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## P20 §1.6 Systems of Linear 1st-Order ODEs

$\dot{x} = A(t)x + g(t)$ $\quad x \in \mathbb{R}^n$

$\dot{x} = A(t)x$ $\quad A(t) = [a_{ij}] \in C(\mathbb{R}, \mathbb{R}^{n\times n})$, $i,j = 1 \sim n$, $t$

$\dot{x} = Ax + g(t)$

$\dot{x} = Ax$ $\quad A \in \mathbb{R}^{n\times n}$

Linearization about a solution $\phi(t)$

$\dot{x} = f(t,x)$ $\quad x \in \mathbb{R}^n$

$\dot{\phi}(t) = f(t,\phi(t))$ $\quad f \in C(\mathbb{R}\times\mathbb{R}^n, \mathbb{R}^n)$

$f: \underset{t}{\mathbb{R}} \times \underset{x}{\mathbb{R}^n} \to \underset{f}{\mathbb{R}^n}$

$\delta x = x - \phi(t)$

$\frac{d}{dt}(\delta x) = \dot{x} - \dot{\phi}(t)$

$= f(t,x) - f(t,\phi)$

$= f(t,\phi+\delta x) - f(t,\phi)$

$= \frac{\partial f}{\partial x}(t,\phi(t))\,\delta x$

$\boxed{+[f(t,\phi+\delta x) - f(t,\phi)] - \frac{\partial f}{\partial x}(t,\phi(t))\,\delta x}$

$f(x)$ $\quad \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & & & \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \cdots & & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{n\times n}$

$f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

Jacobian of $f$
matrix

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When $\|\delta x\| \to 0$

$\Rightarrow \dfrac{\left\| [f(t,\phi+\delta x) - f(t,\phi)] - \frac{\partial f}{\partial x}(t,\phi)\,\delta x \right\|}{\|\delta x\|} \to 0$ $\quad \square^{n\times n}$

$\phi \in \mathbb{R}^n$

$\delta x \in \mathbb{R}^n$

$A(t) = \frac{\partial f}{\partial x}(t,x)\Big|_{x=\phi}$

$\frac{d}{dt}(\delta x) \approx \frac{\partial f}{\partial x}(t,\phi)\,\delta x$

$\frac{d}{dt} z = A(t)\, z$

$\dot{z} = Az$

Linearization about a solution $\phi(t)$

$\dot{x} = f(\ ;x)$

$\phi(t) = x_0$ : constant

$\delta x = x - \phi(t) = x - x_0 = z$

$\frac{d}{dt}(\delta x) = \frac{\partial f}{\partial x}(x)\Big|_{x=\phi=x_0} \delta x$

$\Leftarrow \quad = \underbrace{\frac{\partial f}{\partial x}(x=x_0)}_{A \in \mathbb{R}^{n\times n}} \delta x$

Linearization about a solution $\phi(t)$ an input $\psi(t)$

$\dot{x} = f(t,x,u)$

$\dot{\phi} = f(t,\phi,\psi)$ $\quad x = \phi(t)$, $u = \psi(t)$

$\delta x = x - \phi(t)$

$\delta u = u - \psi(t)$

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$\frac{d}{dt}(\delta x) = \dot{x} - \dot{\phi}$

$= f(t,x,u) - f(t,\phi,\psi)$

$= f(t,\phi+\delta x, \psi+\delta u) - f(t,\phi,\psi)$

$-f(t,\phi,u+\delta u)$

$= \frac{\partial f}{\partial x}(t,\phi,\psi)\,\delta x + F_1(t,\delta x,u)$

$+ \frac{\partial f}{\partial u}(t,\phi,\psi)\,\delta u + F_2(t,\delta u)$

$\frac{\partial f}{\partial x}(t,\phi,\psi) \triangleq A(t) : \mathbb{R} \to \mathbb{R}^{n\times n}$ $\quad (\mathbb{R}^n, \mathbb{R}^n)$

$\frac{\partial f}{\partial u}(t,\phi,\psi) \triangleq B(t) : \mathbb{R} \to \mathbb{R}^{n\times m}$ $\quad (\mathbb{R}^n, \mathbb{R}^m)$

$\dot{z} = A(t)z + B(t)v$


$\theta < \pm 5°$

$\sin\theta \approx \theta$

$\cos\theta \approx 1$

$\delta x: x - 30° = z$

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$\ddot{x} + \frac{g}{\ell}\sin x = 0$

$\begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases}$ $\quad \dot{x}_1 = x_2$, $\quad \dot{x}_2 = -\frac{g}{\ell}\sin x_1$

$\dot{x} = f(x) = \begin{bmatrix} x_2 \\ -\frac{g}{\ell}\sin x_1 \end{bmatrix}$

$\therefore \phi_1(t) \equiv 0$

$\phi_2(t) \equiv 0$ is a solution

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$$A_1(t) = \frac{\partial f}{\partial x}\Big|_{\substack{x_1=\phi_1\\ x_2=\phi_2}} = \begin{bmatrix} \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial x_2} \\ \frac{\partial(-k\sin x_1)}{\partial x_1} & \frac{\partial(-k\sin x_1)}{\partial x_2}\end{bmatrix}_{\substack{x_1=\phi_1\\ x_2=\phi_2}}$$

$$= \begin{bmatrix}0 & 1\\ -k\cos x_1 & 0\end{bmatrix}_{\substack{x_1=0\\ x_2=0}}$$

$$= \begin{bmatrix}0 & 1\\ -k & 0\end{bmatrix}$$

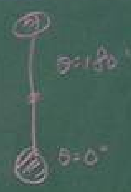
$$\Rightarrow \boxed{\dot z = \begin{bmatrix}0 & 1\\ -k & 0\end{bmatrix} z}$$

$\phi_1(t) = 180°$ ± [illegible]

$\phi_2(t) = 0°$

$$A_2(t) = \begin{bmatrix}0 & 1\\ +k & 0\end{bmatrix}$$

$$\dot z = \begin{bmatrix}0 & 1\\ k & 0\end{bmatrix} z$$

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a solution

$r(0) = r_0$ $\theta(0) = \theta_0$

$\dot r(0) = 0$ $\dot\theta(0) = \omega_0$

$u_1(t) \equiv 0$

$u_2(t) \equiv 0$

$$\begin{cases} \ddot r = r\dot\theta^2 - \frac{k}{r^2} + u_1 \\ \ddot\theta = \frac{-2\dot\theta\dot r}{r} + \frac{1}{r}u_2 \end{cases} \Rightarrow \begin{array}{l} r(t) = r_0 \\ \theta(t) = \theta_0 + \omega_0 t \end{array}$$

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$x_1 = r$ $\quad \dot x_1 = x_2 = f_1$

$x_2 = \dot r$ $\quad \dot x_2 = x_1x_4^2 - \frac{k}{x_1^2} + u_1 = f_2$

$x_3 = \theta \Rightarrow \dot x_3 = x_4 = f_3$

$x_4 = \dot\theta$ $\quad \dot x_4 = -\frac{2x_2x_4}{x_1} + \frac{1}{x_1}u_2 = f_4$

$$A(t) = \frac{\partial f}{\partial x}\Big|_{\substack{r(t),\dot r(t)\\ \theta(t),\dot\theta(t)\\ u_1,u_2}} = \begin{bmatrix}0 & 1 & 0 & 0\\ x_4^2 + \frac{2k}{x_1^3} & 0 & 0 & 2x_1x_4\\ 0 & 0 & 0 & 1\\ \frac{2x_2x_4}{x_1^2} - \frac{1}{x_1^2}u_2 & -\frac{2x_4}{x_1} & 0 & -\frac{2x_2}{x_1}\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1 & 0 & 0\\ \omega_0^2 + 2\omega_0^2 & 0 & 0 & 2r_0\omega_0\\ 0 & 0 & 0 & 1\\ 0 & -\frac{2\omega_0}{r_0} & 0 & 0\end{bmatrix}$$

$$B(t) = \frac{\partial f}{\partial u}\Big|_{\substack{x\\ u}} = \begin{bmatrix}\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2}\\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2}\\ \vdots & \vdots\end{bmatrix} = \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & \frac{1}{x_1}\end{bmatrix}_{\substack{x\\ u}} = \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & \frac{1}{r_0}\end{bmatrix}$$

$z_i = x_i - \phi_i$ $\quad v_i = u_i - \psi_i$

$$\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 3\omega_0^2 & 0 & 0 & 2r_0\omega_0\\ 0 & 0 & 0 & 1\\ 0 & -\frac{2\omega_0}{r_0} & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & \frac{1}{r_0}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

$$\dot z = Az + Bv$$

$r(t) = r_0$

$\theta(t) = \theta_0 + \omega_0 t$

$u_1 = 0,\ u_2 = 0$

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## P28 §1.8 Solutions of Linear State Equations

· Linear Homogeneous Equation

$\dot x = A(t)\,x$, $x(t_0) = x_0$

$A \in C(\mathbb{R}, \mathbb{R}^{n\times n})$

$A: \mathbb{R} \to \mathbb{R}^{n\times n}$

$x(t) = ?$

By Successive Approximation

$\dot x = f(t, x)$, $x(t_0) = x_0$

$$\overset{\phi_{m+1}}{\phi(t)} = x_0 + \int_{t_0}^{t} f(s, \overset{\phi_m}{\phi(s)})\,ds$$

$$\dot\phi = 0 + f(s, \phi(s))\big|_{s=t} = f(t, \phi(t))$$

$\phi_0(t) = x_0$

$\phi_1(t) = x_0 + \int_{t_0}^{t} f(s, \phi_0)\,ds$

$\phi_2(t) = x_0 + \int_{t_0}^{t} f(s, \phi_1)\,ds$

$\vdots$

$\phi_{m+1}(t) = x_0 + \int_{t_0}^{t} f(s, \phi_m)\,ds$

$m \to \infty$ $\phi_m \to \phi$

$\Rightarrow \phi(t) = x_0 + \int_{t_0}^{t} f(s, \phi(s))\,ds$

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$\dot{x} = A(t)x$, $x(t_0) = x_0$

$f(s, \phi_0(s)) = A(s)x_0$

$\phi_0(t,t_0,x_0) = x_0$

$\phi_1(t,t_0,x_0) = x_0 + \int_{t_0}^{t} A(s_1)x_0\,ds_1$

$\phi_2(t,t_0,x_0) = x_0 + \int_{t_0}^{t} A(s_2)\phi_1(s_2,t_0,x_0)\,ds_2$

$\phi_{m-1}$

$\phi_m(t,t_0,x_0) = x_0 + \int_{t_0}^{t} A(s_m)\phi_{m-1}(s_m,t_0,x_0)\,ds_m$

$= x_0 + \int_{t_0}^{t} A(s_m)\left[x_0 + \int_{t_0}^{s_m} A(s_{m-1})\phi_{m-2}(s_{m-1},t_0,x_0)\,ds_{m-1}\right]ds_m$

$= \cdots$

$$= x_0 + \int_{t_0}^{t} A(s_m)x_0\,ds_m + \int_{t_0}^{t} A(s_m)\int_{t_0}^{s_m} A(s_{m-1})x_0\,ds_{m-1} + \int_{t_0}^{t} A(\ )\int_{t_0}^{s_m} A(\ )\int_{t_0}^{s_{m-1}} A(\ ) + \cdots + \int\int\int\int_{t_0}^{s_1} A(s_1)x_0\,ds_1$$

$$= \left[I + \int_{t_0}^{t} A(s_m)\,ds_m + \cdots + \int\int\int\int ds\right] x_0$$

$m \to \infty$, $\phi_m \to \phi$

$\phi(t,t_0,x_0) = \left[\ \right]_{n\times n} x_0$

$\Phi(t,t_0) : \mathbb{R} \to \mathbb{R}^{n\times n}$

Peano-Baker Series

time $t_0$: $x(t_0) = x_0$

$t$: $\phi(t,t_0,x_0)$

$= \Phi(t,t_0)x_0$

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Properties of $\Phi(t,t_0)$

① $t = t_0$, $\Phi(t_0,t_0) = I + \int_{t_0}^{t_0} A \cdots + \int_{t_0}^{t_0}\int_{t_0}^{s} \cdots = I$

$\phi(t_0,t_0,x_0) = \Phi(t_0,t_0)x_0 = Ix_0 = x_0$

② $\frac{d}{dt}\Phi(t,t_0) = 0 + A(t) + A(t)\int_{t_0}^{t} A(s_{m-1})x_0\,ds_{m-1} + \cdots \infty$

$= A(t)\underbrace{\left[I + \int_{t_0}^{t} A(s_{m-1})\cdots\right]}_{\Phi(t,t_0)}$

$\frac{d}{dt}\Phi(t,t_0) = A(t)\Phi(t,t_0)$

$x \in D \subset \mathbb{R}^n \Rightarrow \dot{x} = A(t)x(t)$

$\mathbb{R}^n$, $\dim(D) = n$

$\Phi(t,t_0)$: state transition matrix

$\phi(t,t_0,x_0) = \Phi(t,t_0)x_0$

$\dot{x} = Ax$, $A \in \mathbb{R}^{n\times n}$ LTI

the mth term

$\int_{t_0}^{t} A(s_m)\int_{t_0}^{s_m} A(s_{m-1}) \cdots \int_{t_0}^{s_2} A(s_1)\,ds_1\,ds_{m-1}\cdots ds_m$

$= \int_{t_0}^{t} A \int_{t_0}^{s_m} A \int_{t_0}^{s_{m-1}} A \cdots \int_{t_0}^{s_2} A\,ds_1\,ds_2 \cdots ds_m$

$= A^m \frac{(t-t_0)^m}{m!}$

$\Phi(t,t_0) = I + A(t-t_0) + A^2\frac{(t-t_0)^2}{2!} + \cdots + A^m\frac{(t-t_0)^m}{m!} \cdots = \Phi(t-t_0)$

$\triangleq e^{A(t-t_0)}$ matrix exponential

$\phi(t,t_0,x_0) = \Phi(t,t_0)x_0 = \Phi(t-t_0)x_0$

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P30. Linear Nonhomogeneous Eqn

$\dot{x} = A(t)x + g(t)$, $x(t_0) = x_0$

$A \in C(\mathbb{R}, \mathbb{R}^{n\times n})$

$g \in C(\mathbb{R}, \mathbb{R}^{n})$

the solution

$\phi(t,t_0,x_0) = \Phi(t,t_0)x_0 + \int_{t_0}^{t} \Phi(t,s)g(s)\,ds$

$\int_{t_0}^{t} \Phi(t,s)g(s)\,ds$

$s = t_1$ — $t_0$ $t_1$ $t$

$\int_{t_0}^{t} \Phi(t,t_1)g(t_1)$, $s = t$

① $t = t_0$, $\phi(t,t_0,x_0)$

① $t = t_0$

$\phi(t_0,t_0,x_0) = \Phi(t_0,t_0,x_0)x_0 + \int_{t_0}^{t_0} \cdots$

$= x_0$

② $\frac{d}{dt}\phi(t,t_0,x_0) = \frac{d}{dt}\Phi(t,t_0)x_0 + \frac{d}{dt}\int_{t_0}^{t}\Phi(t,s)g(s)\,ds$

$= A(t)\Phi(t,t_0)x_0$

$\frac{d}{dt}\left[\int_{a(t)}^{b(t)} f(t,s)\,ds\right] = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t} f(t,s)\,ds + \frac{d}{dt}b(t)\cdot f(t,s)\Big|_{s=b(t)} - \frac{d}{dt}a(t)\, f(t,s)\Big|_{s=a(t)}$

$= \int_{t_0}^{t} \frac{\partial}{\partial t}\Phi(t,s)g(s)\,ds + 1\cdot\Phi(t,t)g(t)$

$\int_{t_0}^{t} A(t)\Phi(t,s)g(s)\,ds + g(t)$

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$$= A(t)\left[\Phi(t,t_0)x_0 + \int_{t_0}^{t}\Phi(t,s)g(s)ds\right] + g(t)$$

$$\frac{d}{dt}\phi(t) = A(t)\,\phi(t,t_0,x_0) + g(t)$$

$$\boxed{\dot{x} = A(t)x + g(t)}$$

$x_0 = 0$: $\phi(t,t_0,x_0) = \Phi(t,t_0)\cancel{0} + \int_{t_0}^{t}\Phi(t,s)g(s)ds$

$$= \int_{t_0}^{t}\Phi(t,s)g(s)ds = \phi_p(t)$$

$x_0 \neq 0$, $g(t) = 0$: $\phi(t,t_0,x_0) = \Phi(t,t_0)x_0 + \int \Phi\, 0\, ds$

$$= \phi_h(t)$$

$$\Rightarrow \phi(t,t_0,x_0) = \phi_h(t) + \phi_p(t)$$

$\dot{x} = Ax + g(t)$

$$\Rightarrow \phi(t,t_0,x_0) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-s)}g(s)ds$$

$\dot{x} = Ax + Bu$

$$\Rightarrow \phi(t,t_0,x_0) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-s)}Bu(s)ds$$

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State space (SS) / input output (IO) models


$n \gg m, p$

IO

$y = G\,u = Hu$

↓ transfer function

$$G(s) = \frac{N(s)}{D(s)}$$

S.S. (LTI)

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$x \in \mathbb{R}^n$

$\dot{x} = f(t,x,u)$

$y = g(t,x,u)$

$x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$

$f: \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^m \to \mathbb{R}^n$

$g: \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^m \to \mathbb{R}^p$

Linear time-varying systems

$\dot{x} = A(t)x + B(t)u$

$y = C(t)x + D(t)u$

$A \in C(\mathbb{R}, \mathbb{R}^{n\times n})$

$B \in C(\mathbb{R}, \mathbb{R}^{n\times m})$

$C \in C(\mathbb{R}, \mathbb{R}^{p\times n})$

$D \in C(\mathbb{R}, \mathbb{R}^{p\times m})$

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the solution

$$\phi(t,t_0,x_0) = \Phi(t,t_0)x_0 + \int_{t_0}^{t}\Phi(t,s)g(s)ds$$

$$= \Phi(t,t_0)x_0 + \int_{t_0}^{t}\Phi(t,s)B(s)u(s)ds$$

$\dot{x} = A(t)x \Rightarrow$ S.T.M.

the response

$$y(t) = C(t)\phi(t,t_0,x_0) + D(t)u(t)$$

LTI

$\dot{x} = Ax + Bu$ $\quad$ $\dot{x} = Ax$ $\Rightarrow \Phi(t,t_0) = e^{A(t-t_0)}$

$y = Cx + Du$

the solution

$$\phi(t,t_0,x_0) = \underbrace{e^{A(t-t_0)}x_0}_{\phi_h(t)} + \underbrace{\int_{t_0}^{t} e^{A(t-s)}Bu(s)ds}_{\phi_p(t)}$$

the response

$$y(t) = Ce^{A(t-t_0)}x_0 + \int_{t_0}^{t} Ce^{A(t-s)}Bu(s)ds + Du(t)$$

P49 Linear Systems

Ⓐ $x_0 = 0$

$u_1 \to y_1$

$u_2 \to y_2$

$u \to \square \to y$

$u = \alpha_1 u_1 + \alpha_2 u_2$

$\longrightarrow y = \alpha_1 y_1 + \alpha_2 y_2$ ?

② $u(t) \equiv 0$

$x_{01} \to y_1$

$x_{02} \to y_2$

$x_0 = \beta_1 x_{01} + \beta_2 x_{02}$

$\longrightarrow y = \beta_1 y_1 + \beta_2 y_2$

Ⓐ

$$y(t) = C(t)\int_{t_0}^{t}\Phi(t,s)B(s)\left[\alpha_1 u_1(s) + \alpha_2 u_2(s)\right]ds + D(t)\left[\alpha_1 u_1(t) + \alpha_2 u_2(t)\right]$$

$$= \alpha_1\left[C(t)\int_{t_0}^{t}\Phi(t,s)B(s)u_1(s)ds + D(t)u_1(t)\right] + \alpha_2\left[\quad " \quad u_2 \quad u_2\right]$$

$$= \alpha_1 y_1 + \alpha_2 y_2$$

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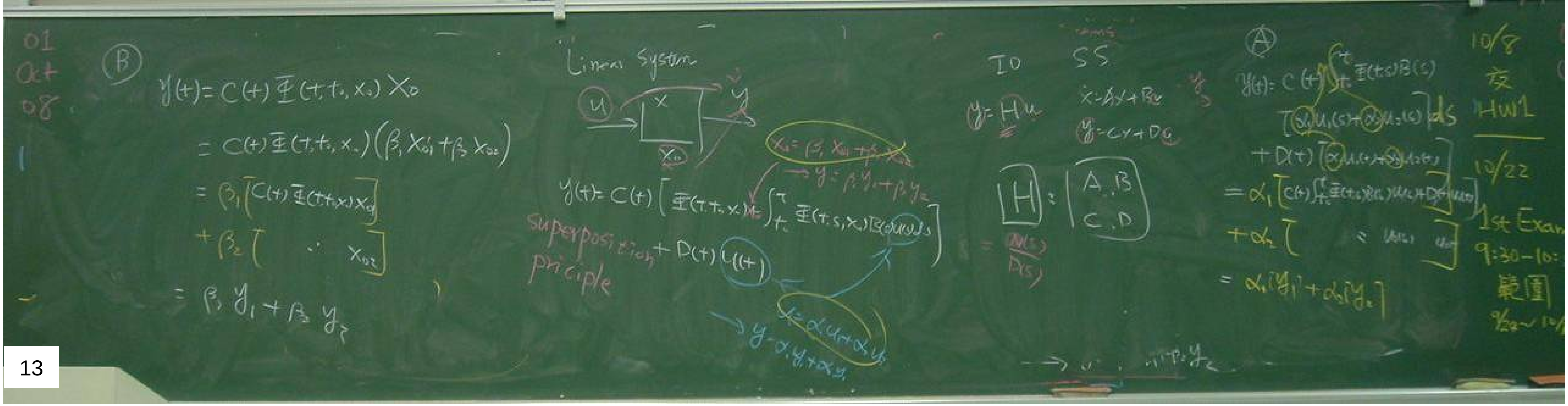
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(B)
$y(t) = C(t)\Phi(t,t_0,x_0)X_0$
$= C(t)\Phi(t,t_0,x_0)(\beta_1 X_{01} + \beta_2 X_{02})$
$= \beta_1 [C(t)\Phi(t,t_0,x_0)X_{01}]$
$+ \beta_2 [\quad \cdots \quad X_{02}]$
$= \beta_1 y_1 + \beta_2 y_2$
Linear System
$y(t) = C(t)\left[\Phi(t,t_0,x_0)X_0 + \int_{t_0}^{t}\Phi(t,s,x_0)B(s)u(s)ds\right] + D(t)u(t)$
$X_0 = \beta_1 X_{01} + \beta_2 X_{02}$
$\rightarrow y = \beta_1 y_1 + \beta_2 y_2$
$u = \alpha_1 u_1 + \alpha_2 u_2$
$\rightarrow y = \alpha_1 y_1 + \alpha_2 y_2$
Superposition principle
IO
SS
$y = Hu$
$\dot{x} = Ax + Bu$
$y = Cx + Du$
$H : \begin{vmatrix} A, B \\ C, D \end{vmatrix}$
$= \frac{N(s)}{D(s)}$
(A)
$y(t) = C(t)\int_{t_0}^{t}\Phi(t,s)B(s)[\alpha_1 u_1(s) + \alpha_2 u_2(s)]ds$
$+ D(t)[\alpha_1 u_1(t) + \alpha_2 u_2(t)]$
$= \alpha_1 [\cdots]$
$+ \alpha_2 [\cdots]$
$= \alpha_1 [y_1] + \alpha_2 [y_2]$
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