Problem 1: SS Model [25%]

- Consider the following continuous-time system:

$$\begin{aligned} \frac{d}{dt}\mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-d) \\ y(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{aligned}$$

- where **A, B, C, D** are constant matrices.
- Assume that a periodic sampling with period *h* is used and the time delay *d* is constant and less than the sampling period *h*.
- Please derive the corresponding discrete-time state-space model if the input *u(t)* is piecewise constant during each sampling period. That is, derive the following form:

$$\begin{aligned} \mathbf{x}(kh+h) &= \mathbf{F}(h,k)\mathbf{x}(kh) & +\mathbf{H}(h,k)u(kh+h) \\ & & +\mathbf{I}(h,k)u(kh) \\ & & +\mathbf{J}(h,k)u(kh-h) \\ y(kh) &= \mathbf{M}(h,k)\mathbf{x}(kh) & +\mathbf{N}(h,k)u(kh) \end{aligned}$$

- and the relationship between **A, B, C, D** and **F, H, I, J**, **M, N**
- where *a(b,c)* denotes that *a* might be a function of *b* and *c*.

Problem 2: TF Model [25%]

- Consider the system described by the difference equation:

$$y(k+2) - 0.3y(k+1) - 0.1y(k) = u(k)$$

- where $u(k)$ is a step at $k = 0$ and $y(k) = 0$ for $k < 0$.

a) Use the z-transform to determine the pulse-transfer function and identity the poles and zeros of the system.

b) Determine the output sequence of the difference equation:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$\Rightarrow \mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau$$

Signal	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n-m]$	z^{-m}
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$

Problem 3: Stability, Controllability, Observability [25%]

a) Is the following system (a) stable, (b) observable, (c) reachable? Justify your answer.

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{bmatrix} \mathbf{x}(k) \quad + \begin{bmatrix} 6 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 2 & -4 \end{bmatrix} \mathbf{x}(k)$$

b) Determine a control sequence such that the system is taken from x(0) = [1 2]^T to the origin.

c) What is the minimum number of steps the solve the problem in (b) and why?

Problem 4: Controller, Observer [25%]

- Given the system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.25 & 0.5 \\ 1 & 2 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k)$$

a) Determine the linear state-feedback controller: $u(k) = -K\mathbf{x}(k)$
such that the closed-loop poles are in 0.5 and 0.8.

b) Determine an observer that estimates the states
such that the observer has the desired characteristic polynomial: *(λ - 0.2)^2*.