Spring 2021

數位控制系統

Digital Control Systems

DCS-35

Input-Output Design

- By Polynomial Approach

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NTU-EE

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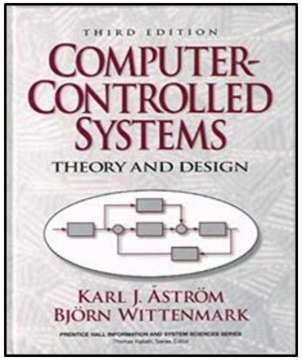
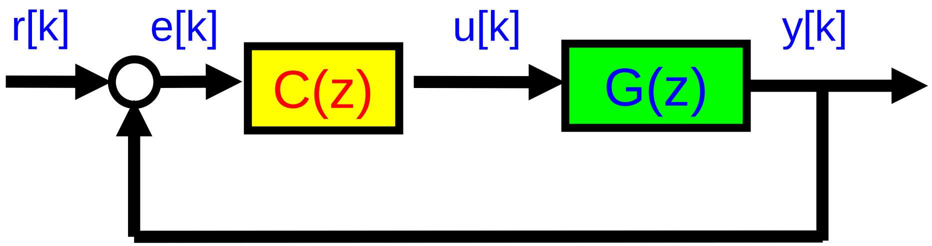
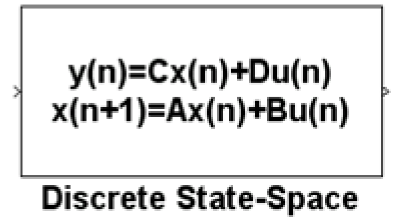


Introduction: Model and Analysis and Design





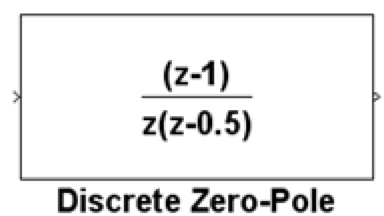
- Plant (DT):
  - Input-Output Model:

$$\frac{Y_d(z)}{\boxed{U_d(z)}} = G_d(z) = \frac{B_d(z)}{A_d(z)}$$

  - State-Space Model:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] + \mathbf{D}u[k] \end{aligned}$$

- Internal model
  - Matrix calculations
- External model
  - Polynomial calculations



- System Properties:
  - Stability
  - Controllability and Reachability
  - Observability and Detectability

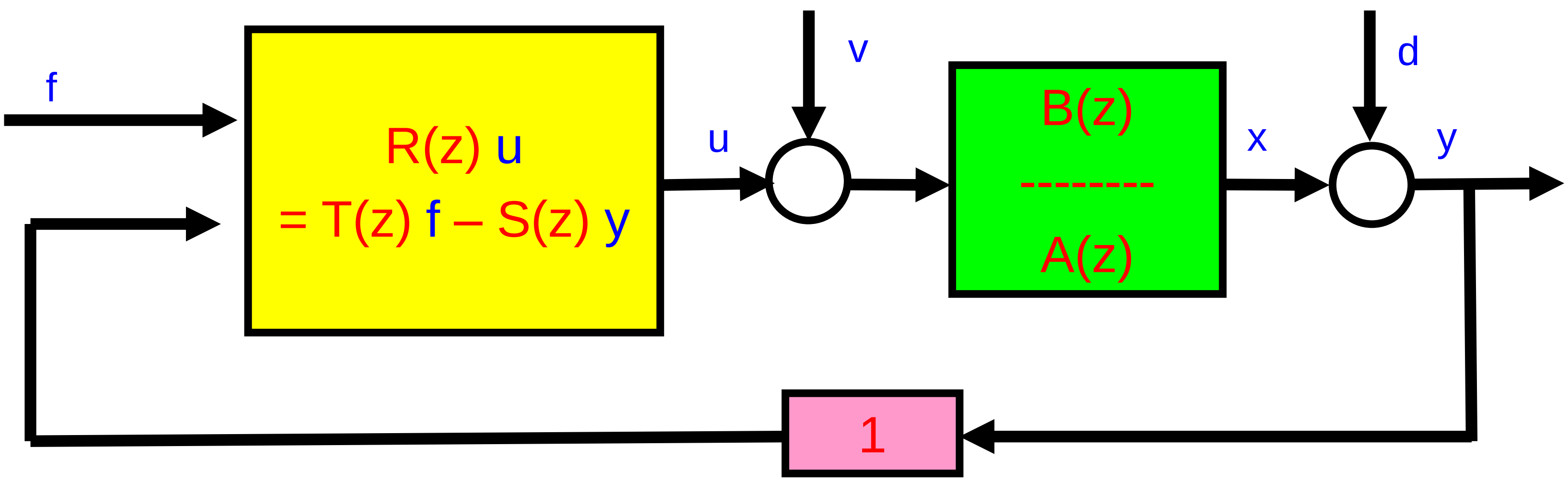
Outline

- Process and Controller Models
  - By Rational Transfer functions
- Poles and Zeros
- Command Signals
- Disturbance Response
- Case Study:
  - Double Integrator

- Control System Design:
  - Command signal following (reference)
  - Load disturbance (actuator)
  - Measurement noise (sensor)
  - Process disturbance (un-modeled dynamics)

- Design Parameters:
  - Closed-loop characteristic polynomial
  - Sampling period

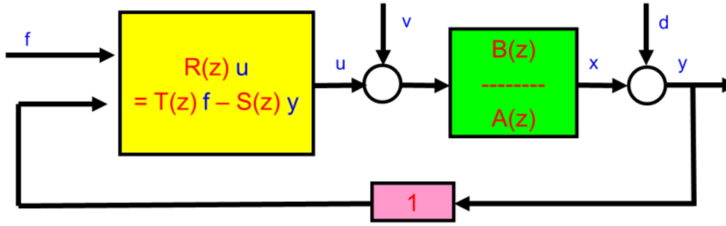
- Block Diagram of a Typical Control System:



$$(z+0.5)(z-1)u$$
$$= (z+0.3)f \; -(z^2-0.2z+1)y$$

$$\frac{(z+1)}{(z^2-0.2z+1)}$$

- The Process Model:



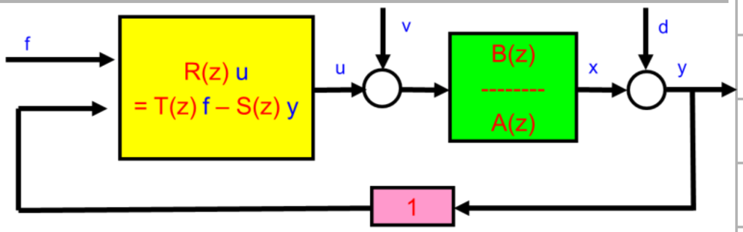
$$A(z)y(k) = B(z)u(k)$$

Or, $y(k) = \dfrac{B(z)}{A(z)}u(k)$

- $A(z) = z^n + a_1 z^{n-1} + \cdots + a_n$
- $B(z) = \qquad b_1 z^{n-1} + \cdots + b_n$

- deg $A(q) >$ deg $B(q)$
- $A(q), B(q)$: no common factors

- The Controller Model:



$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

Or, $u(k) = \frac{T(z)}{R(z)}f(k) - \frac{S(z)}{R(z)}y(k)$

- $G_{\text{ff}}(z) = \frac{T(z)}{R(z)}$
- $G_{\text{fb}}(z) = \frac{S(z)}{R(z)}$

- If causal $\Rightarrow$ deg $R(z) \geq$ deg $S(z)$

$$\Rightarrow \; y(k) = \frac{BT}{AR + BS}f(k)$$

$\Rightarrow$ How to find $T(z), R(z), S(z)$?

- Closed-Loop Characteristic Polynomial:

$$\Rightarrow \; y(k) = \frac{BT}{AR + BS} f(k)$$

$$A_{cl}(z) = A(z)R(z) + B(z)S(z)$$

- $G(z) = \frac{B(z)}{A(z)}$
- $G_{\text{fb}}(z) = \frac{S(z)}{R(z)}$

$\Rightarrow$ Diophantine equation

- Pole-placement design:
  - Find R(z) & S(z),

    such that Diophantine equation is satisfied!

▪ Closed-Loop Characteristic Polynomial:

$$A_{cl}(z) = A(z)R(z) + B(z)S(z)$$

$$= A_c(z) \;\; A_o(z)$$

$$\Rightarrow \;\; A_c(z) = \det(zI - \mathbf{F} + \mathbf{HK})$$

controller polynomial

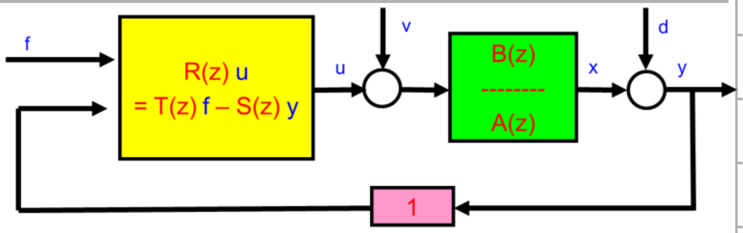
$$\Rightarrow \;\; A_o(z) = \det(zI - \mathbf{F} + \mathbf{LC})$$

observer polynomial

- If controllable $\Rightarrow$ any eig $\rightarrow A_c(z)$
- If observable $\Rightarrow$ any eig $\rightarrow A_o(z)$

Simple Design Problem

- Determining T(z) in G_ff(z):



$$Y(z) = \frac{B(z)T(z)}{A_{cl}(z)}F(z)$$

$$= \frac{B(z)T(z)}{A_c(z)A_o(z)}F(z)$$

- Let $T(z) = t_o A_o(z)$

$$\Rightarrow \ Y(z) = \frac{t_o B(z)}{A_c(z)}F(z)$$

- t_o is for desired static gain

Algorithm 5.1: (Simple Pole-Placement Design)

- Data: $\dfrac{B(z)}{A(z)}$ and $A_{cl}(z)$
  - $A(z), B(z)$ do not have common factors
  - $A_{cl}(z)$ has desired specification
- Step 1: Find $R(z), S(z)$
  - $\deg(S(z)) \leq \deg(R(z))$

Satisfy

$$A(z)R(z) + B(z)S(z) = A_{cl}(z)$$

Algorithm 5.1: (Simple Pole-Placement Design)

- Step 2: Write $A_{cl}(z) = A_c(z)A_o(z)$
  - $\deg(A_o(z)) \leq \deg(R(z))$

  Select $T(z) = t_o A_o(z)$
  - $t_o = \frac{A_c(1)}{B(1)}$
- Controller Law:

$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

- Response to command signals:

$$A_c(z)y(k) = t_o B(z)f(k)$$

- Example 5.1: (Control of a double integrator)

$$\frac{1}{s^2} \quad \Longleftrightarrow \quad \frac{h^2(z+1)}{2(z-1)^2}$$

$$\Rightarrow \quad A(z) = (z-1)^2$$

$$B(z) = \frac{h^2}{2}(z+1)$$

- Diophantine equation

$$A_{cl}(z) = (z-1)^2 R(z) + \frac{h^2}{2}(z+1) S(z)$$

- **Example 5.1: (Control of a double integrator)**

  - Try $R(z) = 1, S(z) = s_0$

    $\Rightarrow$ This is P controller, b/c $G_{\text{fb}} = \dfrac{S(z)}{R(z)}$

    $\Rightarrow A_{cl}(z) = (z^2 - 2z + 1) + \dfrac{s_0 h^2}{2}(z+1)$

    $\Rightarrow$ Impossible for any $A_{cl}(z)$ of 2nd order

- Example 5.1: (Control of a double integrator)

- Try $R(z) = z + r_1, S(z) = s_0 z + s_1$

$\Rightarrow$ This is 1st-order controller,

$$\Rightarrow A_{cl}(z) = (z^2 - 2z + 1)(z + r_1) + \frac{h^2}{2}(z+1)(s_0 z + s_1)$$

$$\Rightarrow z^3 + \left(r_1 + \frac{h^2}{2} s_0 - 2 \right) z^2 + \left(1 - 2r_1 + \frac{h^2}{2}(s_0 + s_1) \right) z + r_1 + s_1 \frac{h^2}{2}$$

$\Rightarrow r_1, s_0, s_1$ for any $A_{cl}(z)$ of 3rd order

▪ Example 5.1: (Control of a double integrator)

- Try $R(z) = z + r_1, S(z) = s_0 z + s_1$

$\Rightarrow$ If $A_{cl}(z) = z^3 + p_1 z^2 + p_2 z + p_3$

$$\Rightarrow \quad \begin{aligned} r_1 + \frac{h^2}{2} s_0 &= p_1 + 2 \\ -2r_1 + \frac{h^2}{2}(s_0 + s_1) &= p_2 - 1 \\ r_1 + s_1 \frac{h^2}{2} &= p_3 \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} r_1 &= \frac{3 + p_1 + p_2 - p_3}{4} \\ s_0 &= \frac{5 + 3p_1 + p_2 - p_3}{2h^2} \\ s_1 &= -\frac{3 + p_1 - p_2 - 3p_3}{2h^2} \end{aligned}$$

- Example 5.1: (Control of a double integrator)

  - $T(z) = t_0 A_o(z)$

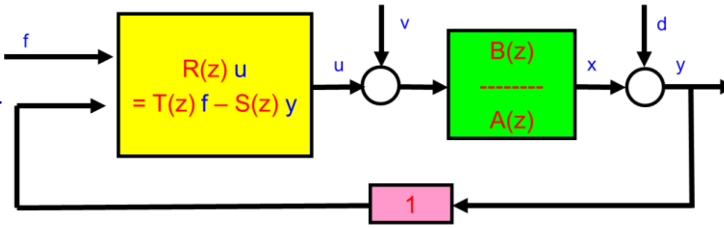
    b/c: $A_{cl}(z) = A_c(z) A_o(z)$

    $\Rightarrow$ $A_c(z)$ of 2nd order

    $A_o(z)$ of 1st order

    And, let $t_0 = \dfrac{A(1)}{B(1)}$

Cancellation of Poles and Zeros:



$$A = A^+ \; A^- \qquad (z-0.1)(z+0.5)(z+2) \quad = \quad (z-0.1)(z+0.5)(z+2)$$

$$B = B^+ \; B^- \qquad (z-0.2)(z-3) \quad = \quad (z-0.2)(z-3)$$

- A^+, B^+ are nice polynomials

  i.e., their roots are inside the unit disc

  i.e., they can be canceled

$$\Rightarrow \quad \frac{BT}{AR+BS}$$

$$\Rightarrow \quad \begin{aligned} R &= B^+ \; \bar{R} \\ S &= A^+ \; \bar{S} \\ T &= A^+ \; \bar{T} \end{aligned} \qquad \Rightarrow \quad \frac{(B^+B^-)(A^+\bar{T})}{(A^+A^-)(B^+\bar{R}) + (B^+B^-)(A^+\bar{S})}$$

$$\Rightarrow \quad \frac{B^-\bar{T}}{A^-\bar{R} + B^-\bar{S}}$$

$$A = A^+ \; A^-$$
$$B = B^+ \; B^-$$
$$R = B^+ \; \bar{R}$$
$$S = A^+ \; \bar{S}$$
$$T = A^+ \; \bar{T}$$

- Closed-Loop Characteristic Polynomial:
  - $A_{cl} = AR + BS$

$$= (A^+A^-)(B^+\bar{R}) + (B^+B^-)(A^+\bar{S})$$

$$= A^+B^+\left(A^-\bar{R} + B^-\bar{S}\right)$$

$$= A^+B^+(\bar{A}_{cl})$$

  - $b/c: A_{cl} = A_c \, A_o \;\; = \left(B^+\bar{A}_c\right)\left(A^+\bar{A}_o\right)$

$$\Rightarrow \; A^-\bar{R} + B^-\bar{S} = \bar{A}_{cl} = \bar{A}_c\bar{A}_o$$

- Minimum-Degree Causal Controller:

$$\Rightarrow \; A^- \bar{R} + B^- \bar{S} = \bar{A}_{cl} = \bar{A}_c \bar{A}_o$$

  - It is unique, if $\deg(\bar{S}) < \deg(A^-)$

- Control Law:

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$R = B^+ \, \bar{R}$$
$$S = A^+ \, \bar{S}$$
$$T = A^+ \, \bar{T}$$

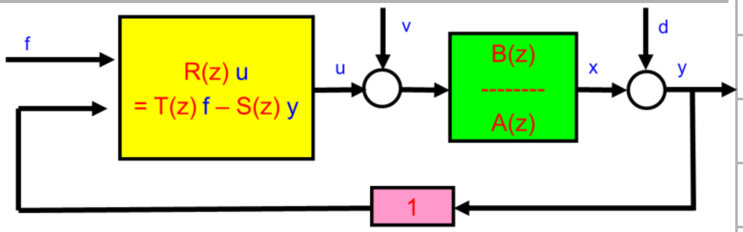
$$Ru = Tf - Sy$$

$$\Rightarrow \; B^+ \bar{R} u = A^+ \bar{T} f - A^+ \bar{S} y$$

$$\Rightarrow u = \frac{A^+}{B^+} \left(\frac{\bar{T}}{\bar{R}} f - \frac{\bar{S}}{\bar{R}} y \right)$$

Transfer Functions

from command to output:



$$y(k) = \frac{BT}{AR + BS} f(k)$$

$$\Rightarrow \frac{BT}{A_{cl}} = \frac{(B^+ B^-)(t_0 A_o)}{A_c A_o}$$

$$= \frac{t_0 B^-}{\bar{A}_c}$$

$$A = A^+ \; A^-$$
$$B = B^+ \; B^-$$
$$R = B^+ \; \bar{R}$$
$$S = A^+ \; \bar{S}$$
$$T = A^+ \; \bar{T}$$
$$T = t_0 \; A_o$$
$$A_c = B^+ \; \bar{A}_c$$
$$A_o = A^+ \; \bar{A}_o$$

- If unstable modes are canceled, they are uncontrollable or unobservable

- Disturbance & Command Signal Response:

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$

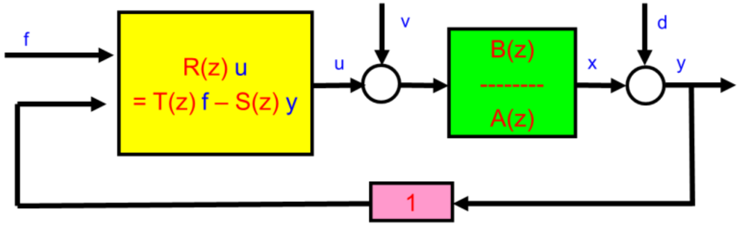
- A^+, B^+ are canceled nice polynomials

- Desired response to command signals:

$$y_m = G_m \, f = \frac{B_m}{A_m} \, f$$

- For perfect model following:

$$B_m = \bar{B}_m B^-$$



- Let:

$$R = A_m\ B^+\ \bar{R}$$
$$S = A_m\ A^+\ \bar{S}$$
$$T = \bar{B}_m\ \bar{A}_o\ \bar{A}_c\ A^+$$

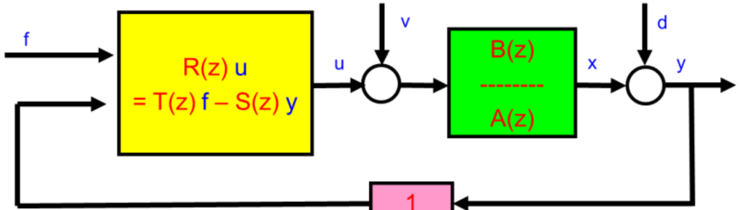
$$A = A^+\ A^-$$
$$B = B^+\ B^-$$
$$B_m = \bar{B}_m\ B^-$$

- Control Law:

$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

$$\Rightarrow\ u(k) = \frac{T(z)}{R(z)}f(k) - \frac{S(z)}{R(z)}y(k)$$

$$\Rightarrow\ u(k) = \frac{A^+}{B^+}\left(\frac{\bar{B}_m\bar{A}_o\bar{A}_c}{A_m\bar{R}}f - \frac{\bar{S}}{\bar{R}}y\right)$$



$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$
$$R = A_m \, B^+ \, \bar{R}$$
$$S = A_m \, A^+ \, \bar{S}$$
$$T = \bar{B}_m \, \bar{A}_o \, \bar{A}_c \, A^+$$

- From the Diophantine equation:

$$\bar{A}_o \, \bar{A}_c = A^- \, \bar{R} + B^- \, \bar{S}$$

- Hence:

$$\Rightarrow \; \frac{\bar{B}_m}{A_m \bar{R}} (\bar{A}_o \bar{A}_c) = \frac{\bar{B}_m (A^- \, \bar{R} + B^- \, \bar{S})}{A_m \, \bar{R}}$$

$$= \frac{\bar{B}_m \, A^-}{A_m} + \frac{\bar{B}_m \, B^- \, \bar{S}}{A_m \, \bar{R}}$$

$$= \frac{B_m \, A^-}{A_m \, B^-} + \frac{B_m \, \bar{S}}{A_m \, \bar{R}}$$

$$y_m = \frac{B_m}{A_m} \, f$$

$$\Rightarrow \; u(k) = \frac{A^+}{B^+} \left(\left(\frac{B_m \, A^-}{A_m \, B^-} + \frac{B_m \, \bar{S}}{A_m \, \bar{R}} \right) f - \frac{\bar{S}}{\bar{R}} y \right)$$

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$
$$R = A_m \, B^+ \, \bar{R}$$
$$S = A_m \, A^+ \, \bar{S}$$
$$T = \bar{B}_m \, \bar{A}_o \, \bar{A}_c \, A^+$$

- Control Law:

$$\Rightarrow \quad u(k) = \frac{A^+}{B^+}\left(\left(\frac{B_m \, A^-}{A_m \, B^-} + \frac{B_m \, \bar{S}}{A_m \, \bar{R}}\right) f - \frac{\bar{S}}{\bar{R}} y\right)$$

$$y_m = \frac{B_m}{A_m} f \qquad \to f = \frac{A_m}{B_m} y_m$$

$$\Rightarrow \quad u = \frac{B_m \, A}{A_m \, B} f + \frac{A^+ \, \bar{S}}{B^+ \, \bar{R}} (y_m - y)$$

- Feedforward:

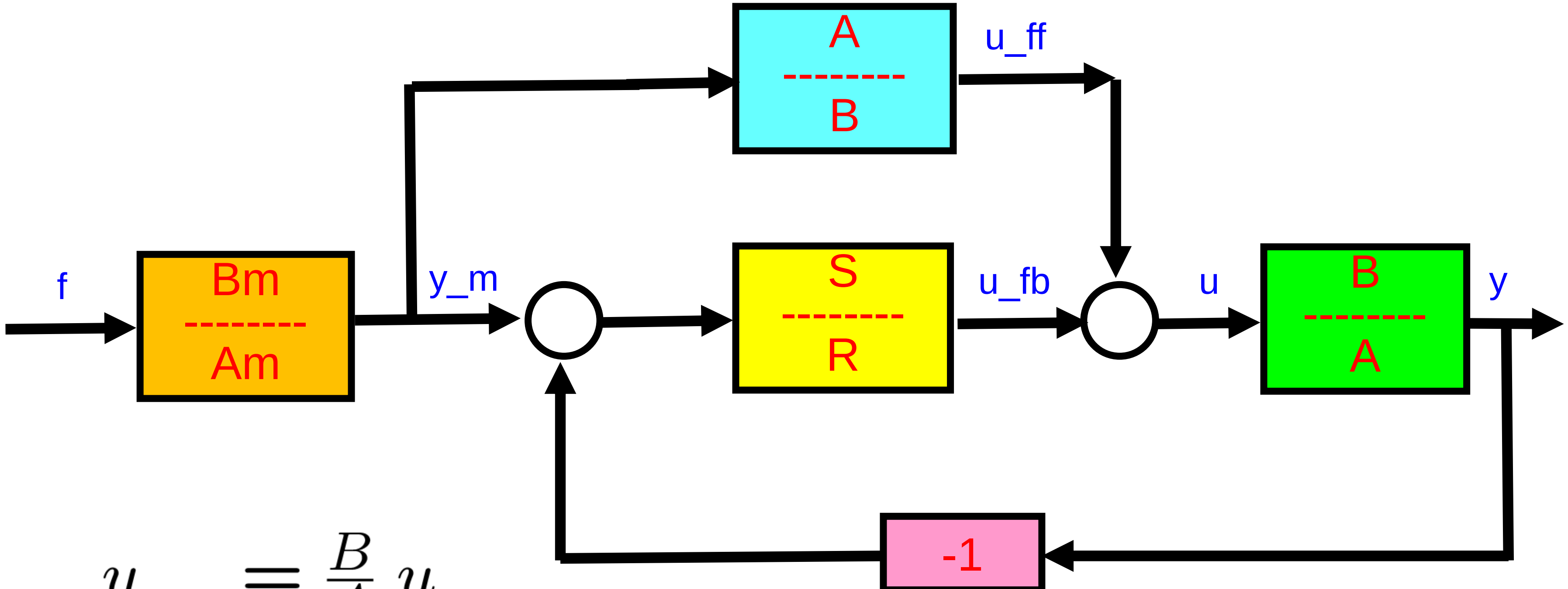
$$G_{\mathrm{ff}} = \frac{B_m \, A}{A_m \, B} \qquad = \frac{\bar{B}_m \, A}{A_m \, B^+}$$

- Feedback from e = (y_m – y):

$$G_{\mathrm{fb}} = \frac{A^+ \, \bar{S}}{B^+ \, \bar{R}}$$

- Control Law:

$$\Rightarrow \quad u = \frac{B_m \; A}{A_m \; B} f + \frac{A^+ \; \bar{S}}{B^+ \; \bar{R}} (y_m - y)$$



$$\Rightarrow \quad \begin{aligned} y &= \frac{B}{A} u \\ y_m &= \frac{B_m}{A_m} f \end{aligned}$$

$$\Rightarrow \quad y \rightarrow y_m \qquad \Rightarrow \quad y = \frac{B_m}{A_m} f$$

- Example 5.5 (motor with zero cancellation):

$$G(z) = \frac{K(z-b)}{(z-1)(z-a)}$$

$$G_m(z) = \frac{z(1+p_1+p_2)}{(z^2+p_1 z+p_2)}$$

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$
$$R = A_m \, B^+ \, \bar{R}$$
$$S = A_m \, A^+ \, \bar{S}$$
$$T = \bar{B}_m \, \bar{A}_o \, \bar{A}_c \, A^+$$

- Cancel the zero z = b:

$$B^+ = z - b$$

$$B^- = K \qquad \bar{B}_m = B_m / K$$

$$A^+ = 1 \qquad A_o = 1 \qquad \bar{A}_c = A_m$$

- Example 5.5 (motor with zero cancellation):

- Control Law:

$$A_{cl} = AR + BS$$

$$\Rightarrow \quad A_m = A\bar{R} + B^- \bar{S}$$

Try $\bar{R} = r_0, \bar{S} = s_0 z + s_1$

$$\Rightarrow \quad \begin{aligned} r_0 &= 0 \\ s_0 &= \frac{1+a+p_1}{K} \\ s_1 &= -\frac{p_2-a}{K} \end{aligned}$$

And $T(z) = A_o(z)\bar{B}_m$

$$\Rightarrow \quad \frac{z(1+p_1+p_2)}{K} = t_0 z$$

$$\begin{aligned} A &= A^+ \; A^- \\ B &= B^+ \; B^- \\ B_m &= \bar{B}_m \; B^- \\ R &= A_m \; B^+ \; \bar{R} \\ S &= A_m \; A^+ \; \bar{S} \\ T &= \bar{B}_m \; \bar{A}_o \; \bar{A}_c \; A^+ \end{aligned}$$

- Example 5.5 (motor with zero cancellation):

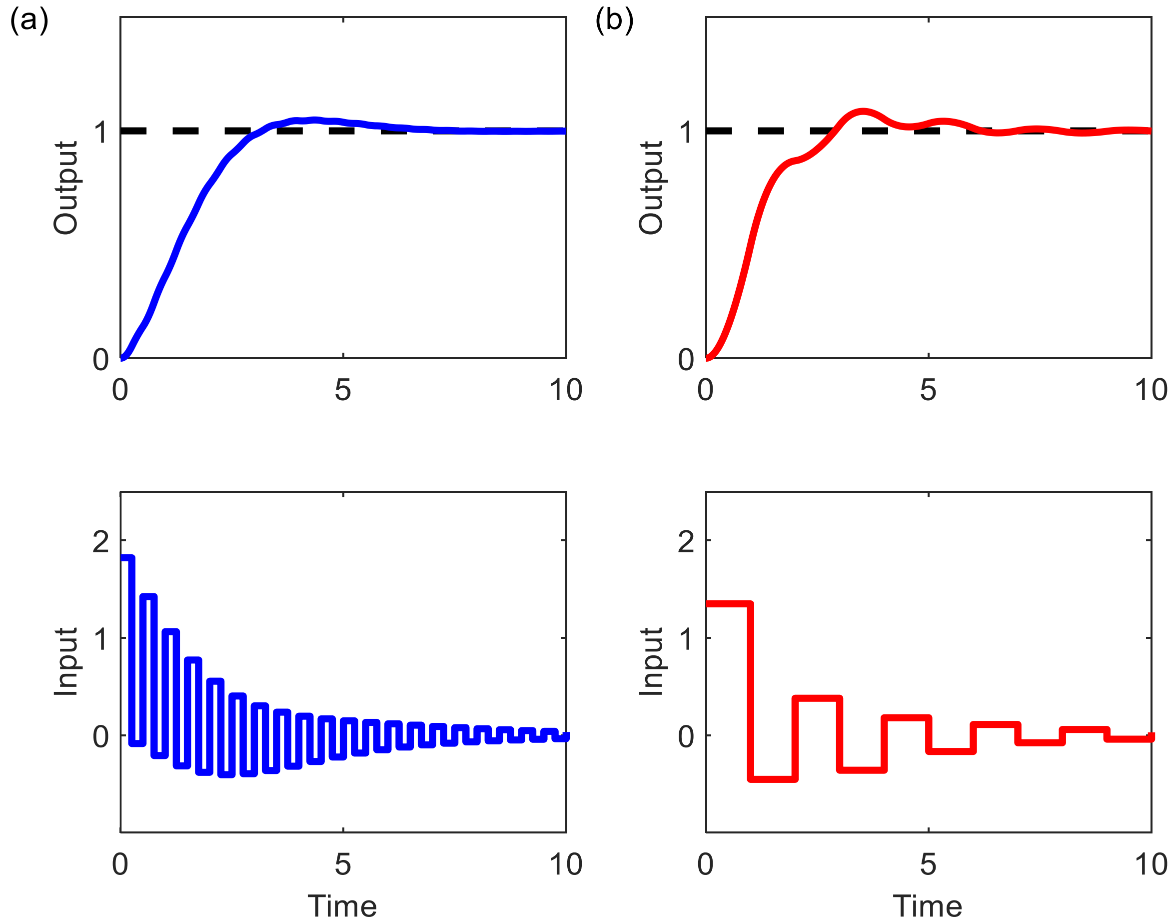
- Control Law:

$$A = A^{+} A^{-}$$
$$B = B^{+} B^{-}$$
$$B_m = \bar{B}_m B^{-}$$
$$R = A_m B^{+} \bar{R}$$
$$S = A_m A^{+} \bar{S}$$
$$T = \bar{B}_m \bar{A}_o \bar{A}_c A^{+}$$

$$\Rightarrow \; u(k) = \frac{A^{+}}{B^{+}} \left(\frac{\bar{B}_m \bar{A}_o \bar{A}_c}{A_m \bar{R}} f - \frac{\bar{S}}{\bar{R}} y \right)$$

$$\Rightarrow \; u(k) = t_0 f(k) - s_0 y(k) - s_1 y(k-1) + b u(k-1)$$

- Example 5.5 (motor with zero cancellation):
- Step response for a motor with pole-placement control
- Specification: $\zeta = 0.7$, $\omega = 1$
- (a) $h = 0.25$, (b) $h = 1.0$.



- Example 5.6 (motor w/o zero cancellation):

$$G(z) = \frac{K(z-b)}{(z-1)(z-a)}$$

$$G_m(z) = \frac{(1+p_1+p_2)}{(1-b)} \frac{(z-b)}{(z^2+p_1 z+p_2)}$$

$$B^+ = 1$$

$$B^- = K(z-b)$$

$$A^+ = 1$$

$$A_c = A_m$$

$$A_o = z$$

$$\bar{B}_m = \frac{1+p_1+p_2}{K(1-b)}$$

- Example 5.6 (motor w/o zero cancellation):

- Control Law:

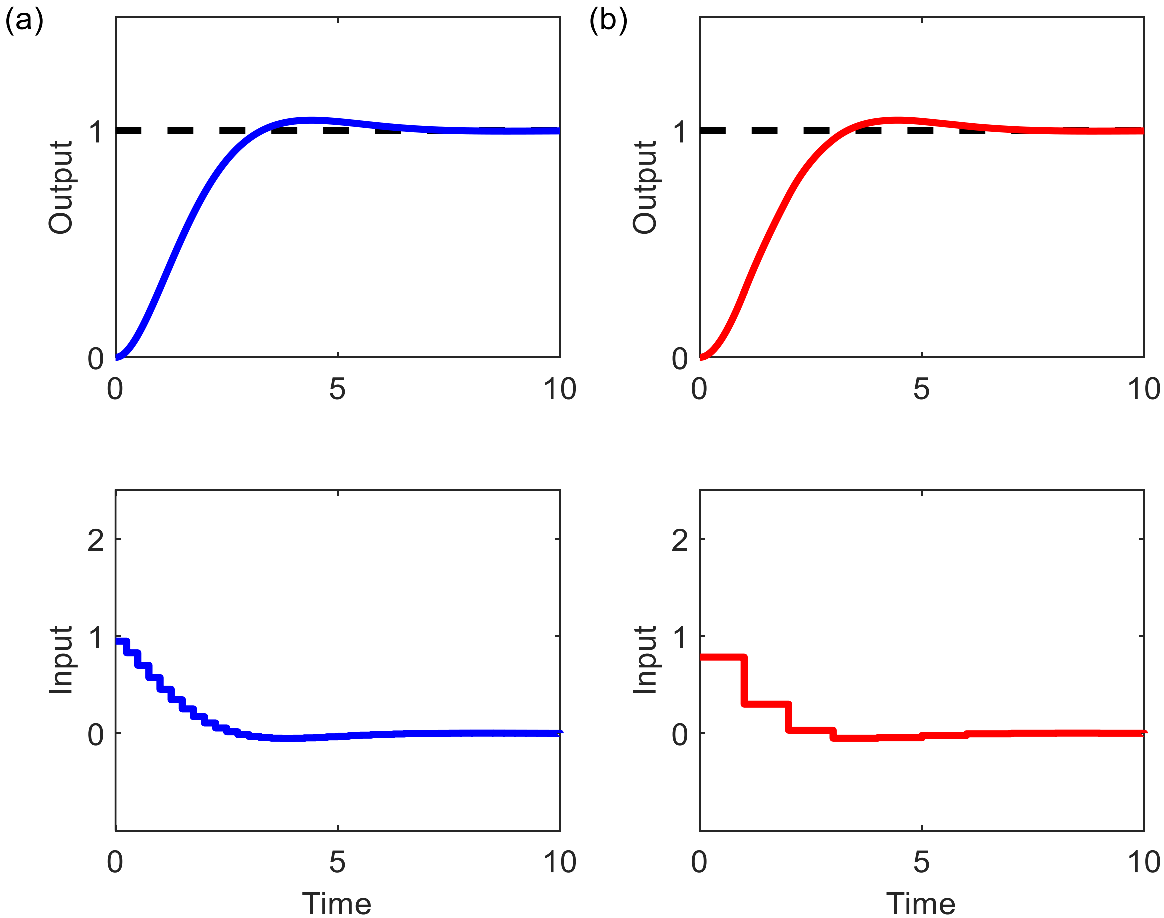
$$AR + B^{-}S = A_m A_o$$

$$\deg S = 1, \deg R = 1$$

Try $R = z + r_1, S = s_0 z + s_1$

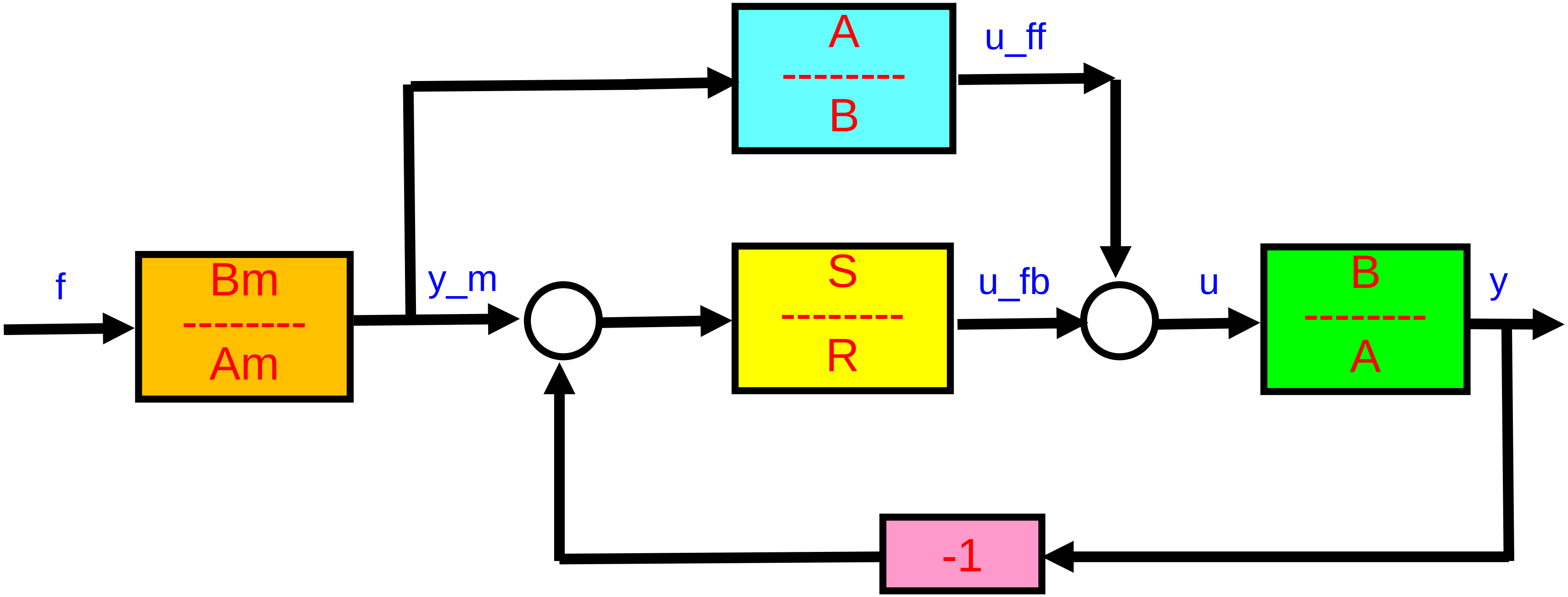
$$u(k) = t_0 u_c(k) - s_0 y(k) - s_1 y(k-1) - r_1 u(k-1)$$

- Example 5.6 (motor wi/0 zero cancellation):
- Step response for a motor with pole-placement control
- Specification: ζ = 0.7, ω = 1
- (a) h = 0.25, (b) h = 1.0.



- Algorithm 5.3 (General Pole-Placement Design):

- Block Diagram:



- Algorithm 5.3 (General Pole-Placement Design):
- Data:
  - Process Model: $\dfrac{B(z)}{A(z)}$ $A(z), B(z)$: no common factors
  - Closed-Loop Chac. Poly.: $A_{cl}(z)$
  - Desired Response: $\dfrac{B_m(z)}{A_m(z)}$
  - $R_d(z), S_d(z)$ specify $R(z), S(z)$

- Algorithm 5.3 (General Pole-Placement Design):
- Pole Excess Condition:
  - $\deg A_m(z) - \deg B_m(z) \geq \deg A(z) - \deg B(z)$
  - $\deg A_m(z) + \deg B(z) \geq \deg A(z) + \deg B_m(z)$

$$G_{\text{ff}} = \frac{B_m \; A}{A_m \; B}$$

- Model Following Condition:
  - $B_m = B^- \bar{B}_m$
- Degree Condition:
  - $\deg A_{cl} = 2 \deg A + \deg A_m + \deg R_d + \deg S_d - 1$

- Algorithm 5.3 (General Pole-Placement Design):
- Step 1:
  - $A = A^+ \, A^-$
  - $B = B^+ \, B^-$

A^+, B^+: can be canceled by the controller

- Step 2:
  - Solve the Diophantine eqn:

$$A^- \, R_d \, \bar{R} + B^- \, S_d \, \bar{S} = \bar{A}_{cl}$$

- Algorithm 5.3 (General Pole-Placement Design):
- Step 3:

$$R = A_m \, B^+ \, R_d \, \bar{R}$$
$$S = A_m \, A^+ \, S_d \, \bar{S}$$
$$T = \bar{B}_m \, A^+ \, \bar{A}_{cl}$$
$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$
$$A_{cl} = A^+ \, B^+ \, A_m \, \bar{A}_{cl}$$

- the controller:

$$Ru = Tu_c - Sy$$

- the C.L. Chac. Poly.:

$$A_{cl} = A^+ \, B^+ \, A_m \, \bar{A}_{cl}$$

- Algorithm 5.3 (General Pole-Placement Design):

- Calculating the Control Law:

$$
\begin{aligned}
R &= A_m\, B^+\, R_d\, \bar{R} \\
S &= A_m\, A^+\, S_d\, \bar{S} \\
T &= \bar{B}_m\, A^+\, \bar{A}_{cl} \\
A &= A^+\, A^- \\
B &= B^+\, B^- \\
B_m &= \bar{B}_m\, B^- \\
A_{cl} &= A^+\, B^+\, A_m\, \bar{A}_{cl}
\end{aligned}
$$

$$A^-\, R_d\, \bar{R} + B^-\, S_d\, \bar{S} = \bar{A}_{cl}$$

- $AR^0 + BS^0 = A^0_{cl}$
- $AU + BV = 0$ $\quad U, V$: min-degree sol
- Define $\begin{cases} R = XR^0 + YU \\ S = XS^0 + YV \end{cases}$ $\quad X$: stable monic poly.

$$\Rightarrow AR + BS = XA^0_{cl} = A_{cl}$$

Process Model:

- $G(s) = \frac{K}{s^2}$ $\qquad K = 1$
- $G(z) = \frac{h^2}{2} \frac{z+1}{(z-1)^2}$

- Specifications:

- Closed-Loop Polynomial A_c:

  - $A_c(s) \;\Rightarrow\; s^2 + 2\zeta w s + w^2$
  - $A_c(z) = z^2 - 2z e^{-\zeta w h} \cos\left(wh\sqrt{1-\zeta^2}\right) + e^{-2\zeta w h}$

    $= z^2 + a_{c1} z + a_{c2}$

- Closed-Loop Polynomial A_o:

  - $A_o(s) \;\Rightarrow\; (s+\alpha)^2$
  - $A_o(z) = (z - e^{-\alpha h})^2$

    $= z^2 + a_{o1} z + a_{o2}$

- Controller Design:

- The Diophantine Eqn:

$$(z-1)^2 R(z) + \frac{h^2}{2}(z+1) S(z) = A_o(z) A_c(z) = A_{cl}(z)$$

- For Integral Action:
  - $R(z)$ should have $(z-1)$

- For Minimum-Degree Solution:
  - $R(z) \Rightarrow$ 2nd order
  - $S(z) \Rightarrow$ 2nd order

- Controller Design:
  - $R(z) = (z+r)(z-1) = z^2 + r_1 z + r_2$
  - $S(z) = s_0 z^2 + s_1 z + s_2$
- Straightforward calculations give:

$$s_0 = \frac{A_{cl}(1) - 2A'_{cl}(1) + 2A''_{cl}(1)}{4h^2}$$

$$s_1 = \frac{-A_{cl}(1) + 2A'_{cl}(1) - A''_{cl}(1)}{h^2}$$

$$s_2 = \frac{7A_{cl}(1) - 6A'_{cl}(1) + 2A''_{cl}(1)}{4h^2}$$

$$r_1 = \frac{A_{cl}(-1)}{8}$$

$$r_2 = 1 - \frac{A_{cl}(-1)}{8}$$

- And:
  - $T(z) = \frac{A_c(1)A_o(z)}{B(1)} = \frac{(1+a_{c1}+a_{c2})A_o(z)}{h^2}$

- Nominal Design:

- Closed-Loop Parameters:

$$\begin{aligned} \zeta &= 0.707 \\ w &= 0.2 \\ \alpha &= 2 \\ h &= 1 \end{aligned}$$

$$e^{-\alpha h} = 0.135$$

- Simulation Study:

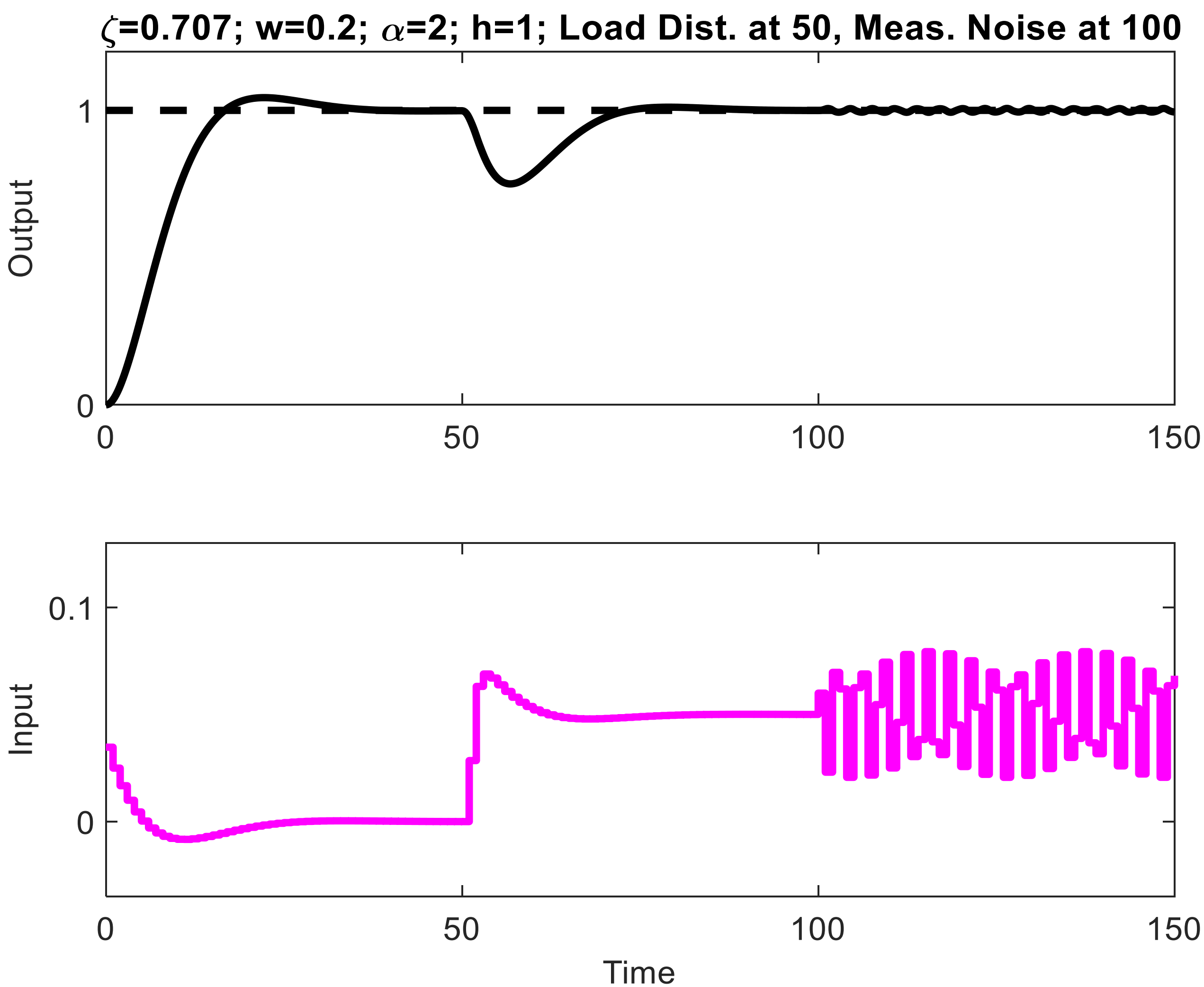
Command Signal:	Unit Step
Load Disturbance:	Step of -0.05 at time 50
Measurement Noise:	$0.01 \sin 2t$ at time 100

- Frequency Folding:

Nyquist Frequency:	$0.5 \text{ Hz} = \pi \text{ rad/s}$
Measurement Noise:	2 rad/s

- Simulation Study:

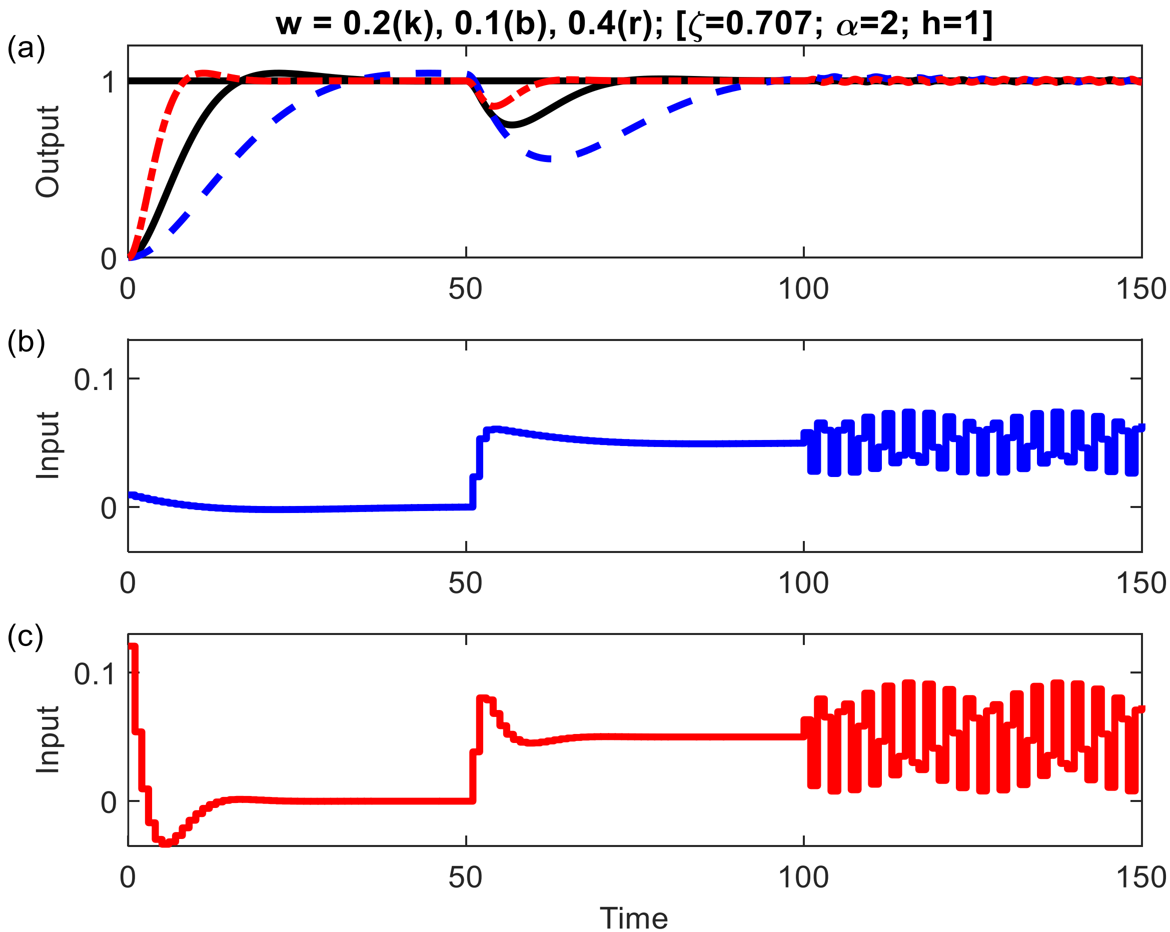
$$\zeta = 0.707$$
$$w = 0.2$$
$$\alpha = 2$$
$$h = 1$$



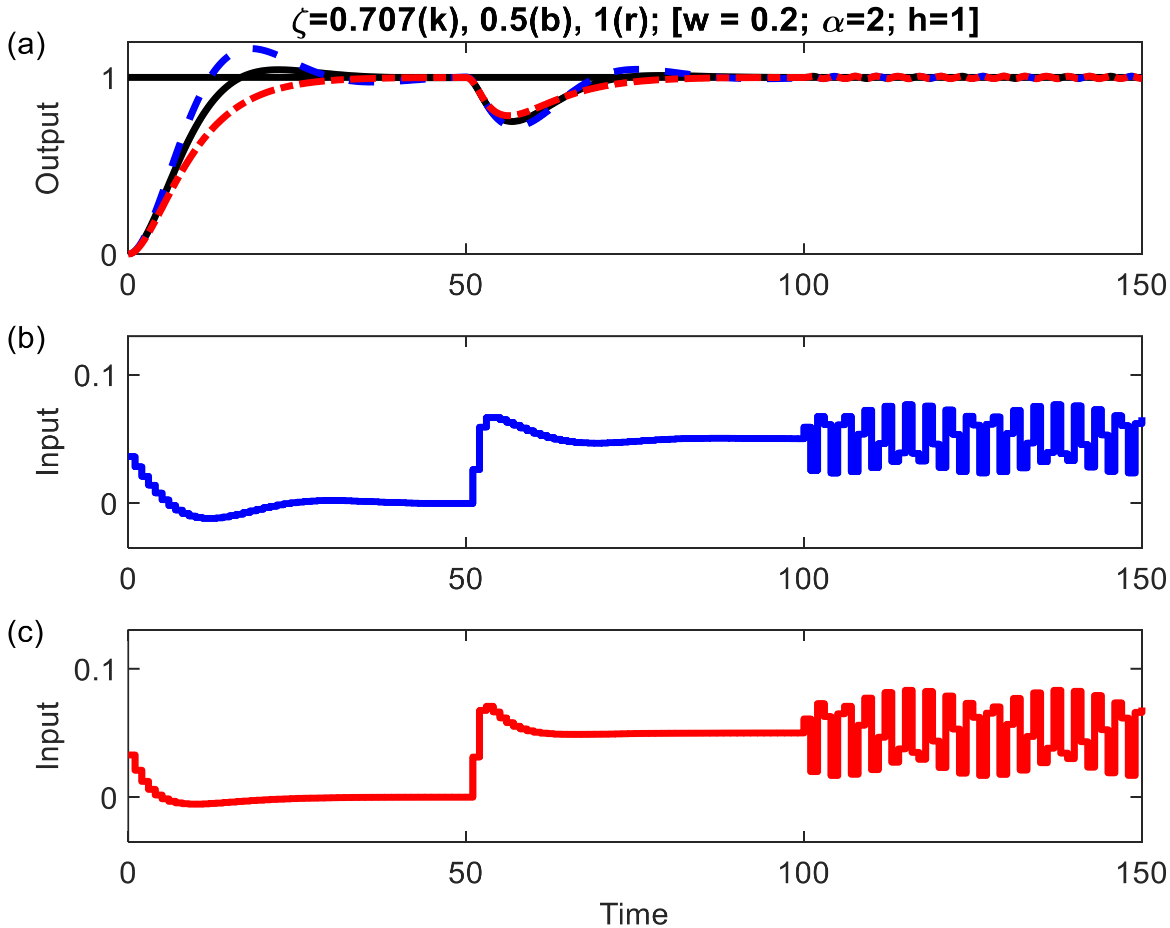
Disturb at 50

Noise at 100

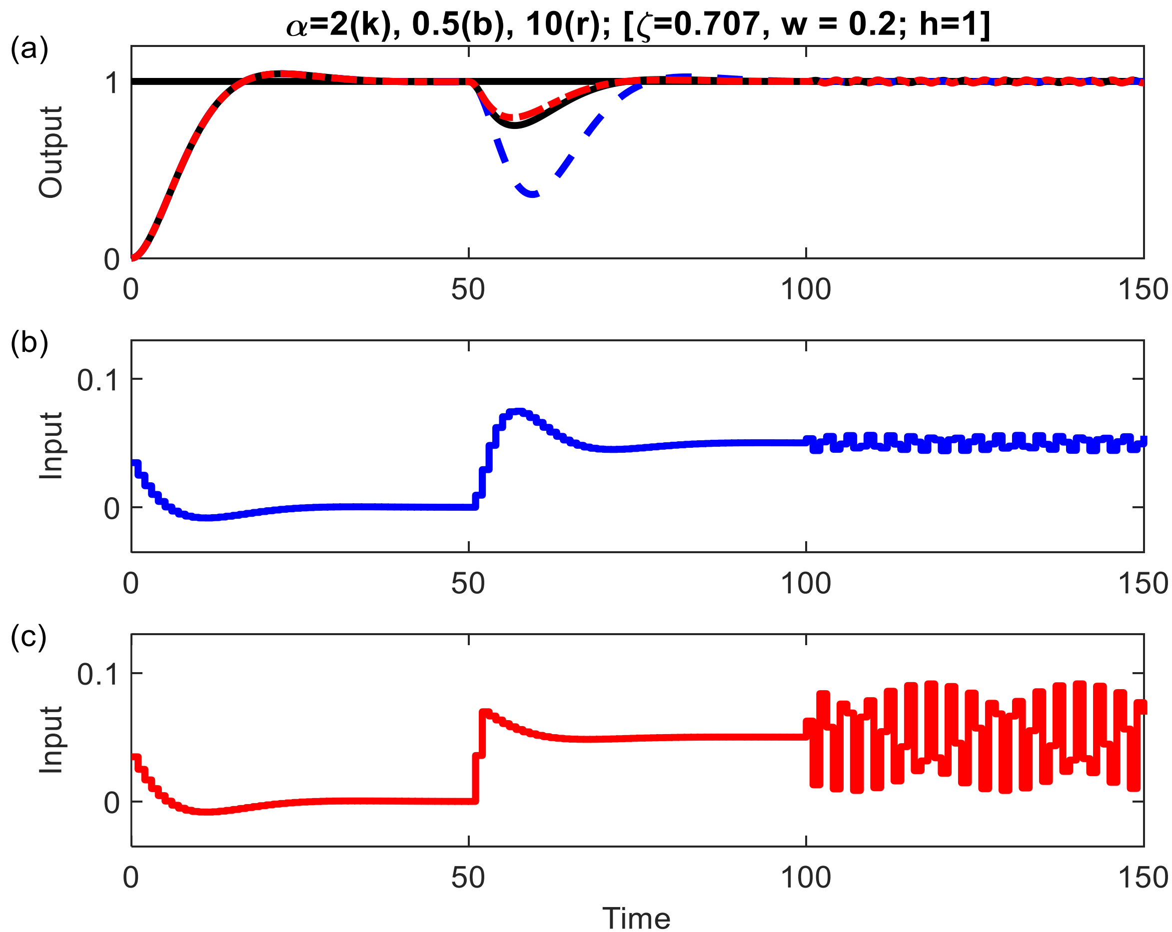
- Changing Natural Frequency ω:
- (a) ω = 0.2, 0.1, 0.4; [ζ = 0.707; α = 2; h = 1]
- (b) control signal when ω = 0.1
- (c) control signal when ω = 0.4



- Changing Damping Ratio ζ:
- (a) ζ = 0.707, 0.5, 1.0; [ω = 0.2; α = 2; h = 1]
- (b) control signal when ζ = 0.5
- (c) control signal when ζ = 1.0



- Changing Observer Poles α: $z = e^{-\alpha h}$
- (a) α = 2, 0.5, 10; [ζ = 0.707; ω = 0.2; *h* = 1]
- (b) control signal when α = 0.5
- (c) control signal when α = 10



- Changing Sampling Period h: $z = e^{-\alpha h}$
- (a) h = 1, 2, 0.1; [ζ = 0.707; ω = 0.2; α = 2]
- (b) control signal when h = 2
- (c) control signal when h = 0.1