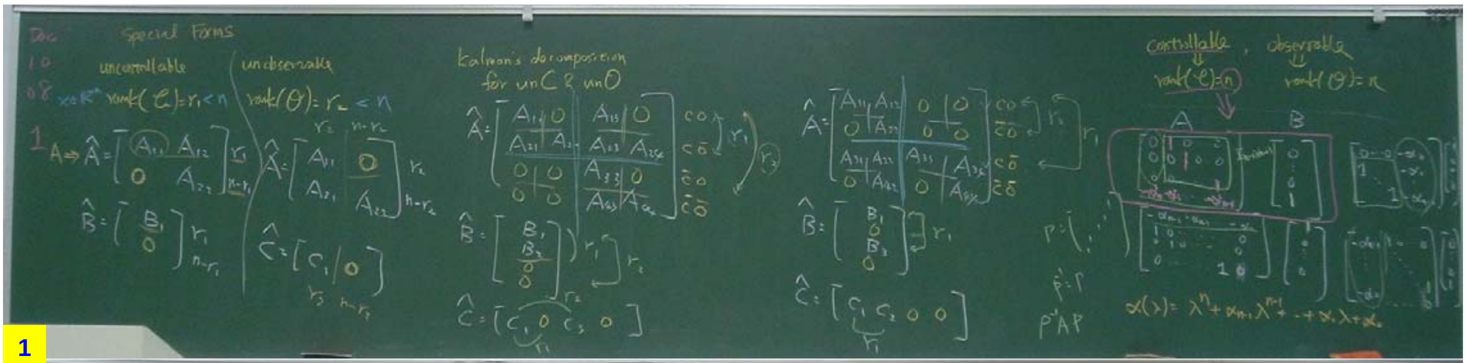
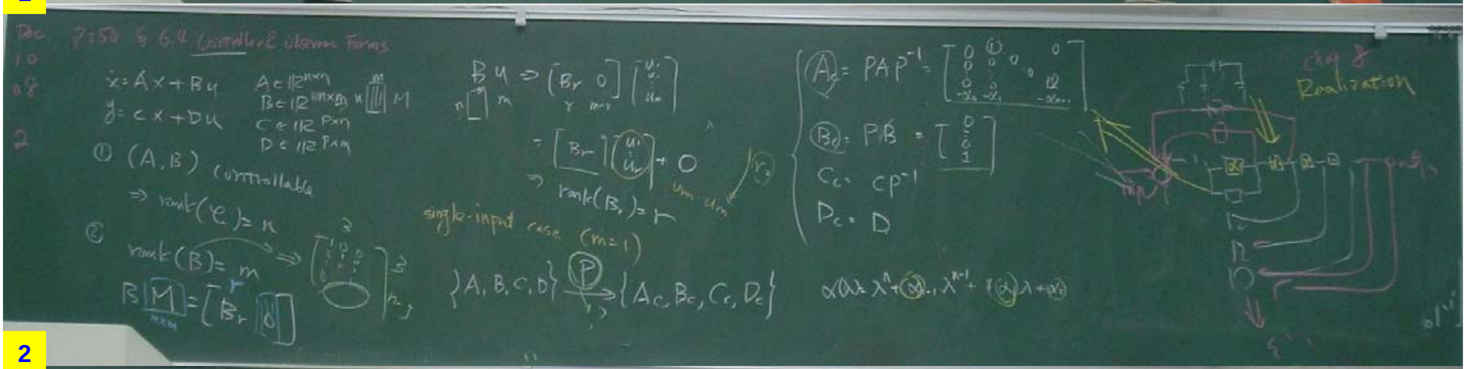
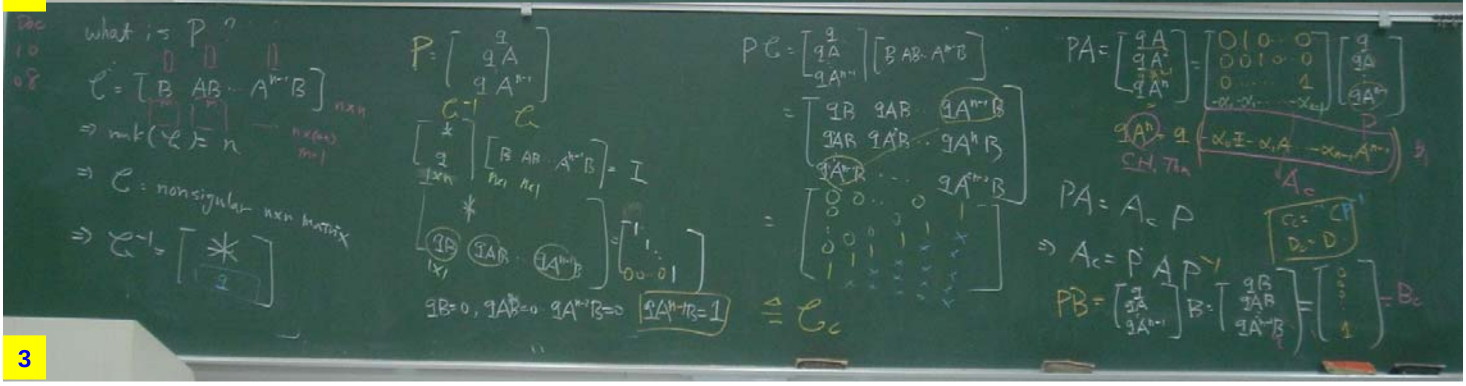
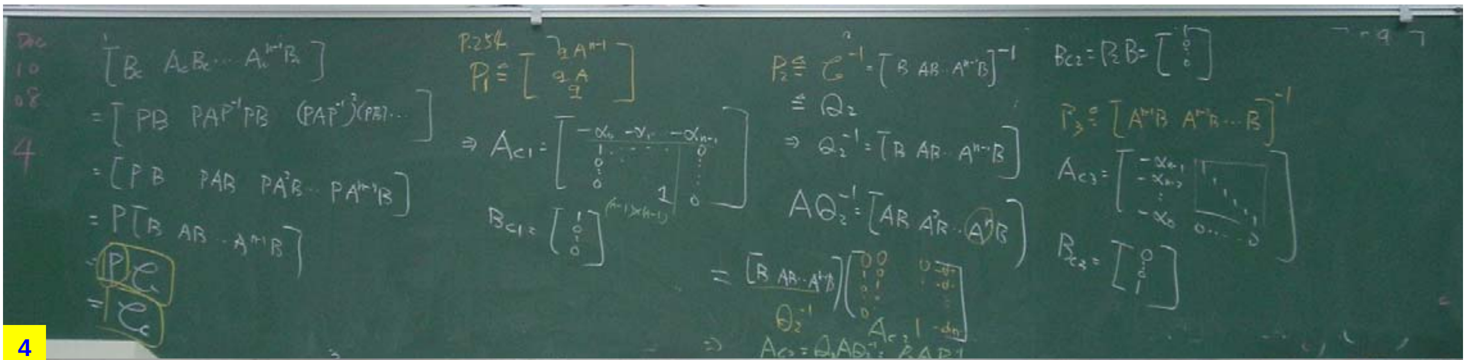
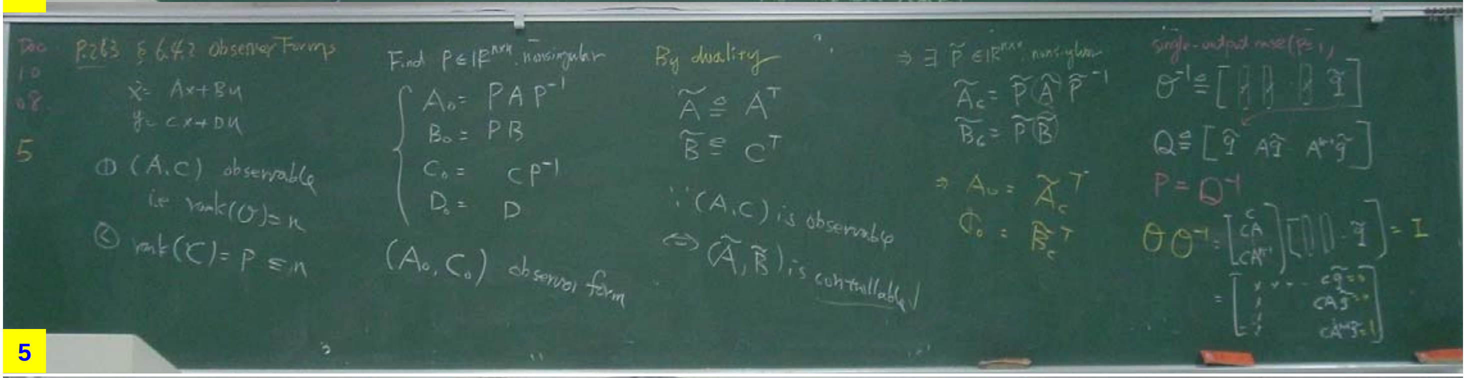
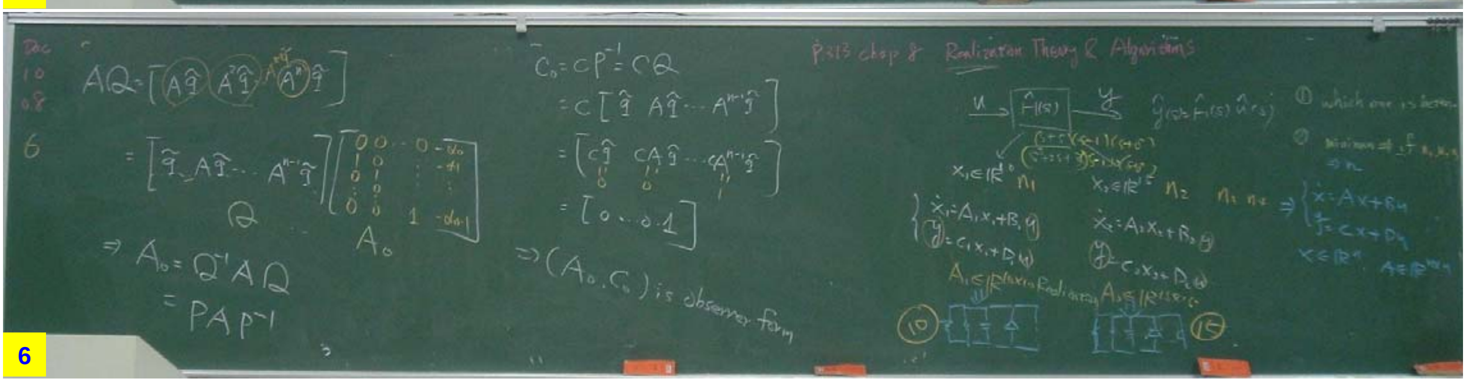

Dec 10 08
Special Forms
uncontrollable
unobservable
Kalman's decomposition for unC & unO
controllable, observable
rank(𝒞) = n
rank(𝒪) = n
$\alpha(\lambda) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \dots + \alpha_1\lambda + \alpha_0$
1

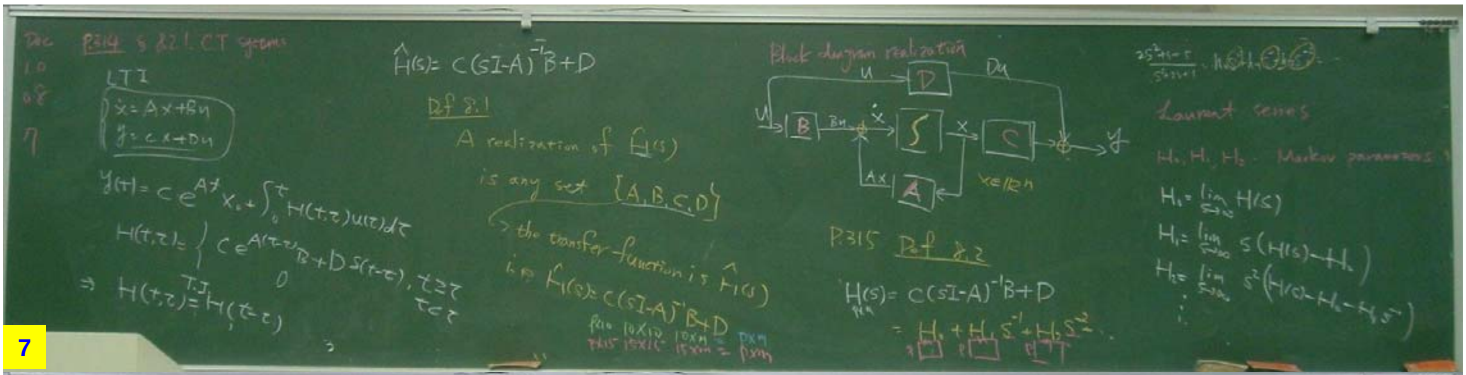

Dec 10 08
P.250 § 6.4 Controller & observer Forms
$\dot{x} = Ax + Bu$
$y = Cx + Du$
① (A, B) controllable
⇒ rank(𝒞) = n
② rank(B) = m
single-input case (m=1)
$\{A, B, C, D\} \xrightarrow{P} \{A_c, B_c, C_c, D_c\}$
$A_c = PAP^{-1}$
$B_c = PB$
$C_c = CP^{-1}$
$D_c = D$
Chap 8 Realization
2


Dec 10 08
what is P ?
$\mathcal{C} = [B \; AB \cdots A^{n-1}B]$
⇒ rank(𝒞) = n
⇒ 𝒞 : nonsingular n×n matrix
$PA = A_c P$
$\Rightarrow A_c = PAP^{-1}$
C.H. Thm
$C_c = CP^{-1}$
$D_c = D$
$qB = 0, \; qAB = 0, \; qA^{n-2}B = 0 \quad qA^{n-1}B = 1$
3

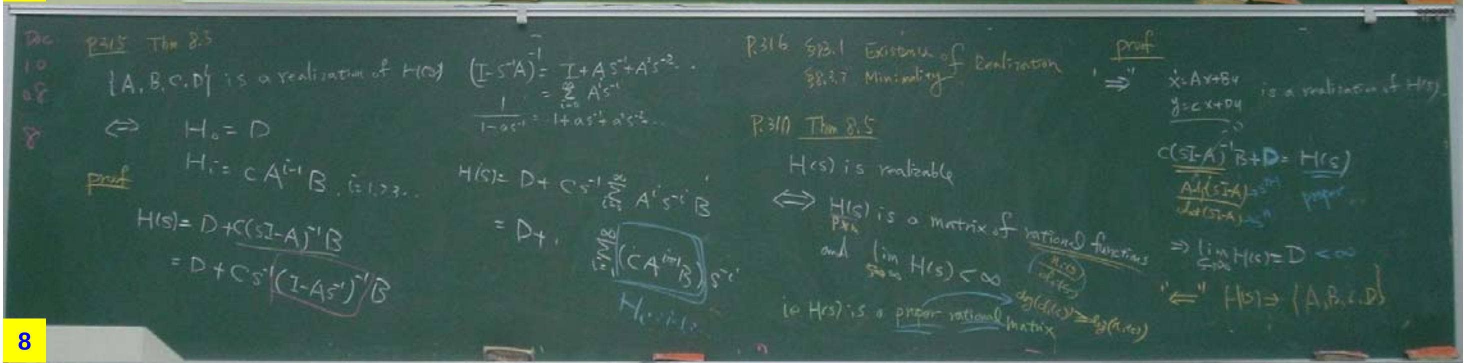

Dec 10 08
$[B_c \; A_cB_c \cdots A_c^{n-1}B_c]$
$= P[B \; AB \cdots A^{n-1}B]$
$= P\mathcal{C}$
$= \mathcal{C}_c$
P.254
$P_2 \triangleq \mathcal{C}^{-1} = [B \; AB \cdots A^{n-1}B]^{-1}$
$\triangleq Q_2$
$AQ_2^{-1} = [AB \; A^2B \cdots A^nB]$
$P_3 \triangleq [A^{n-1}B \; A^{n-2}B \cdots B]^{-1}$
4


Dec 10 08
P.263 § 6.4.2 observer Forms
$\dot{x} = Ax + Bu$
$y = Cx + Du$
① (A, C) observable i.e. rank(𝒪) = n
② rank(C) = p ≤ n
Find $P \in \mathbb{R}^{n\times n}$ nonsingular
$A_o = PAP^{-1}$
$B_o = PB$
$C_o = CP^{-1}$
$D_o = D$
(A_o, C_o) observer form
By duality
$\tilde{A} \triangleq A^T$
$\tilde{B} \triangleq C^T$
∵ (A, C) is observable ⟺ $(\tilde{A}, \tilde{B})$ is controllable
single-output case (p=1)
$P = Q^{-1}$
5

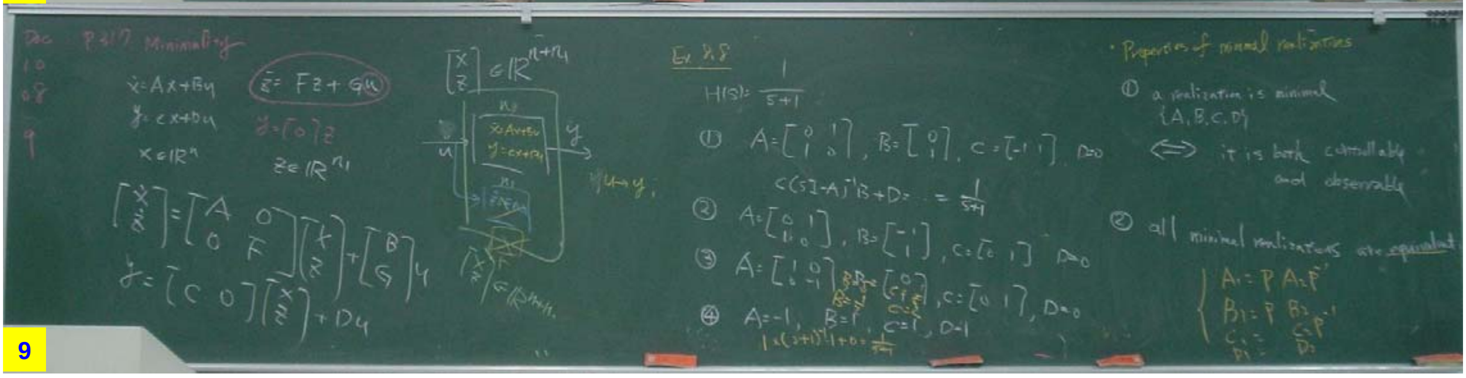

Dec 10 08
$AQ = [A\hat{q} \; A^2\hat{q} \cdots A^n\hat{q}]$
$\Rightarrow A_o = Q^{-1}AQ$
$= PAP^{-1}$
$C_o = CP^{-1} = CQ$
$= [0 \cdots 0 \; 1]$
⇒ (A_o, C_o) is observer form
P.313 Chap 8 Realization Theory & Algorithms
$\dot{x} = Ax + Bu$
$y = Cx + Du$
6


P.314 §8.2.1 CT systems
LTI
Def 8.1
A realization of Ĥ(s)
is any set {A, B, C, D}
the transfer function is Ĥ(s)
Block diagram realization
P.315 Def 8.2
Laurent series
Markov parameters

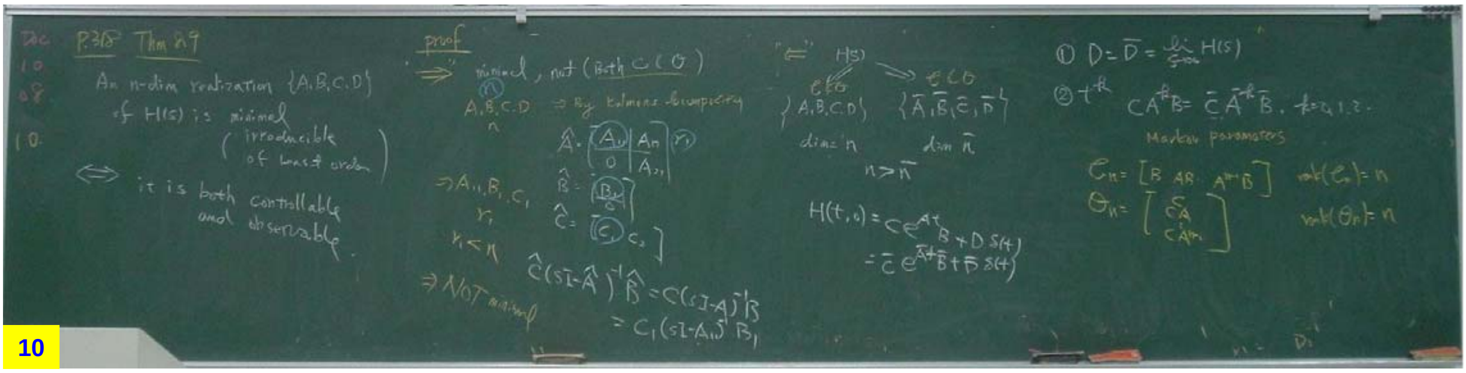
7


P.315 Thm 8.3
{A, B, C, D} is a realization of H(s)
proof
P.316 §8.3.1 Existence of Realization
§8.3.2 Minimality
P.316 Thm 8.5
H(s) is realizable
is a matrix of rational functions
proper rational matrix
proof
is a realization of H(s)

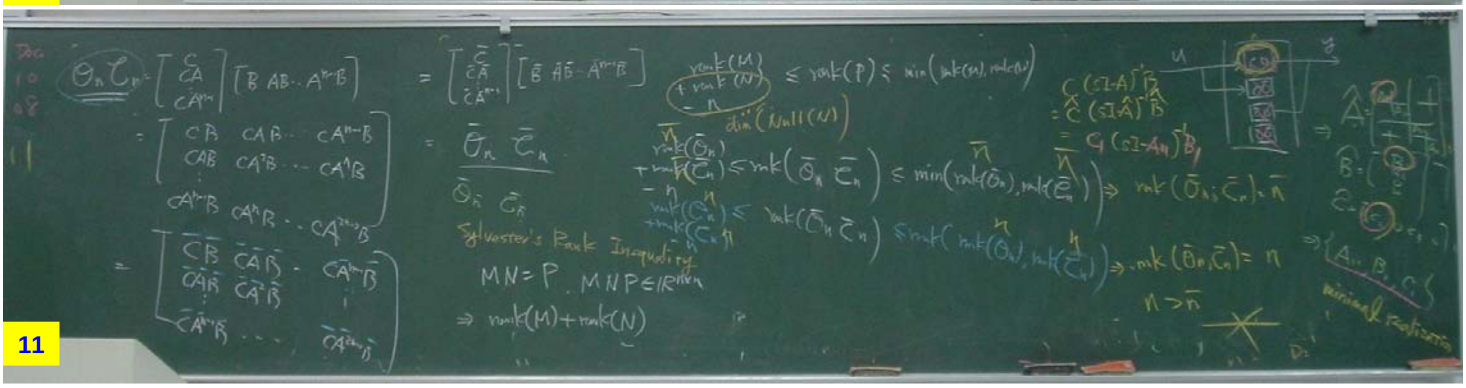
8


P.317 Minimality
Ex 8.8
Properties of minimal realizations
a realization is minimal
{A, B, C, D}
it is both controllable and observable
all minimal realizations are equivalent

9


P.318 Thm 8.9
An n-dim realization {A, B, C, D}
of H(s) is minimal
(irreducible of least order)
it is both controllable and observable
proof
NOT minimal
Markov parameters

10


Sylvester's Rank Inequality
minimal realization

11

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