Homework 4: Stability, Controllability, Observability

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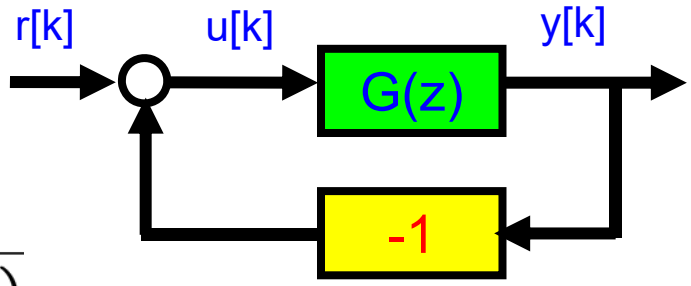
DCS03-HW-12

- Problem 4-1:
  - Consider the system and let:



$$G(z) = \frac{K}{z(z-0.2)(z-0.4)}$$

  - where $K > 0$.
  - Determine the values of K for which the closed-loop system is stable.

- Problem 4-2:
  - Is the following system (a) stable, (b) observable, (c) controllable?

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 6 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 2 & -4 \end{bmatrix} \mathbf{x}(k)$$

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- Problem 4-3:
  - Consider the system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(k) \qquad x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

  a) Determine a control sequence such that the system is taken from the initial state, $x(0)$, to the origin.

  b) Which is the minimum number steps that solve the problem in (a).

  c) Explain why it is not possible to find a sequence of control signals such that the state [1, 1, 1]^T is reached from the origin.

- Problem 4-4:
  - Given the system: $(q^2 + 0.4q)y[k] = u[k]$

  a) For which values of K in the proportional controller: $u[k] = K[r(k) - y(k)]$ is the closed-loop system stable?

  b) Determine the stationary error $r(k) - y(k)$ when $r(k)$ is a step function and when $K = 0.5$ in the controller in (a).