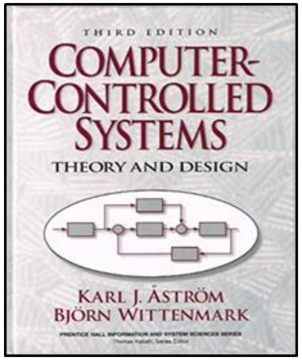
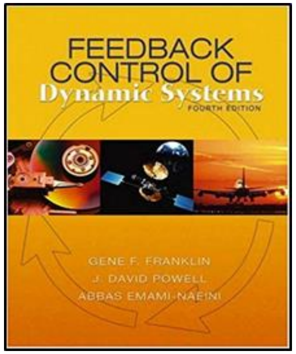
Spring 2021

數位控制系統
Digital Control Systems

DCS-32
Discretized Controller –
Discrete Design





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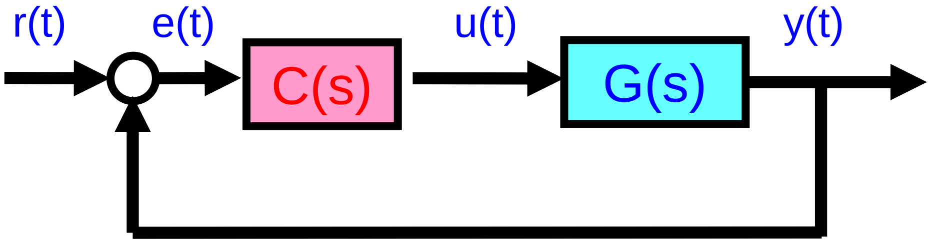
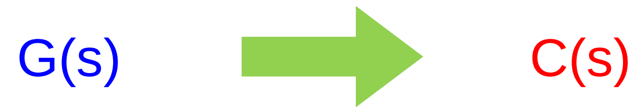
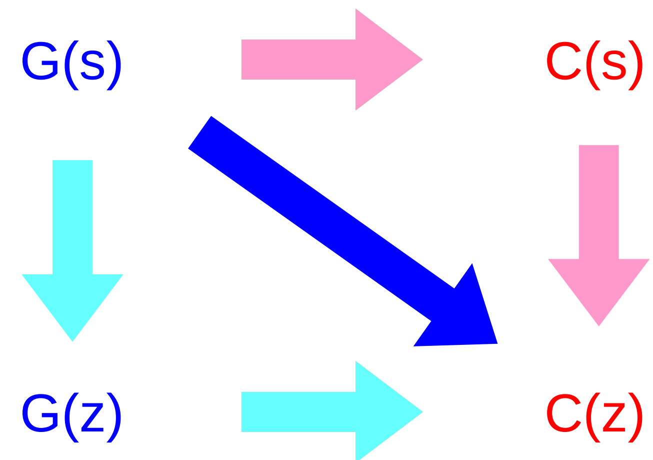
Feb – Jun, 2021

Introduction: CT and DT Plant-Controller





- Direct Design

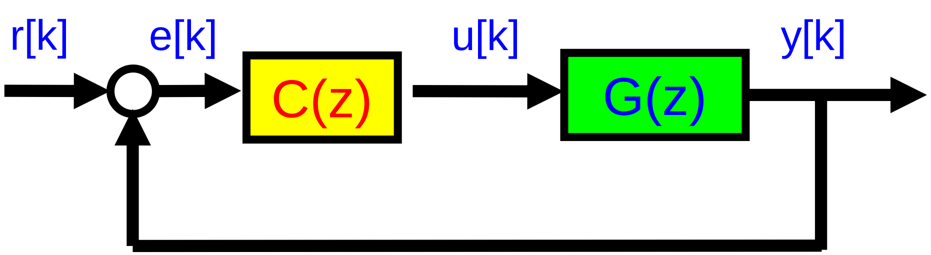


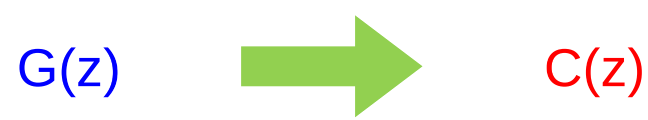
- Discrete Design

- Transform CT plant into DT plant
- By DT plant, design DT controller

- Emulation

- By CT plant, design CT controller
- Transform CT controller into DT controller





- Basic principles of low-order controller design

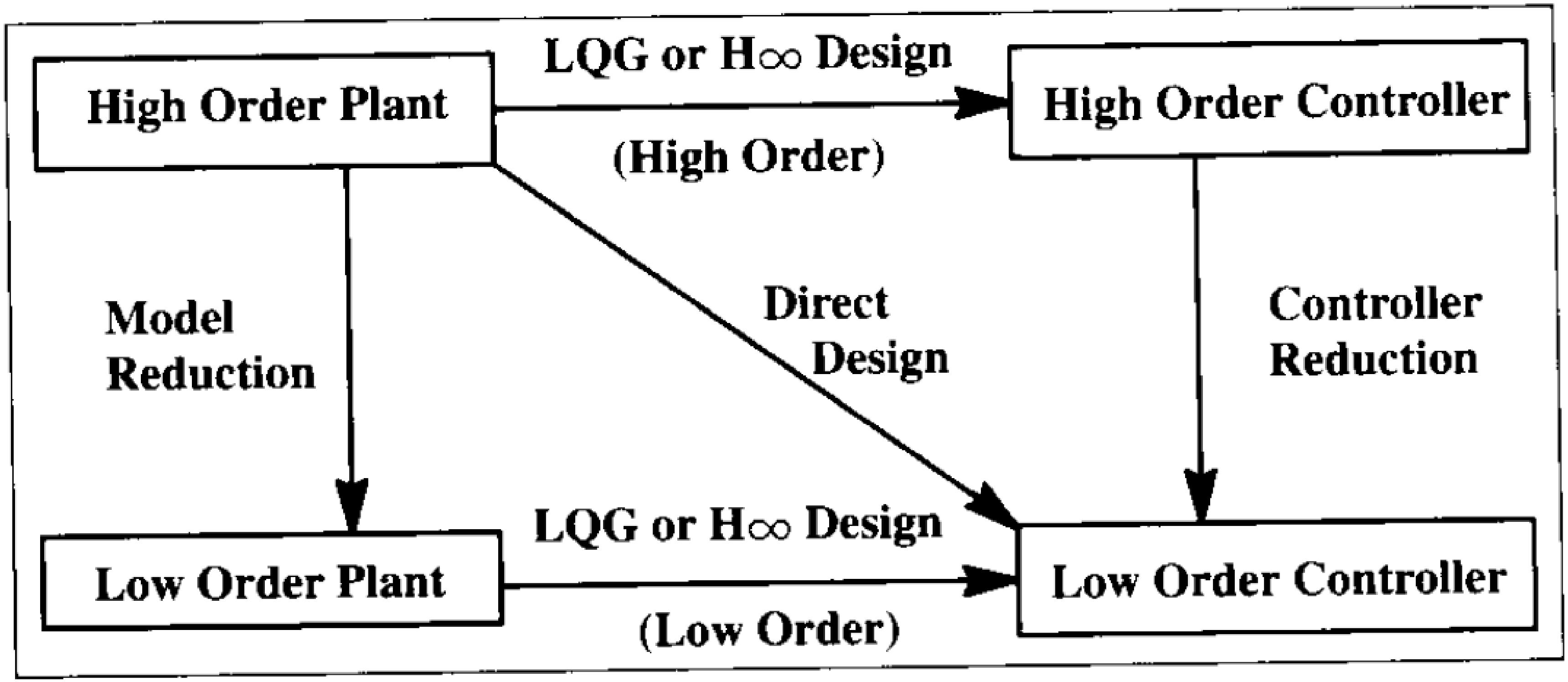


Fig. 1. Basic principles of low order controller design.

B.D.O. Anderson, "Controller Design: Moving from Theory to Practice," IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993

Anderson 1993

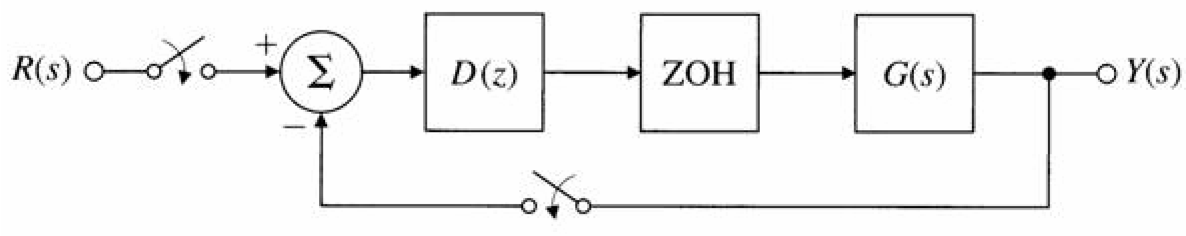
- Study in Digital Control Systems
  - Controller Design of Digital Control Systems
    - **Design Process**
      - > Discrete Design:
        - » CT plant -> DT plant -> DT controller
      - > Emulation:
        - » CT plant -> CT controller -> DT controlle
      - > Direct Design: (B.D.O. Anderson, 1992 Bode Prize Lecture)
        - » CT plant -> DT controller

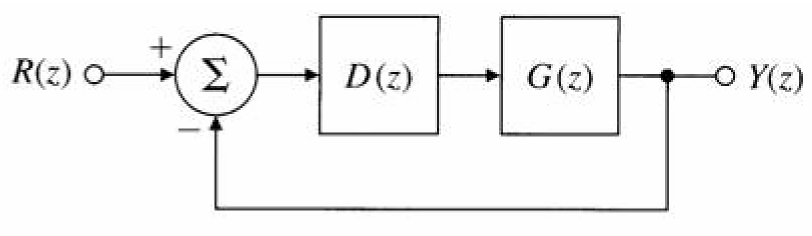
- Discrete Design
  - By Transfer Function
  - By State Space

- Design by Emulation
  - Tustin's Method or bilinear approximation
  - Matched Pole-Zero method (MPZ)
  - Modified Matched Pole-Zero method (MMPZ)
  - Digital PID-Controllers

- Techniques for Enhancing the Performance

- The Exact Discrete Equivalent:





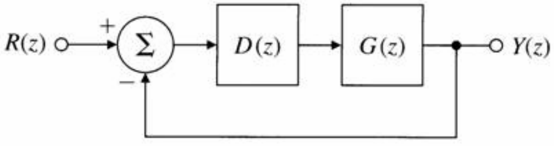
$$G(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$$\Rightarrow \frac{Y(z)}{R(z)} = \frac{D(z)G(z)}{1 + D(z)G(z)}$$

Franklin et al. 2002

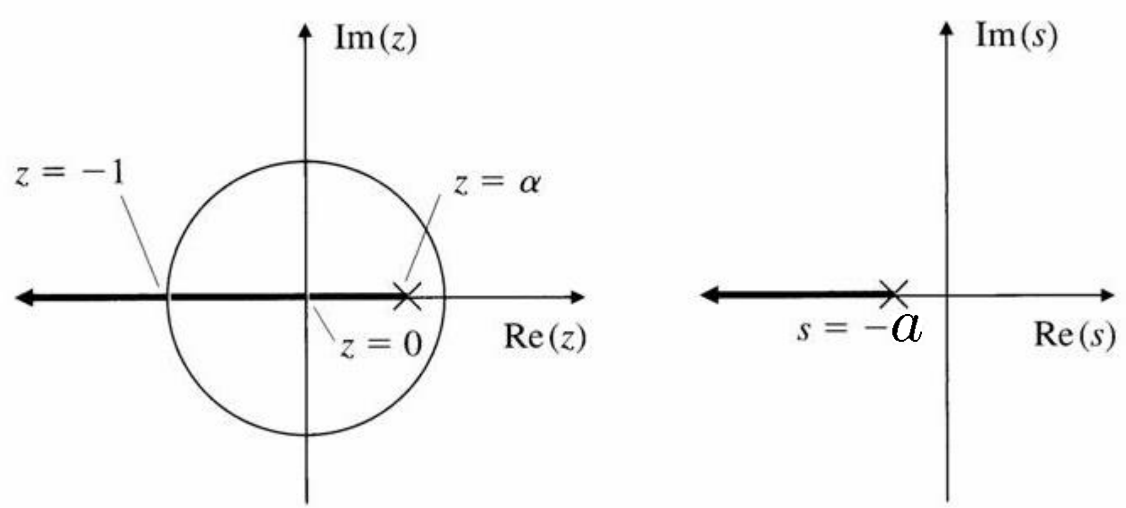
- Discrete Root Locus: $G(s) = \dfrac{a}{s+a}$ & $D(z) = K$

$$\Rightarrow G(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{a}{s(s+a)}\right\}$$



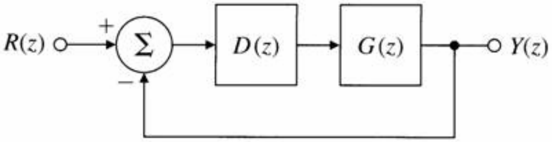
$$= (1 - z^{-1})\left[\frac{(1-e^{-ah})z^{-1}}{(1-z^{-1})(1-e^{-ah}z^{-1})}\right]$$

$$= \frac{1-e^{-ah}}{z-e^{-ah}} \qquad \alpha = e^{-ah}$$



Franklin et al. 2002

▪ Feedback Properties:



- Proportional: $u[k] = K\, e[k] \quad \Rightarrow \quad D(z) = K$

- Derivative: $u[k] = K\, T_D \left[e[k] - e[k-1]\right]$

$$\Rightarrow \quad D(z) = K\, T_D \left(1 - z^{-1}\right) = k_D \frac{z-1}{z}$$

- Integral: $u[k] = u[k-1] + \dfrac{K_p}{T_I}\, e[k]$

$$\Rightarrow \quad D(z) = \frac{K}{T_I} \left(\frac{1}{1 - z^{-1}}\right) = k_I \frac{z}{z-1}$$

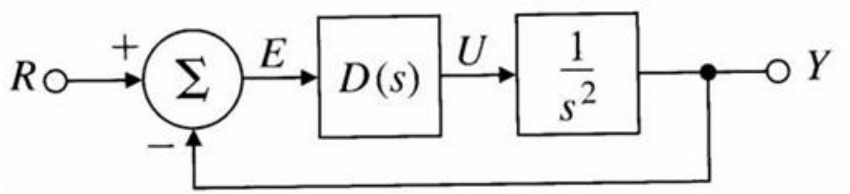
- Lead Compensation:

$$u[k+1] = \beta u[k] + K \left[e[k+1] - \alpha e[k]\right]$$

$$\Rightarrow \quad D(z) = K \frac{1 - \alpha z^{-1}}{1 - \beta z^{-1}}$$

Franklin et al. 2002

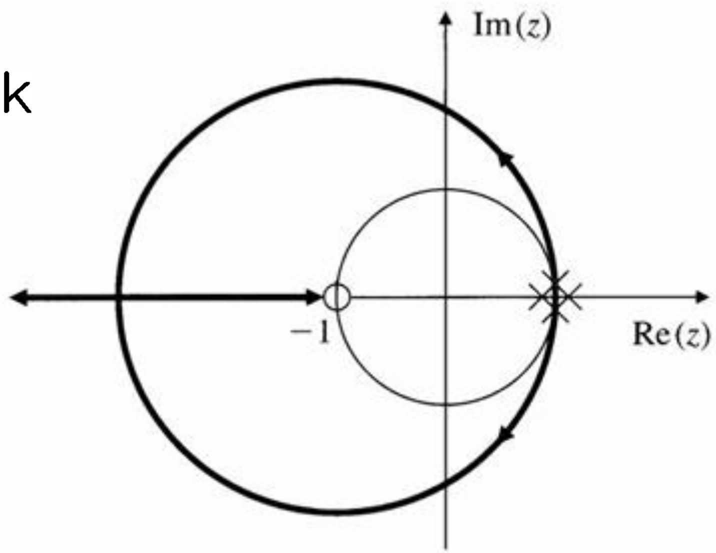
- Space Station Digital Controller example:



$$h = 1$$

$$G(z) = \frac{h^2}{2}\left[\frac{z+1}{(z-1)^2}\right] = \frac{1}{2}\left[\frac{z+1}{(z-1)^2}\right]$$

with proportional feedback



Franklin et al. 2002

- Space Station Digital Controller example:
  - P + D feedback

$$\Rightarrow \quad U(z) = K\left[1 + T_D\left(1 - z^{-1}\right)\right] E(z)$$

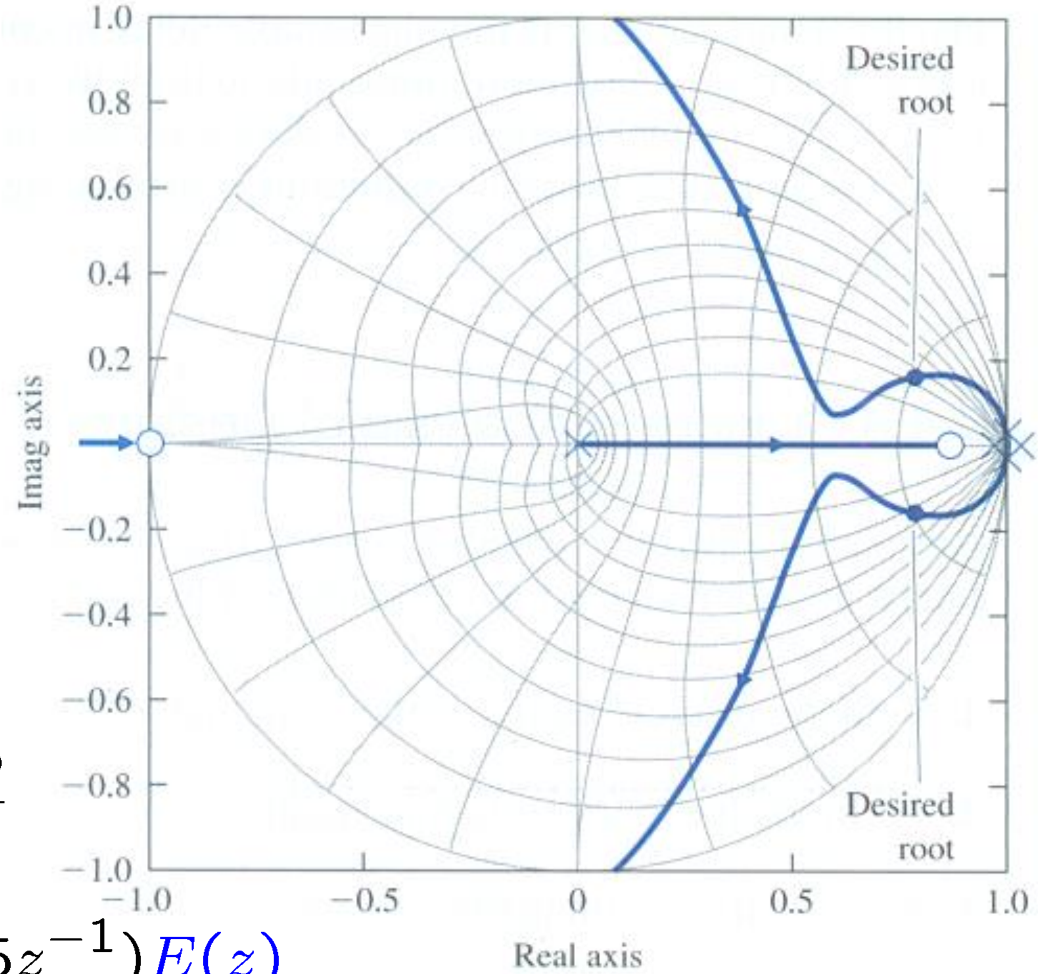
$$\Rightarrow \quad D(z) = K\,\frac{z - \alpha}{z}$$

$$w_n \approx 0.3 \text{ rad/sec}$$

$$\zeta = 0.7$$

$$\Rightarrow \quad z = 0.78 \pm 0.18\,j$$

- Space Station Digital Controller example:
  - P + D feedback



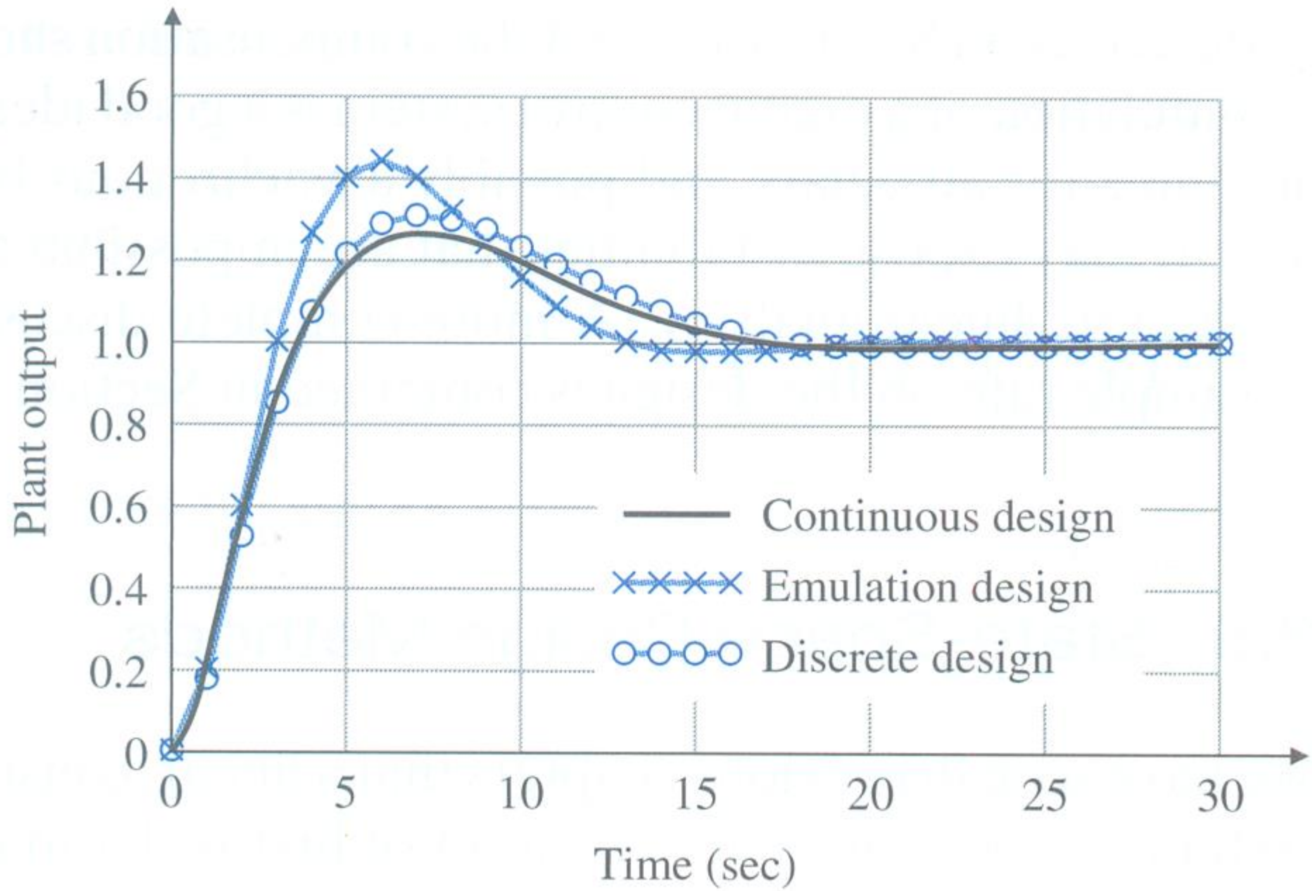
$$\Rightarrow \quad D(z) = 0.374 \frac{z - 0.85}{z}$$

$$\Rightarrow \quad U(z) = 0.374(1 - 0.85z^{-1})E(z)$$

$$\Rightarrow \quad u[k+1] = 0.374e[k+1] - 0.318e[k]$$

Franklin et al. 2002

- Step Response of the continuous & digital systems:



ζ	w_n
0.705	0.324
0.645	0.441
0.733	0.306

Franklin et al. 2002

- State-Space Model:

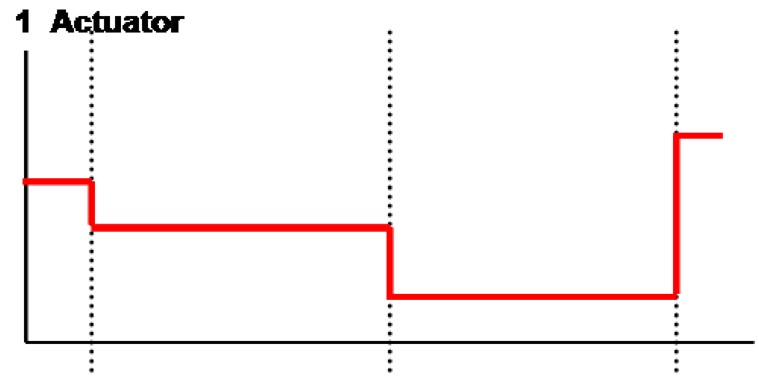
$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)\\ y(t) &= \mathbf{C}\mathbf{x}(t) + Du(t)\end{aligned}$$

$$\Rightarrow \quad \mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau$$

Let $t = kh + h$ & $t_0 = kh$

$$\Rightarrow \quad \mathbf{x}(kh+h) = e^{\mathbf{A}(h)}\mathbf{x}(kh) + \int_{kh}^{kh+h} e^{\mathbf{A}(kh+h-\tau)}\mathbf{B}u(\tau)d\tau$$

▪ State-Space Model:



Let $u(\tau)$ be piecewise constant through h

$$u(\tau) = u(kh), \quad kh \le \tau < kh + h$$

Let $\eta = kh + h - \tau$

$$\Rightarrow \mathbf{x}(kh+h) = e^{\mathbf{A}h}\mathbf{x}(kh) + \left(\int_0^h e^{\mathbf{A}\eta} d\eta\right) \mathbf{B} u(kh)$$

Let $\mathbf{F} = e^{\mathbf{A}h}$ & $\mathbf{H} = \left(\int_0^h e^{\mathbf{A}\eta} d\eta\right) \mathbf{B}$

Then,
$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] + Du[k] \end{aligned}$$

Franklin et al. 2002

- Discrete Transfer Function:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] \end{aligned}$$

$$\begin{aligned} z\mathbf{X}(z) &= \mathbf{F}\mathbf{X}(z) + \mathbf{H}U(z) \\ Y(z) &= \mathbf{C}\mathbf{X}(z) \end{aligned}$$

$$\left(z\mathbf{I} - \mathbf{F}\right)\mathbf{X}(z) = \mathbf{H}\,U(z)$$

$$\Rightarrow \frac{Y(z)}{U(z)} = G(z) = \mathbf{C}\left(z\mathbf{I} - \mathbf{F}\right)^{-1}\mathbf{H}$$

Franklin et al. 2002

- Discrete SS Model of 1/s^2:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

$$\mathbf{F} = e^{\mathbf{A}h} = \mathbf{I} + \mathbf{A}h + \frac{\mathbf{A}^2 h^2}{2!} + \cdots$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} h + \frac{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^2 h^2}{2!} + \cdots$$
$$= \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$$

Franklin et al. 2002

- Discrete SS Model of 1/s^2:

$$\mathbf{H} = \left(\int_0^h e^{\mathbf{A}\eta} d\eta\right)\mathbf{B} = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k h^{k+1}}{(k+1)!}\mathbf{B}$$

$$= \left(\mathbf{I} + \mathbf{A}\frac{h}{2!}\right) h\mathbf{B}$$

$$= \left(\begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \frac{h^2}{2}\right) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} h^2/2 \\ h \end{bmatrix}$$

$$\Rightarrow \; G(z) = \frac{Y(z)}{U(z)} = \mathbf{C}\left(z\mathbf{I} - \mathbf{F}\right)^{-1}\mathbf{H} = \frac{h^2}{2}\left[\frac{z+1}{(z-1)^2}\right]$$

Franklin et al. 2002

- Pole Placement by State Feedback:

$$\alpha(z) = \det\left(z\mathbf{I} - \mathbf{F}\right)$$

If the system is controllable

$$\mathcal{C} = \left[\begin{array}{cccccc} \mathbf{H} & \mathbf{FH} & \mathbf{F}^2\mathbf{H} & \cdots & \mathbf{F}^{n-1}\mathbf{H} \end{array}\right] \quad \text{is full-rank}$$

$$u[k] = -\mathbf{K}\mathbf{x}[k]$$

$$\Rightarrow \det\left(z\mathbf{I} - \mathbf{F} + \mathbf{H}\mathbf{K}\right) = \alpha_c(z)$$

Franklin et al. 2002

Discrete Full-Order Estimator:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] \end{aligned}$$

$$\bar{\mathbf{x}}[k+1] = \mathbf{F}\bar{\mathbf{x}}[k] + \mathbf{H}u[k] + \mathbf{L}\left[y[k] - \mathbf{C}\bar{\mathbf{x}}[k]\right]$$

$$\tilde{\mathbf{x}}[k+1] = \left(\mathbf{F} - \mathbf{L}\mathbf{C}\right)\tilde{\mathbf{x}}[k] \qquad (\tilde{\mathbf{x}} = \mathbf{x} - \bar{\mathbf{x}})$$

If the system is observable, $\mathcal{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{F} \\ \mathbf{C}\mathbf{F}^2 \\ \vdots \\ \mathbf{C}\mathbf{F}^{n-1} \end{bmatrix}$ is full-rank

$$\Rightarrow \det\left(z\mathbf{I} - \mathbf{F} + \mathbf{L}\mathbf{C}\right) = \alpha_e(z)$$

Franklin et al. 2002

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