Spring 2021

數位控制系統

Digital Control Systems

DCS-33
Discretized Controller – Design by Emulation

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NTU-EE

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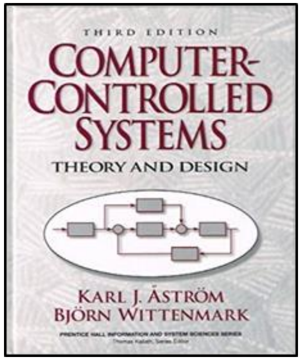
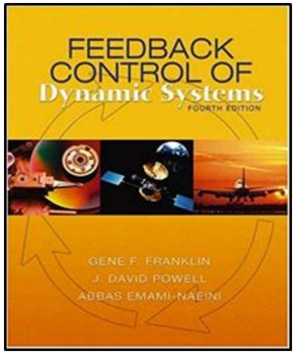
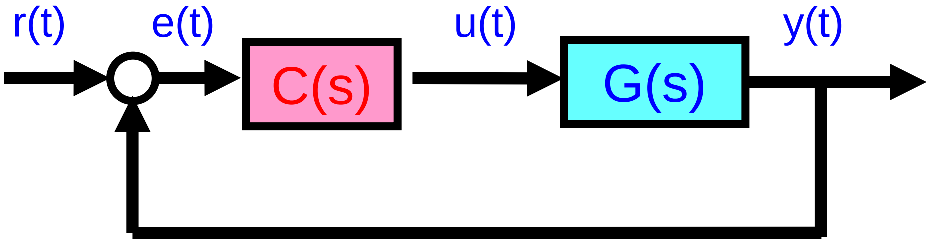




Introduction: CT and DT Plant-Controller

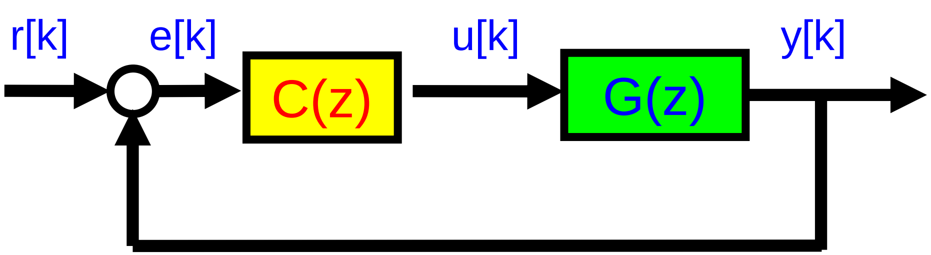


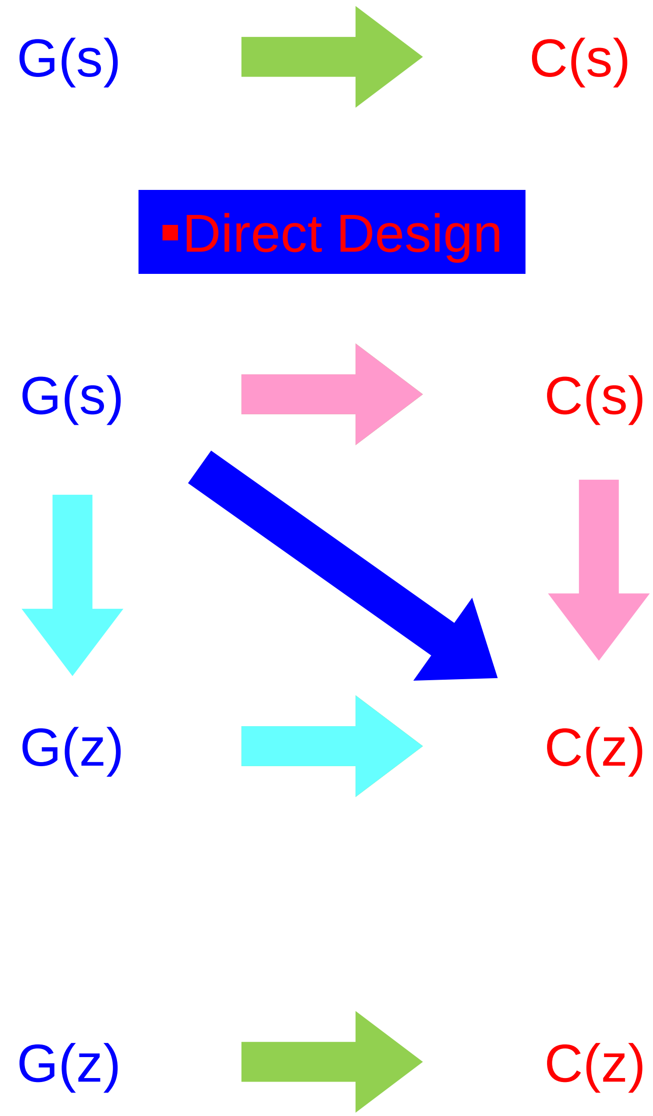
- Discrete Design

- Transform CT plant into DT plant
- By DT plant, design DT controller

- Emulation

- By CT plant, design CT controller
- Transform CT controller into DT controller





- Basic principles of low-order controller design

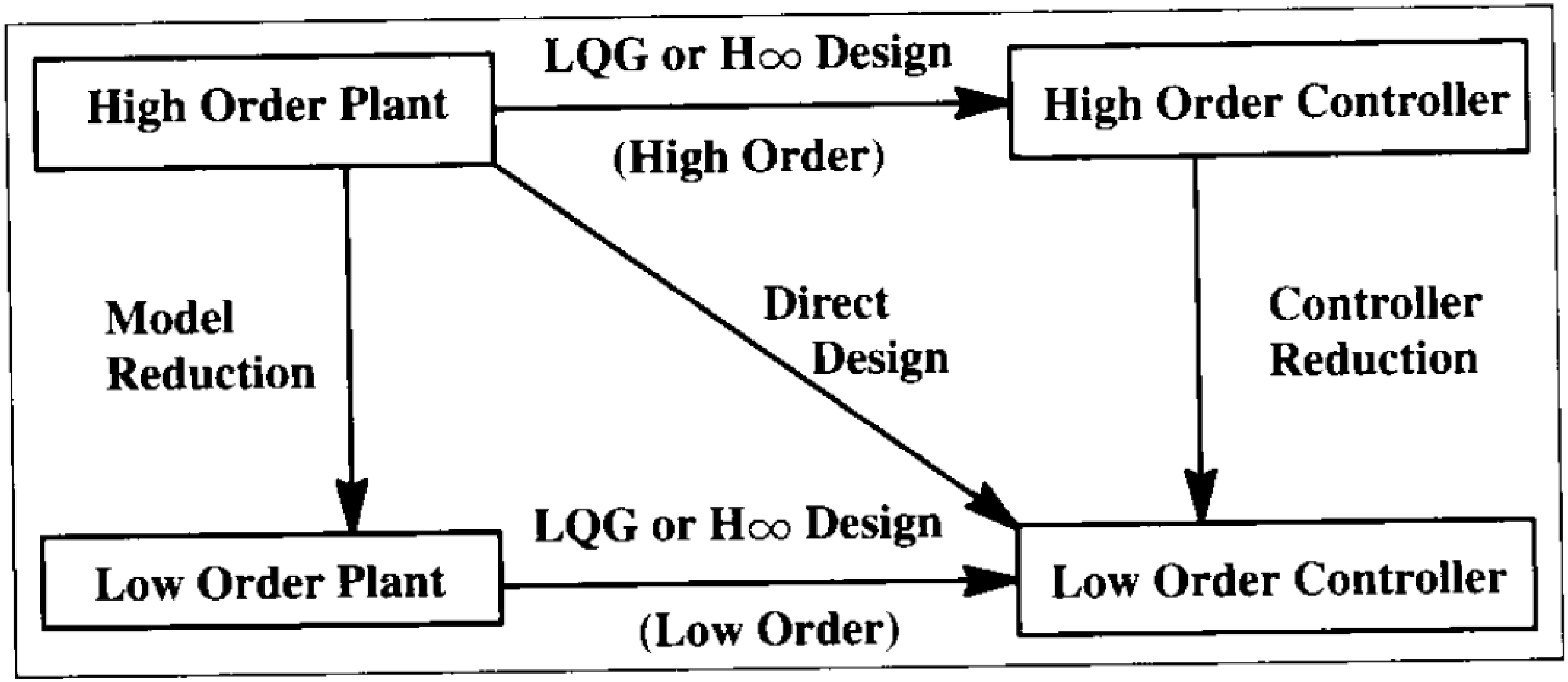


Fig. 1. Basic principles of low order controller design.

B.D.O. Anderson, “Controller Design: Moving from Theory to Practice,” IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993

Anderson 1993

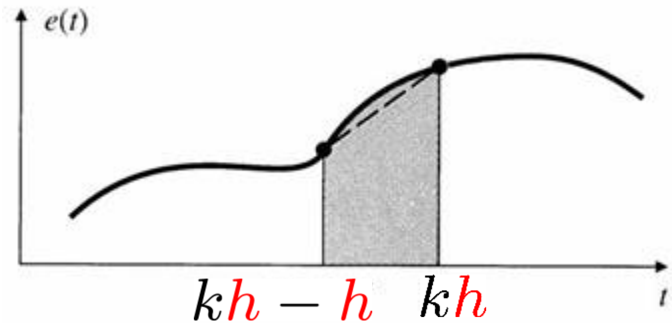
Study in Digital Control Systems

- Controller Design of Digital Control Systems
  - **Design Process**
    - > Discrete Design:
      - » CT plant -> DT plant -> DT controller
    - > Emulation:
      - » CT plant -> CT controller -> DT controlle
    - > Direct Design: (B.D.O. Anderson, 1992 Bode Prize Lecture)
      - » CT plant -> DT controller

Outline

- Discrete Design
  - By Transfer Function
  - By State Space
- Design by Emulation
  - Tustin's Method or bilinear approximation
  - Matched Pole-Zero method (MPZ)
  - Modified Matched Pole-Zero method (MMPZ)
  - Digital PID-Controllers
- Techniques for Enhancing the Performance

- Tustin's method:



$$\frac{U(s)}{E(s)} = D(s) = \frac{1}{s}$$

$$\Rightarrow \quad u[kh] = \int_0^{kh-h} e(t)dt + \int_{kh-h}^{kh} e(t)dt$$

$$= u[kh-h] + \text{ area under } e(t) \text{ over last } h$$

$$\Rightarrow \quad u[k] = u[k-1] + \frac{h}{2}\left[e[k-1] + e[k]\right]$$

$$\Rightarrow \quad \frac{U(z)}{E(z)} = D(z) = \frac{h}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) = \frac{1}{\frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Franklin et al. 2002

- Tustin's method:

$$\frac{U(s)}{E(s)} = D(s) = \frac{a}{s+a}$$

$$\Rightarrow \frac{U(z)}{E(z)} = D(z) = \frac{a}{\frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+a}$$

$$\Rightarrow s = \frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

for every occurance of s in any $D(s)$
yields a $D(z)$ on the trapezoidal integration

Franklin et al. 2002

- Tustin's method:

$$D(s) = 10\,\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$$w_s = 25 \times w_{\text{BW}} = 25 \times 10 = 250 \text{ rad/sec}$$

$$f_s = w_s/(2\pi) \approx 40 \text{ Hz}$$

$$h = \frac{1}{f_s} = \frac{1}{40} = 0.025 \text{ sec}$$

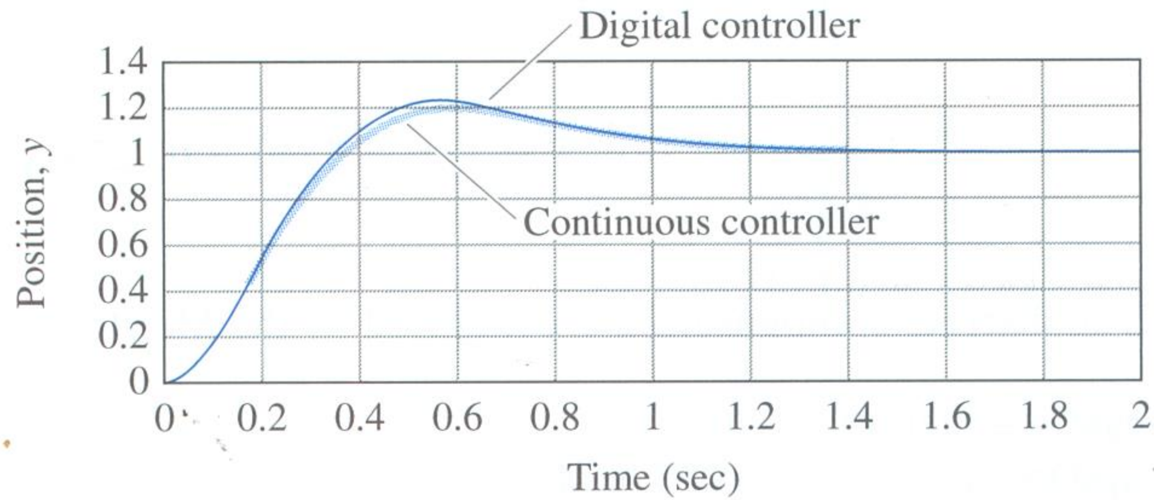
```
sysDs = tf( 10*[0.5 1], [0.1 1] );
sysDz = c2d( sysDs, 0.025, 'tustin');
```

$$\Rightarrow \quad D(z) = \frac{45.56 - 43.33z^{-1}}{1 - 0.7778z^{-1}}$$

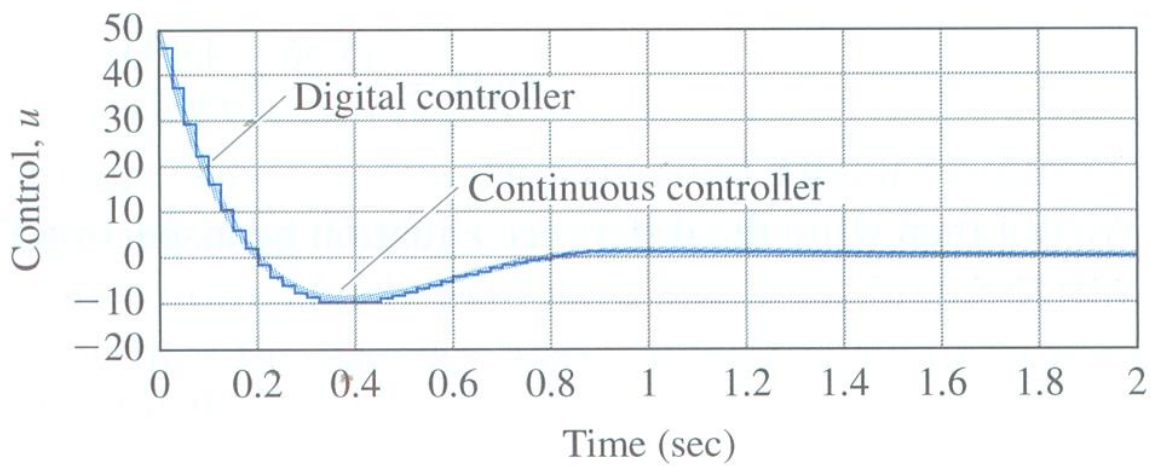
$$\Rightarrow \quad u[k] = 0.7778u[k-1] + 45.56e[k] - 43.33e[k-1]$$

Franklin et al. 2002

- Tustin's method:



(a)



(b)

Franklin et al. 2002

MATLAB's command: c2d

C2D Conversion of continuous-time systems to discrete time.

SYSD = C2D(SYSC, TS, METHOD)
converts the continuous system SYSC to a discrete-time system SYSD with sample time TS.
The string METHOD selects the discretization method among the following:

'zoh'	Zero-order hold on the inputs.
'foh'	Linear interpolation of inputs (triangle appx.)
'tustin'	Bilinear (Tustin) approximation.
'prewarp'	Tustin approximation with frequency prewarping. The critical frequency Wc is specified last as in C2D(SysC, Ts, 'prewarp', Wc)
'matched'	Matched pole-zero method (for SISO systems only).

- MathWorks: c2d: https://www.mathworks.com/help/control/ref/c2d.html

Differentiation & Tustin Approximation:

- Forward difference: (Euler's method)

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t+h) - x(t)}{h} = \frac{q-1}{h}\, x(t)$$

- Backward difference:

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t) - x(t-h)}{h} = \frac{q-1}{qh}\, x(t)$$

Astrom & Wittenmark 1997

- Differentiation & Tustin Approximation:

  - Trapezoidal method: (Tustin, bilinear)

$$\frac{\dot{x}(t+h)+\dot{x}(t)}{2} \approx \frac{x(t+h)-x(t)}{h}$$

$$\frac{q+1}{2} p\, x(t) \approx \frac{q-1}{h}\, x(t)$$

$$p\, x(t) \approx \frac{2}{h} \cdot \frac{q-1}{q+1}\, x(t)$$

Astrom & Wittenmark 1997

▪ Properties of Approximations:

$$G_d(z) \;=\; G_c(s)$$

$$p\,x(t) \approx \begin{cases} \dfrac{q-1}{h}\,x(t) & \text{Forward} \\[2ex] \dfrac{q-1}{qh}\,x(t) & \text{Backward} \\[2ex] \dfrac{2}{h}\cdot\dfrac{q-1}{q+1}\,x(t) & \text{Trapezoidal} \end{cases}$$

- Forward
- Backward
- Trapezoidal

$$s = \frac{z-1}{h} \quad \text{(Forward difference or Euler's method)}$$

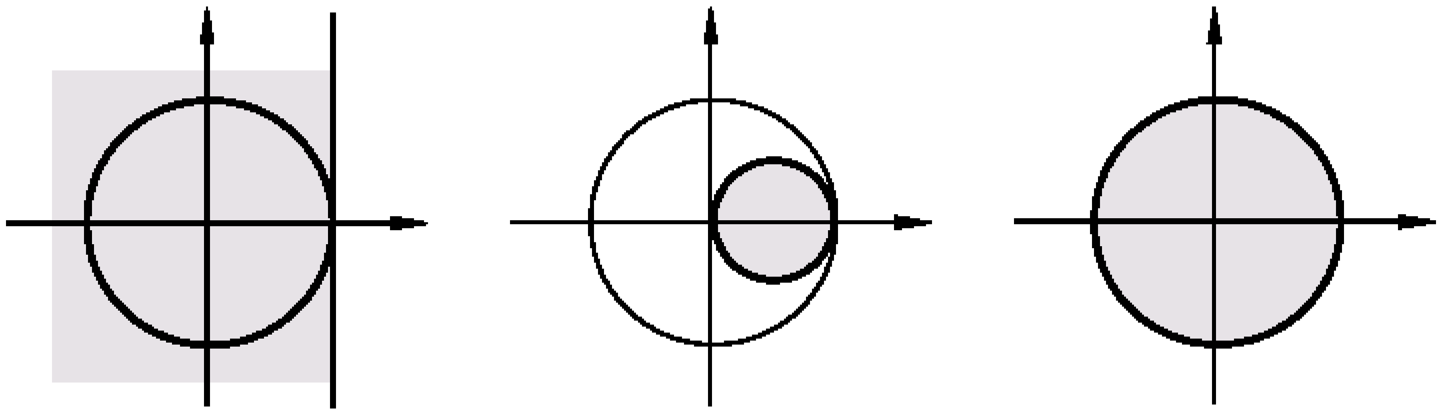
$$s = \frac{z-1}{zh} \quad \text{(Backward difference)}$$

$$s = \frac{2}{h}\,\frac{z-1}{z+1} \quad \text{(Tustin's or bilinear approximation)}$$

- What is the difference between the above approximations and $z = e^{sh}$?

Astrom & Wittenmark 1997

▪ Stability of Approximations:



Forward differences

Backward differences

Tustin

$$s = \frac{z-1}{h} \qquad s = \frac{z-1}{zh} \qquad s = \frac{2}{h}\,\frac{z-1}{z+1}$$

$$z = e^{sh} \qquad \approx 1 + sh \qquad \approx \frac{1}{1-sh} \qquad \approx \frac{1+sh/2}{1-sh/2}$$

Astrom & Wittenmark 1997

- Approximations introduce frequency distortion

$$G_d(z) = G_c(s)\big|_{z=e^{sh}} \qquad G(z) = (1-z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$$= (1-e^{-sh})\frac{1}{s}G_c(s) \qquad s = \frac{2}{h}\frac{z-1}{z+1} \qquad \begin{aligned} z &= e^{iwh} \\ s &= iw \end{aligned}$$

$$\Rightarrow \quad G_d(z=e^{iwh}) = \frac{1}{iw}(1-e^{-iwh})G_c\left(s = \frac{2}{h}\cdot\frac{e^{iwh}-1}{e^{iwh}+1}\right)$$

$$\Rightarrow \quad s = \frac{2}{h}\cdot\frac{e^{iwh}-1}{e^{iwh}+1} = \frac{2}{h}\cdot\frac{e^{iwh/2}-e^{-iwh/2}}{e^{iwh/2}+e^{-iwh/2}}$$

$$= \frac{2i}{h}\cdot\frac{(e^{iwh/2}-e^{-iwh/2})/(2i)}{(e^{iwh/2}+e^{-iwh/2})/(2)}$$

$$s = i\cdot w' \qquad = i\cdot\left(\frac{2}{h}\tan\left(\frac{wh}{2}\right)\right)$$

$$G_d(z=e^{iwh}) = G_c(s=iw) \qquad G_c\left(i\cdot\left(\frac{2}{h}\tan\left(\frac{wh}{2}\right)\right)\right) = G_c\left(i\cdot w'\right)$$

Astrom & Wittenmark 1997

- Approximations introduce frequency distortion

$$G_d(z) = G_c(s)\big|_{z=e^{sh}}$$

$$z = e^{iw} \iff s = iw'$$

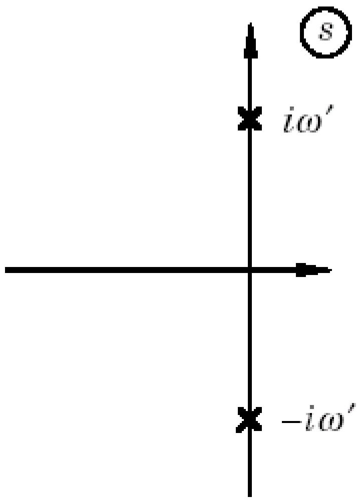
$$\Rightarrow \; w' = \frac{2}{h}\tan\left(\frac{wh}{2}\right)$$

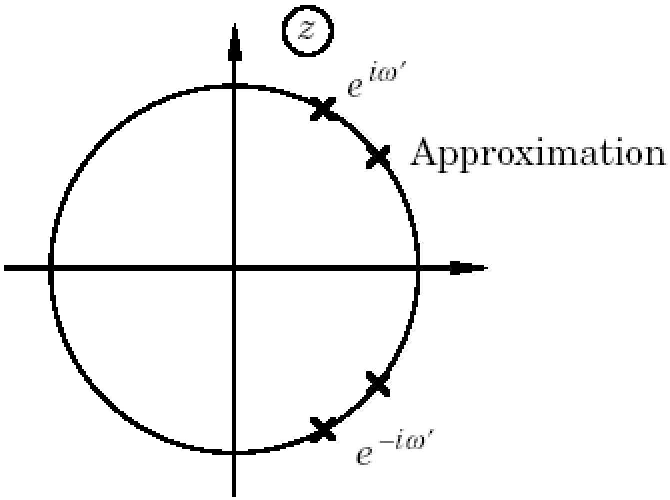
$$\Rightarrow \; w = \frac{2}{h}\tan^{-1}\left(\frac{w'h}{2}\right)$$

$$\approx \; w' - \frac{(w')^3 h^2}{12}$$

Astrom & Wittenmark 1997

- Approximations introduce frequency distortion





$$z = e^{iw}$$

$$w \approx w' - \frac{(w')^3 h^2}{12}$$

Astrom & Wittenmark 1997

Tustin with Prewarping:

$$s' = \frac{w_1}{\tan(w_1 h/2)} \cdot \frac{z-1}{z+1} \qquad\qquad s = \frac{2}{h}\,\frac{z-1}{z+1}$$

$$\Rightarrow \; G_d(e^{iw_1 h}) = G_c(iw_1)$$

- No distortion 0 (always true) and at w_1
- Still distortion at other frequencies

Astrom & Wittenmark 1997

- Matched Pole-Zero (MPZ) method:

$$z = e^{sh}$$

1. Map poles and zeros according to the relation

2. If the numerator is of lower order than the denominator, add powers of (z+1) to the numerator until numerator and denominator are of equal order

3. Set the DC or low-frequency gain of D(z) = that of D(s)

Matched Pole-Zero (MPZ) method:

- Case 1:

$$D(s) = K_c \frac{s+a}{s+b}$$

$$\Rightarrow \quad D(z) = K_d \frac{z-e^{-ah}}{z-e^{-bh}}$$

- By the Final Value Theorem:

$$K_c \frac{a}{b} = K_d \frac{1-e^{-ah}}{1-e^{-bh}}$$

$$\text{or} \quad K_d = K_c \frac{a}{b} \left(\frac{1-e^{-bh}}{1-e^{-ah}} \right)$$

Franklin et al. 2002

Matched Pole-Zero (MPZ) method:

- Case 2:

$$D(s) = K_c \frac{s+a}{s(s+b)}$$

$$\Rightarrow \quad D(z) = K_d \frac{(z+1)(z-e^{-ah})}{(z-1)(z-e^{-bh})}$$

- By the Final Value Theorem:

$$\Rightarrow \quad K_d = K_c \frac{a}{2b}\left(\frac{1-e^{-bh}}{1-e^{-ah}}\right)$$

- Matched Pole-Zero (MPZ) method:
  - The same power of z in the num & den of D(z):

$$\frac{U(z)}{E(z)} = D(z) = K_d \frac{1 - \alpha z^{-1}}{1 - \beta z^{-1}}$$

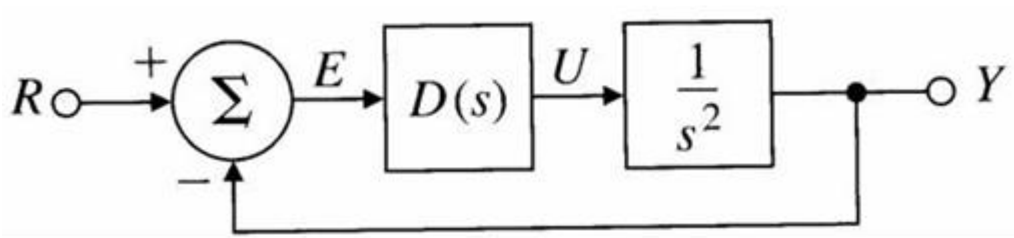
$$\alpha = e^{-ah} \;\; \& \;\; \beta = e^{-bh}$$

$$\Rightarrow \;\; u[k] = \beta u[k-1] + K_d \Big[e[k] - \alpha e[k-1] \Big]$$

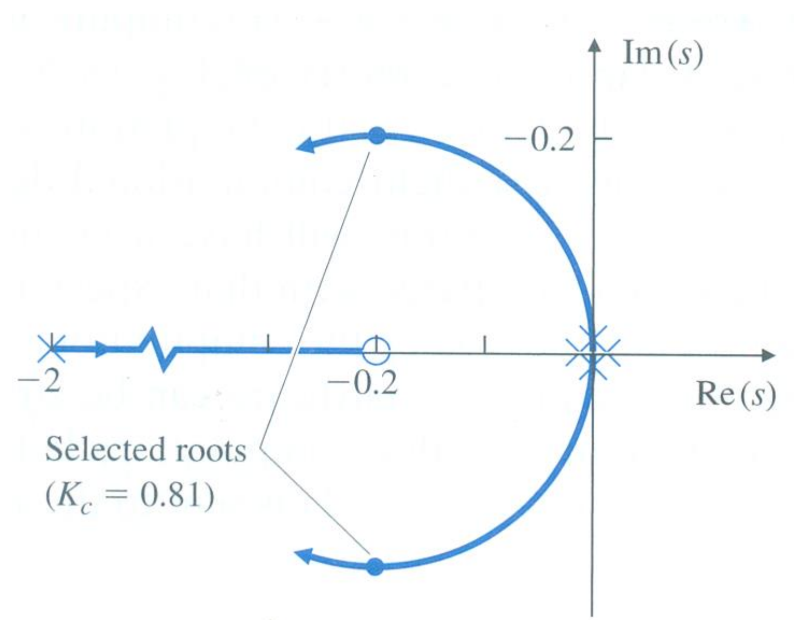
Franklin et al. 2002

Matched Pole-Zero (MPZ) method:

- Space station attitude digital controller



$$w_n \approx 0.3 \text{ rad/sec}$$
$$\zeta = 0.7$$



$$\Rightarrow \quad D(s) = 0.81\,\frac{s+0.2}{s+2}$$

Franklin et al. 2002

- Matched Pole-Zero (MPZ) method:
  - Space station attitude digital controller

$$w_s = 0.3 \times 20 = 6 \quad \text{rad/sec}$$

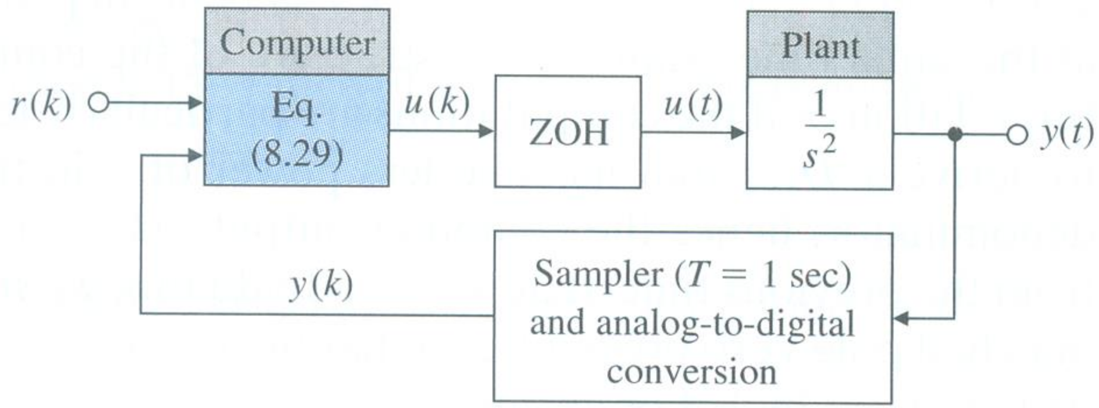
$$\Rightarrow \quad h \approx 1 \text{ sec}$$

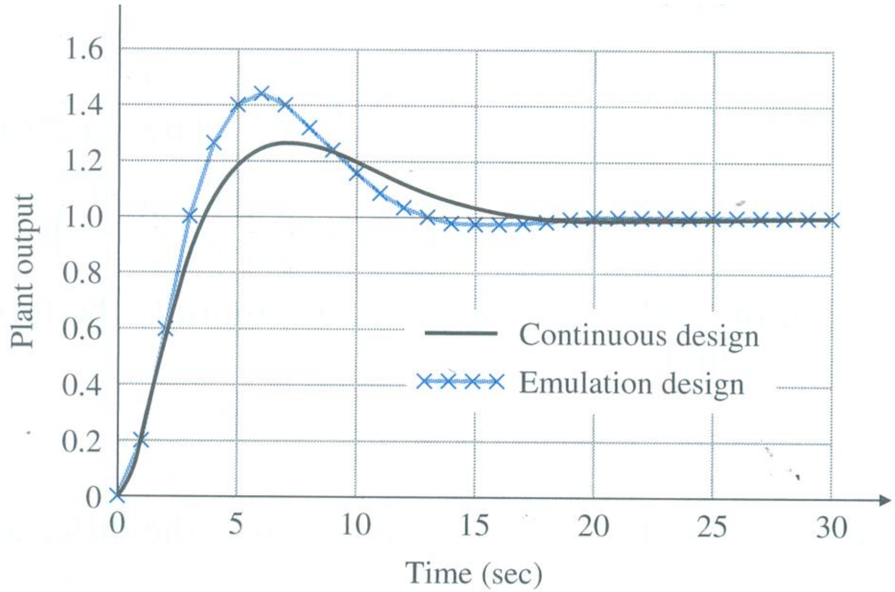
$$\Rightarrow \quad D(z) = 0.389 \frac{z - 0.82}{z - 0.135} = \frac{0.389 - 0.319 z^{-1}}{1 - 0.135 z^{-1}}$$

$$\Rightarrow \quad u[k] = 0.135 u[k-1] + 0.389 e[k] - 0.319 e[k-1]$$

▪ Matched Pole-Zero (MPZ) method:

- Space station attitude digital controller





Franklin et al. 2002

- Modified Matched Pole-Zero (MMPZ) method:
  - u[k+1] depends only on e[k], but not e[k+1]

$$D(s) = K_c \frac{s+a}{s(s+b)}$$

$$\Rightarrow \quad D(z) = K_d \frac{(z-e^{-ah})}{(z-1)(z-e^{-bh})}$$

$$\Rightarrow \quad K_d = K_c \frac{a}{b}\left(\frac{1-e^{-bh}}{1-e^{-ah}}\right)$$

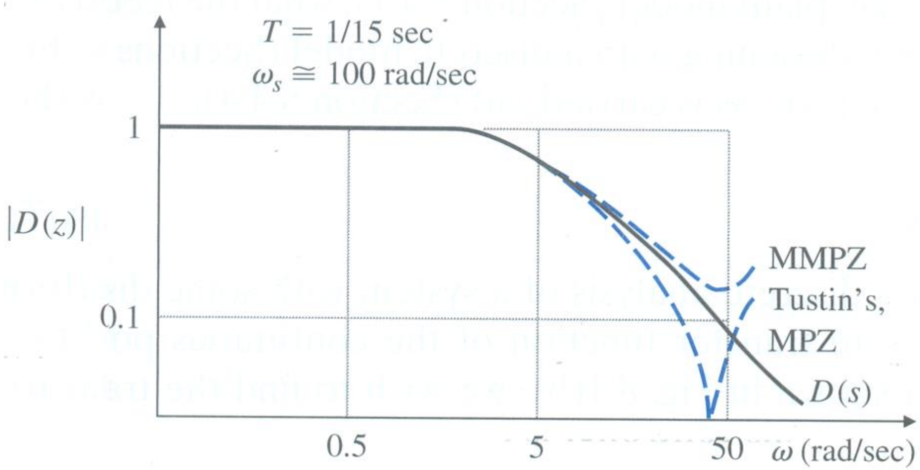
$$\Rightarrow \quad u[k] = (1+e^{-bh})u[k-1] - e^{-bh}u[k-2] + K_d\left[e[k-1] - e^{-ah}e[k-2]\right]$$

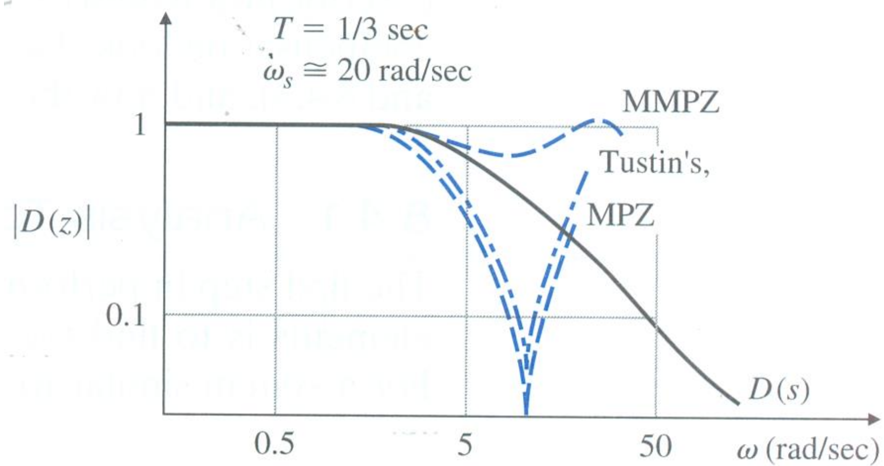
Franklin et al. 2002

▪ Comparison of Digital Approximation Methods:

$$D(s) = \frac{5}{s+5}$$

	w_s	w_s
Method	100 rad/sec	20 rad/sec
MPZ	$0.143\frac{z+1}{z-0.715}$	$0.405\frac{z+1}{z-0.189}$
MMPZ	$0.285\frac{1}{z-0.715}$	$0.811\frac{1}{z-0.189}$
Tustin's	$0.143\frac{z+1}{z-0.713}$	$0.454\frac{z+1}{z-0.0914}$





Franklin et al. 2002

- The "textbook" version of the PID-controller:

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int^t e(s)\,ds + T_d\,\frac{de(t)}{dt}\right) \qquad e = u_c - y$$

- $e = u_c - y$:
  difference between command and output
- K: gain or proportional gain
- T_i: integration time or reset time
- T_d: derivative time

Astrom & Wittenmark 1997

- Modification of Linear Response:
  - A pure derivative cannot be,

    and should not be, implemented:
    - Because amplification of measurement noise
    - Derivative gain should be limited

$$sT_d \approx \frac{sT_d}{1 + sT_d/N}$$

$$s = iw$$

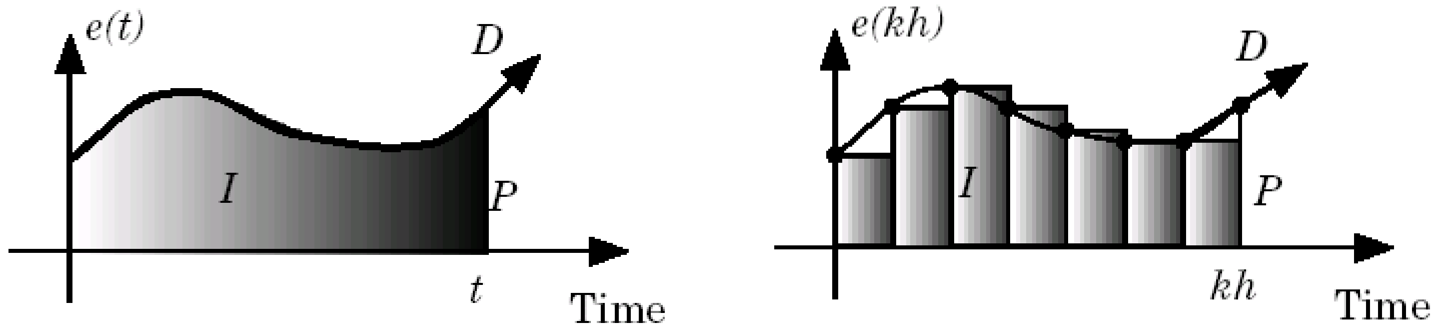
- ω small ---> s Td
- ω large ---> N
- $N = 3 \sim 20$

- Two-input-&-one-output controller:

$$U(s) = K\left(bU_c(s) - Y(s) + \frac{1}{sT_i}\left(U_c(s) - Y(s)\right) - \frac{sT_d}{1 + sT_d/N}Y(s)\right)$$

Astrom & Wittenmark 1997

- Discrete-time PID:



- One popular approximation:

P-part: $P(t) = K(bu_c(t) - y(t))$ (No Approximation)

I-part: $I(kh+h) = I(kh) + \frac{Kh}{T_i} e(kh)$ (Forward)

D-part: $$D(kh) = \frac{T_d}{T_d + Nh} D(kh-h) - \frac{KT_dN}{T_d+Nh}\left(y(kh) - y(kh-h)\right)$$ (Backward)

Astrom & Wittenmark 1997

- Discrete-time PID:

$$u(kh) = P(kh) + I(kh) + D(kh)$$

  - Can be written as:

$$R(q)u(kh) = T(q)u_c(kh) - S(q)y(kh)$$

with $R(q) = (q-1)(q-a_d)$,

$S(q)$, and $T(q)$ of 2nd order

- Coefficients in different approximations:

	Special	Tustin	Ramp Equivalence
s_0	$K(1+b_d)$	$K(1+b_i+b_d)$	
s_1	$-K(1+a_d+2b_d-b_i)$	$-K\Big(1+a_d+2b_d-b_i(1-a_d)\Big)$	
s_2	$K(a_d+b_d-b_ia_d)$	$K(a_d+b_d-b_ia_d)$	
t_0	Kb	$K(b+b_i)$	
t_1	$-K(b(1+a_d)-b_i)$	$-K\Big(b(1+a_d)-b_i(1-a_d)\Big)$	
t_2	$Ka_d(b-b_i)$	$Ka_d(b-b_i)$	
a_d	$\dfrac{T_d}{Nh+T_d}$	$\dfrac{2T_d-Nh}{2T_d+Nh}$	$\exp\left(-\dfrac{Nh}{T_d}\right)$
b_d	Na_d	$\dfrac{2NT_d}{2T_d+Nh}$	$\dfrac{T_d}{h}(1-a_d)$
b_i	$\dfrac{h}{T_i}$	$\dfrac{h}{2T_i}$	$\dfrac{h}{2T_i}$

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