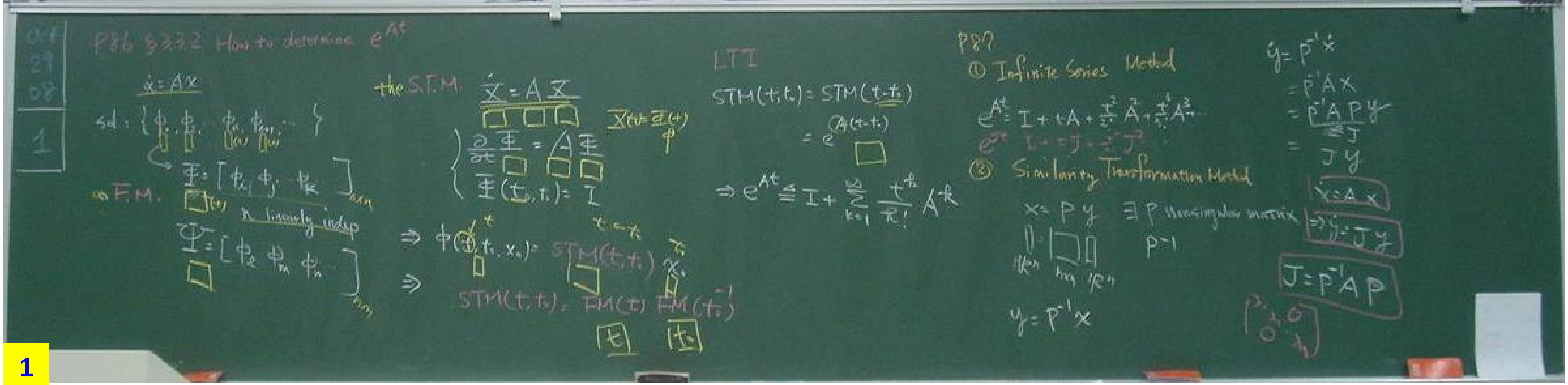
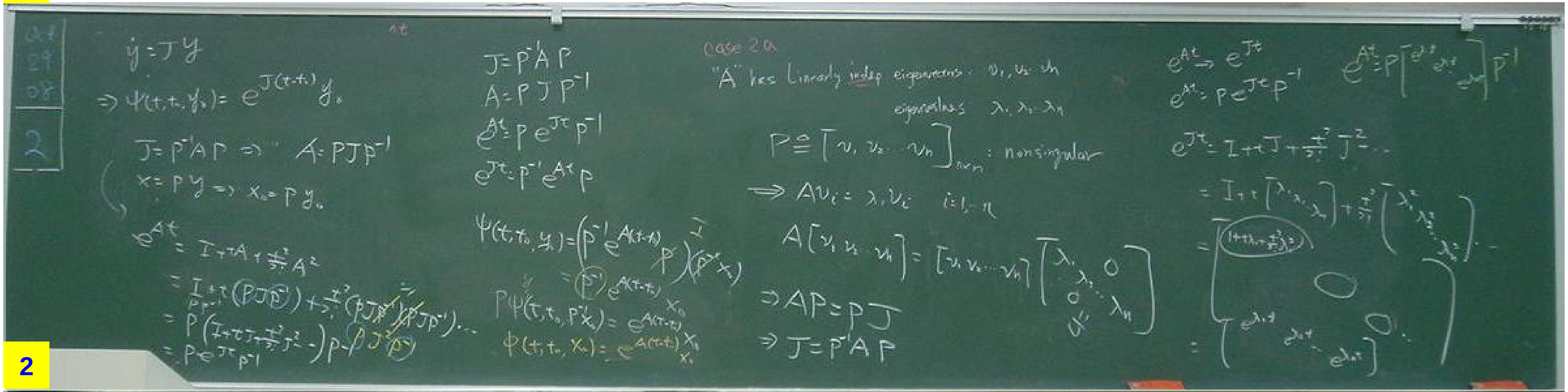

Oct 29 08
1
P86 §3.3.2 How to determine e^{At}
ẋ = Ax
the S.T.M.
LTI
STM(t,t₀) = STM(t−t₀)
P87
① Infinite Series Method
e^{At} = I + tA + t²/2! A² + t³/3! A³ ...
② Similarity Transformation Method
x = Py
∃ P nonsingular matrix
y = P⁻¹x
ẏ = P⁻¹ẋ
ẋ = Ax
ẏ = Jy
J = P⁻¹AP
STM(t,t₀) = FM(t) FM⁻¹(t₀)
1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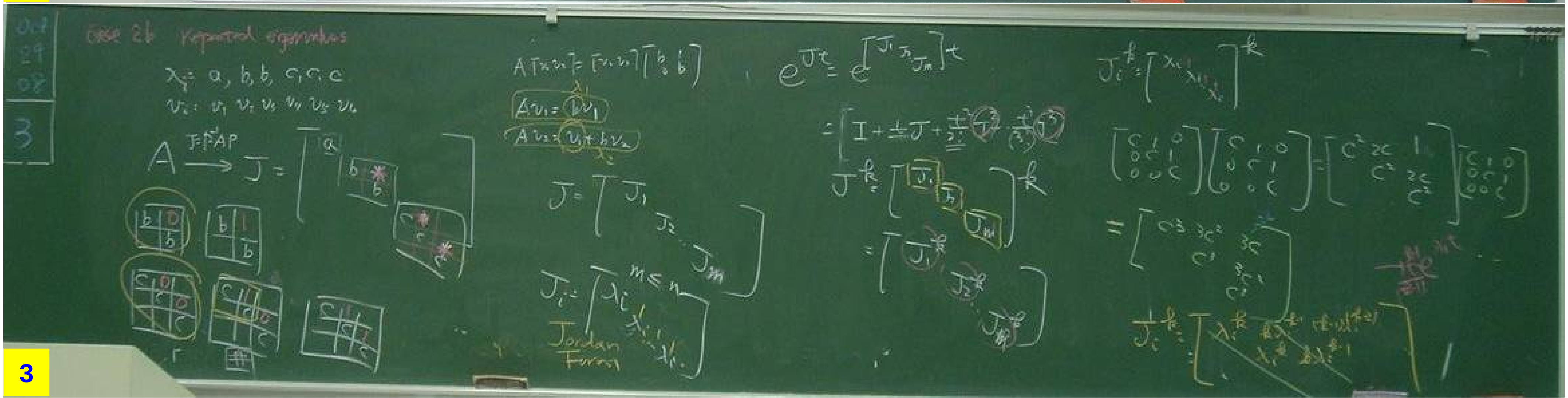
2
ẏ = Jy
J = P⁻¹AP ⇒ A = PJP⁻¹
x = Py ⇒ x₀ = Py₀
e^{At} = I + tA + t²/2! A²
J = P⁻¹AP
A = PJP⁻¹
e^{At} = Pe^{Jt}P⁻¹
e^{Jt} = P⁻¹e^{At}P
case 2a
"A" has linearly indep eigenvectors: v₁, v₂ ... vₙ
eigenvalues λ₁, λ₂ ... λₙ
P ≜ [v₁ v₂ ... vₙ]ₙₓₙ : nonsingular
⇒ Avᵢ = λᵢvᵢ i = 1 ... n
⇒ AP = PJ
⇒ J = P⁻¹AP
e^{At} ⇒ e^{Jt}
e^{At} = Pe^{Jt}P⁻¹
e^{Jt} = I + tJ + t²/2! J² ...
2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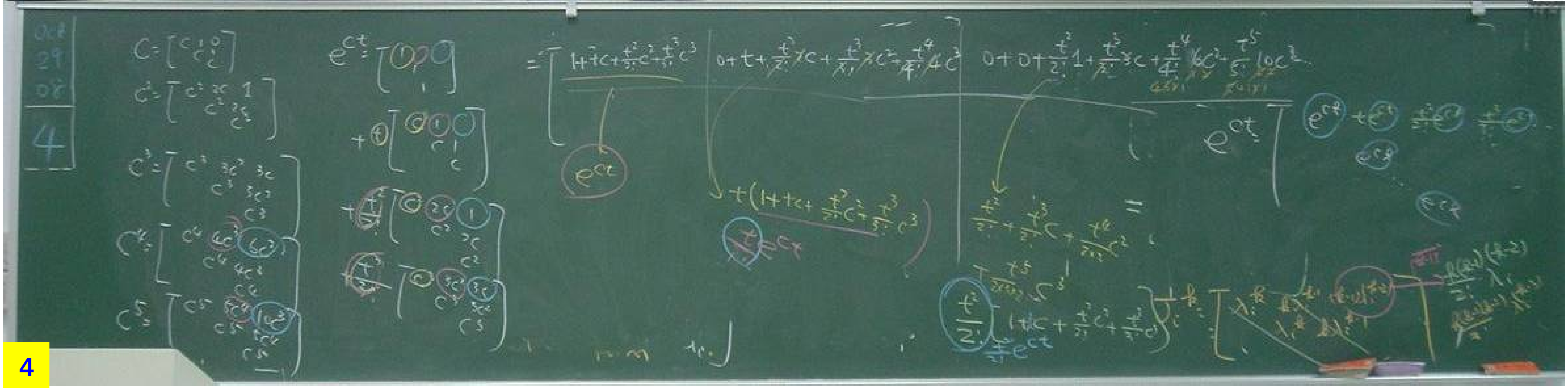
3
case 2b Repeated eigenvalues
λᵢ = a, b, b, c, c, c
vᵢ = v₁ v₂ v₃ v₄ v₅ v₆
Av₁ = bv₁
Av₂ = v₁ + bv₂
J = [J₁ J₂ ... Jₘ]
m ≤ n
Jordan Form
3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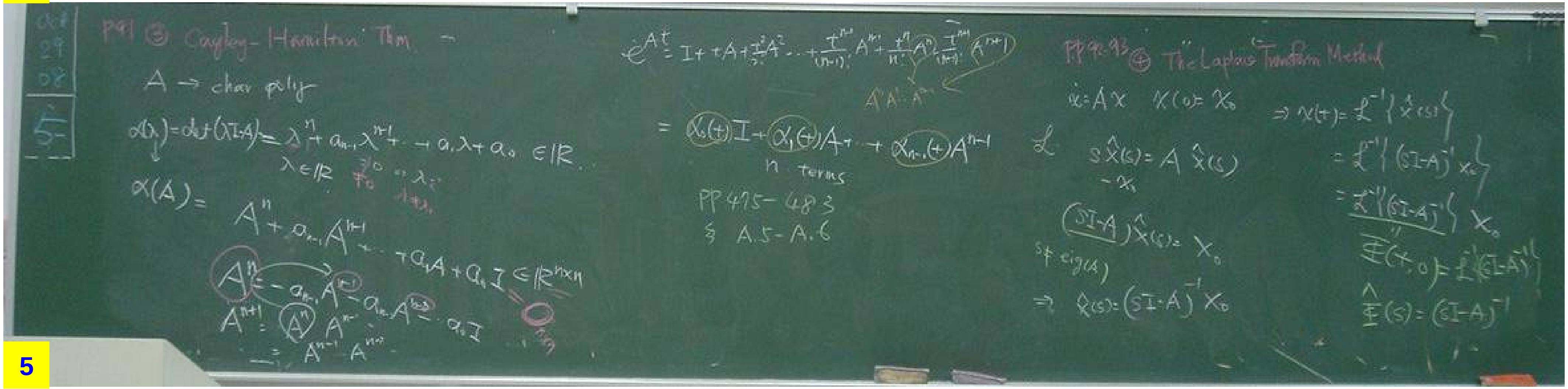
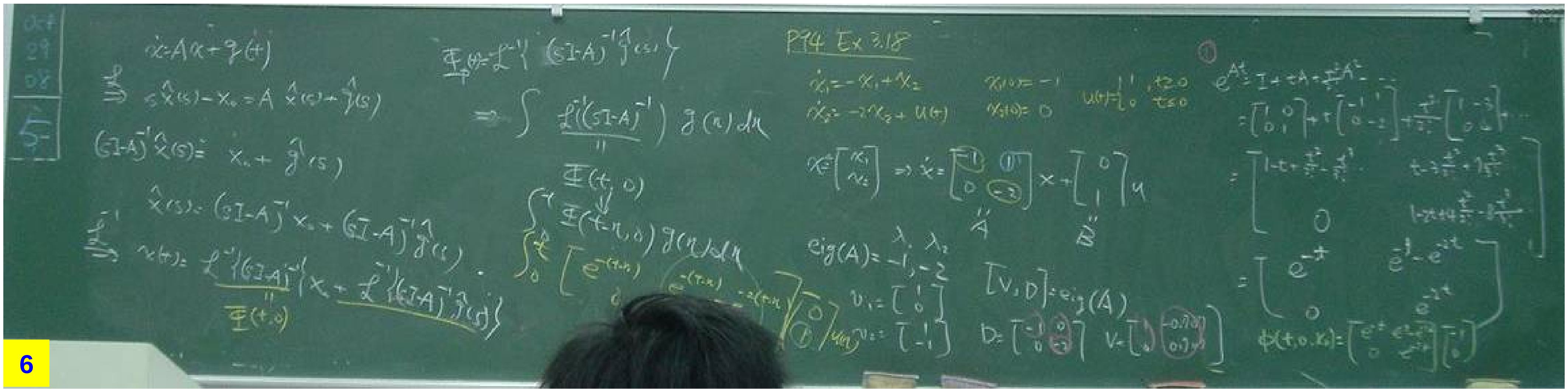
4
e^{ct}
t

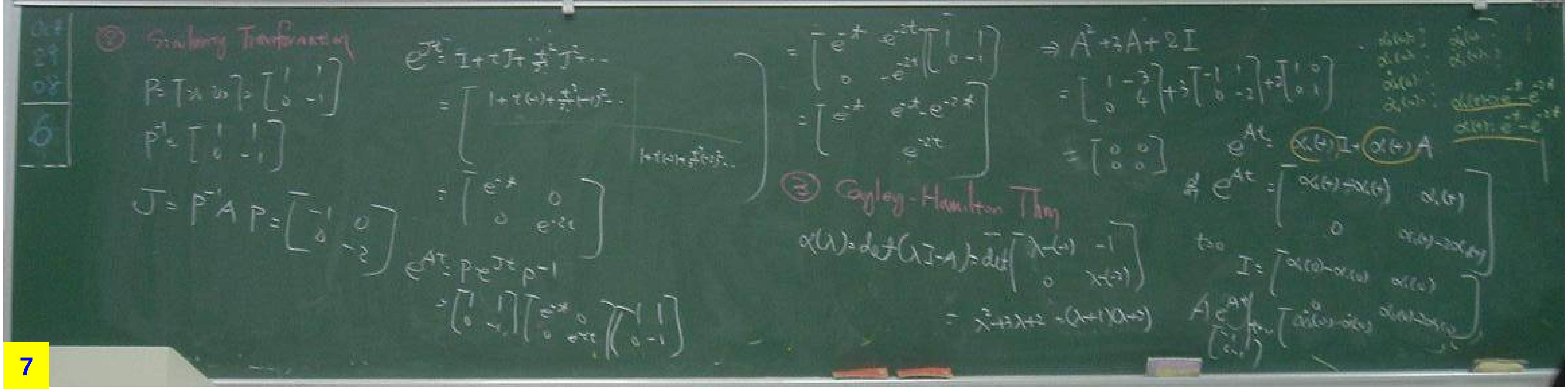
4

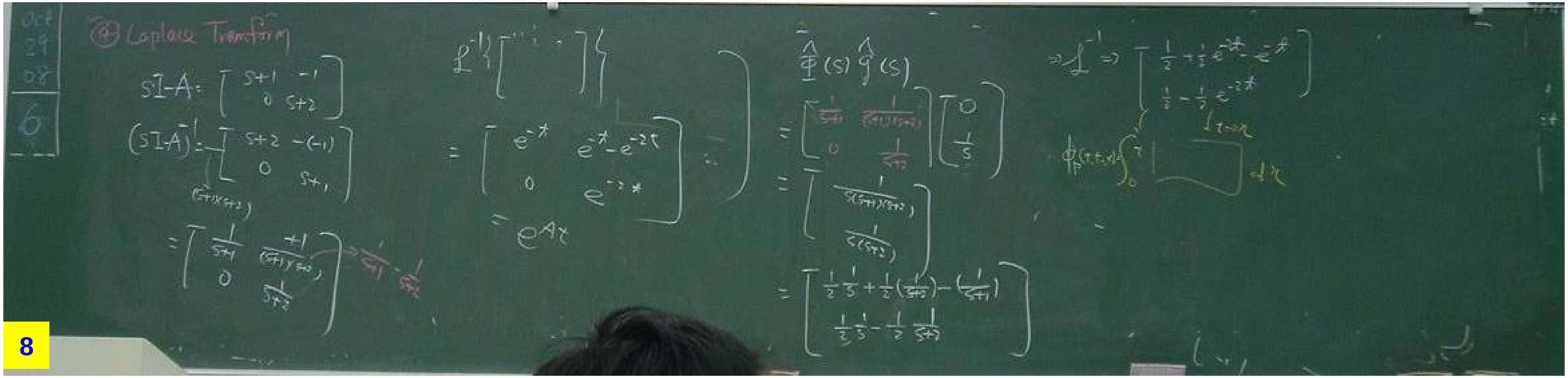

Oct 29 08
5
P91 ③ Cayley-Hamilton Thm
A → char poly
$\alpha(\lambda)=\det(\lambda I-A)=\lambda^n+a_{n-1}\lambda^{n-1}+\dots+a_1\lambda+a_0 \in \mathbb{R}$
$\alpha(A)=A^n+a_{n-1}A^{n-1}+\dots+a_1A+a_0I \in \mathbb{R}^{n\times n}$
$e^{At}=I+tA+\frac{t^2}{2!}A^2+\dots$
$=\alpha_0(t)I+\alpha_1(t)A+\dots+\alpha_{n-1}(t)A^{n-1}$
n terms
PP 475-483
§ A.5-A.6
PP 92-93 ④ The Laplace Transform Method
$\dot{x}=Ax \quad x(0)=x_0$
$\Rightarrow \hat{x}(s)=(sI-A)^{-1}x_0$
$\hat{\Phi}(s)=(sI-A)^{-1}$

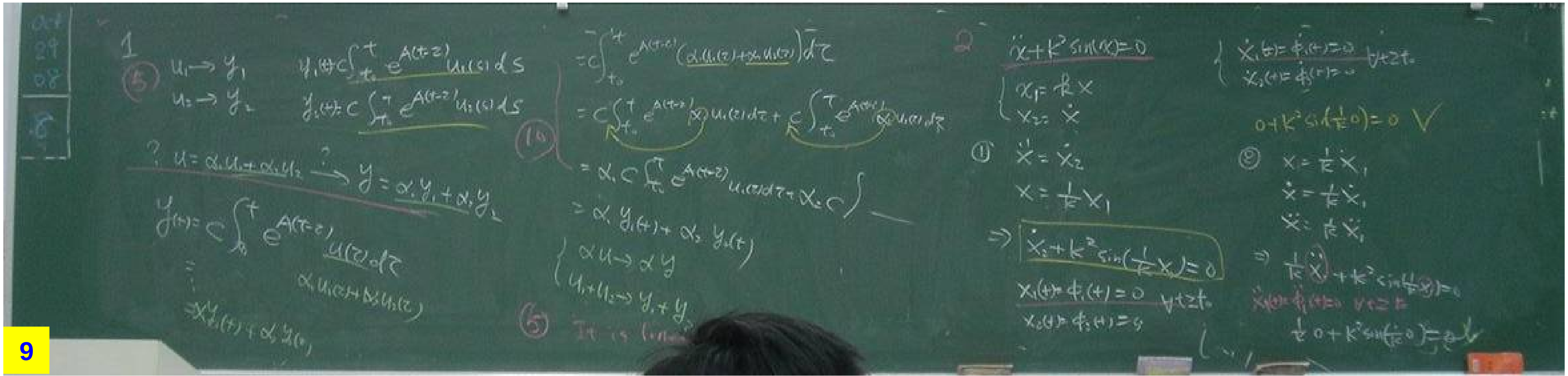
5

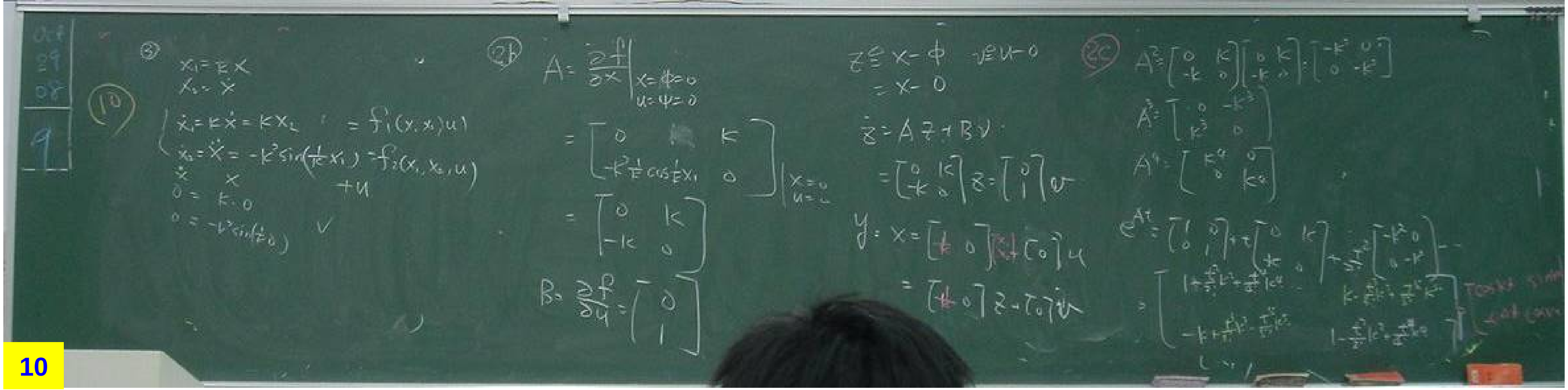

Oct 29 08
5
$\dot{x}=Ax+g(t)$
P94 Ex 3.18
$\dot{x}_1=-x_1+x_2$
$\dot{x}_2=-2x_2+u(t)$
$eig(A)=-1,-2$
$[V,D]=eig(A)$

6

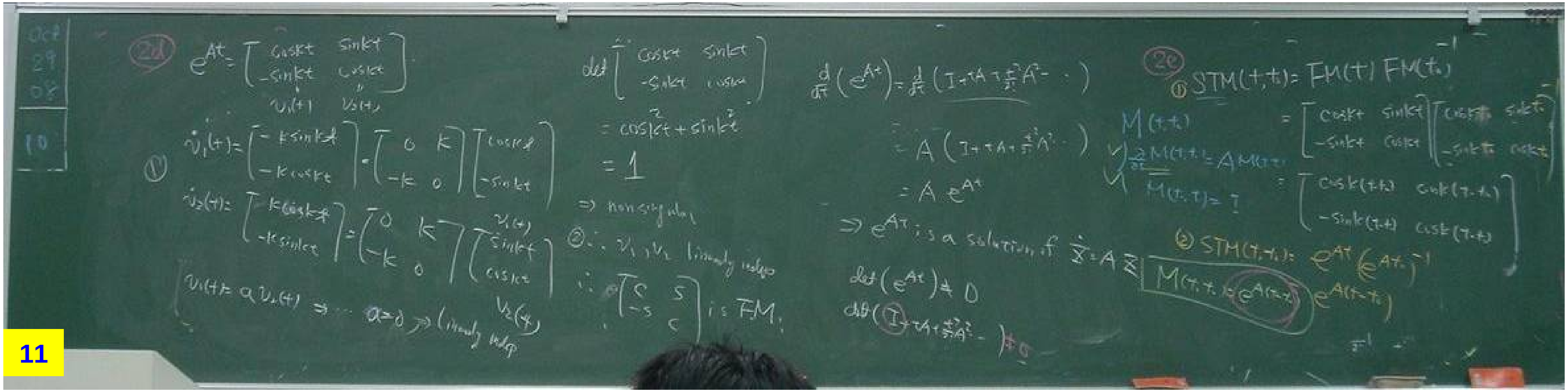

② Similarity Transformation
③ Cayley-Hamilton Thm
7


④ Laplace Transform
8


9



10



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