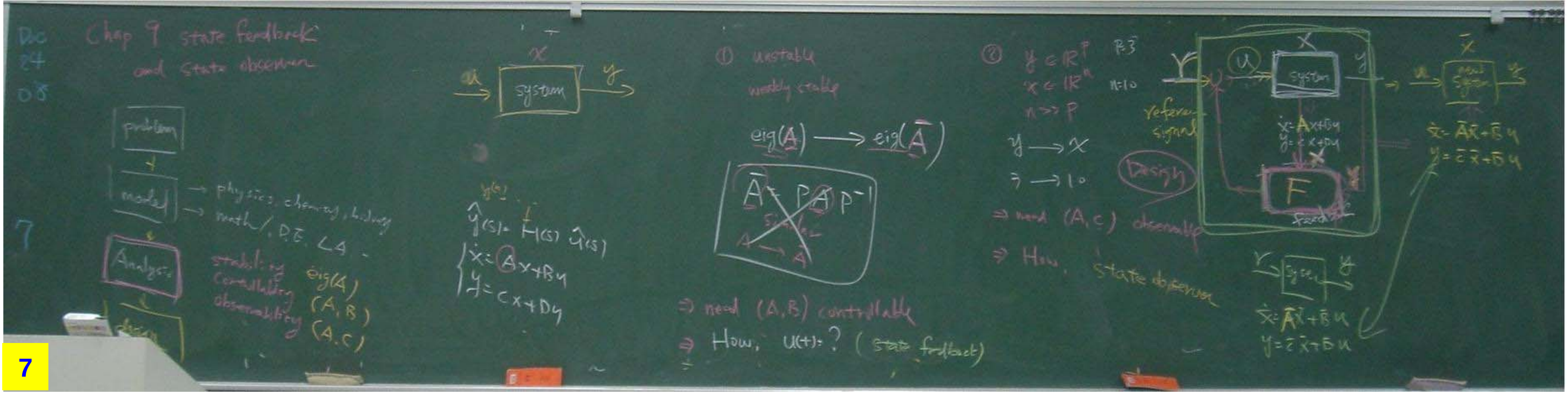

Dec 24 08
3
Wr = QᵀΛrQ
diagonal
det(σ²I − WrWo)
= det(σ²I − RᵀRWo)
= det(σ²I − RWoRᵀ)
RWoRᵀ = UΣ²Uᵀ
UᵀU = I
R, U, Σ
P ≜ Σ^{1/2} Uᵀ (Rᵀ)⁻¹
P⁻¹ ≜ RᵀU Σ^{-1/2}
W̄r = P Wr Pᵀ
= Σ
W̄o = (Pᵀ)⁻¹ Wo P⁻¹
⇒ W̄o = W̄r = Σ
(A, B, C, D) ⇒ Wr, Wo
Ā = PAP⁻¹
B̄ = PB
C̄ = CP⁻¹
D̄ = D
3


singular value decomposition
diagonal
4


left pseudo inverse
right pseudo inverse
5


6

7


Chap 9 state feedback
and state observer
problem
model
Analysis
→ physics, chemistry, biology
→ math / P.D.E. ...
stability eig(A)
controllability (A, B)
observability (A, C)
u
system
x
y
$\hat{y}(s) = H(s)\,\hat{u}(s)$
$\dot{x} = Ax + Bu$
$y = Cx + Du$
① unstable
weakly stable
eig(A) ⟶ eig($\bar{A}$)
$\bar{A} = PAP^{-1}$
⇒ need (A, B) controllable
⇒ How, u(t) = ? (state feedback)
② $y \in \mathbb{R}^p$
$x \in \mathbb{R}^n$
n >> p
y ⟶ x
⇒ need (A, C) observable
⇒ How, state observer
reference signal
Design
F
feedback
new system
$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$
$y = \bar{C}\bar{x} + \bar{D}u$

8

9