

Spring 2021

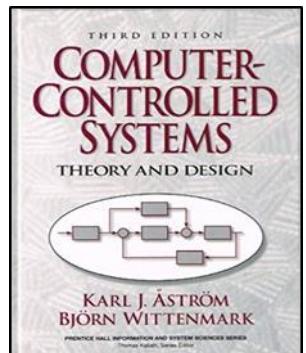
數位控制系統
Digital Control Systems

DCS-21
A Design Example

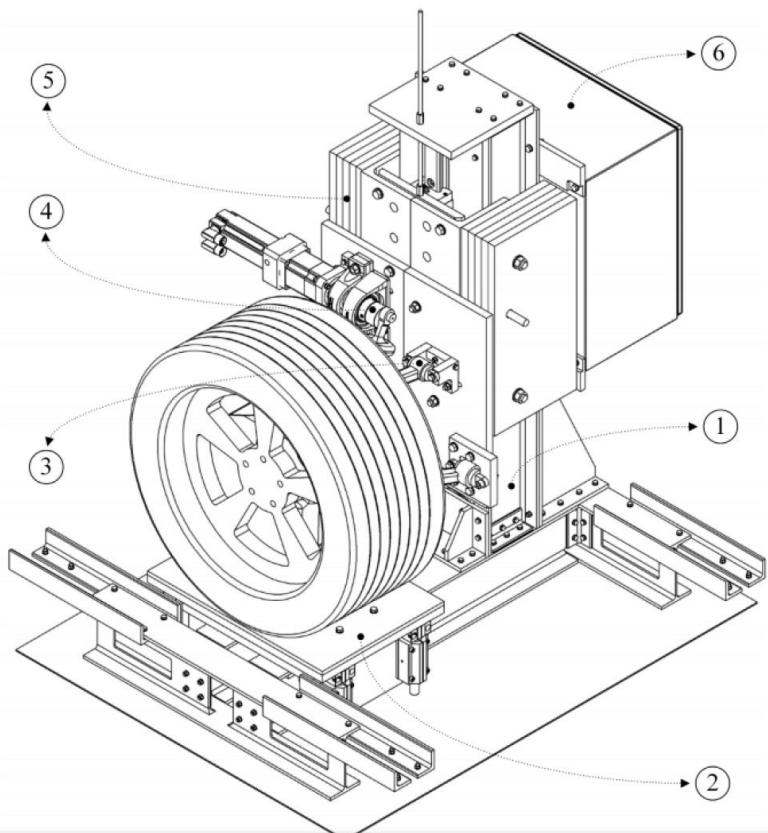
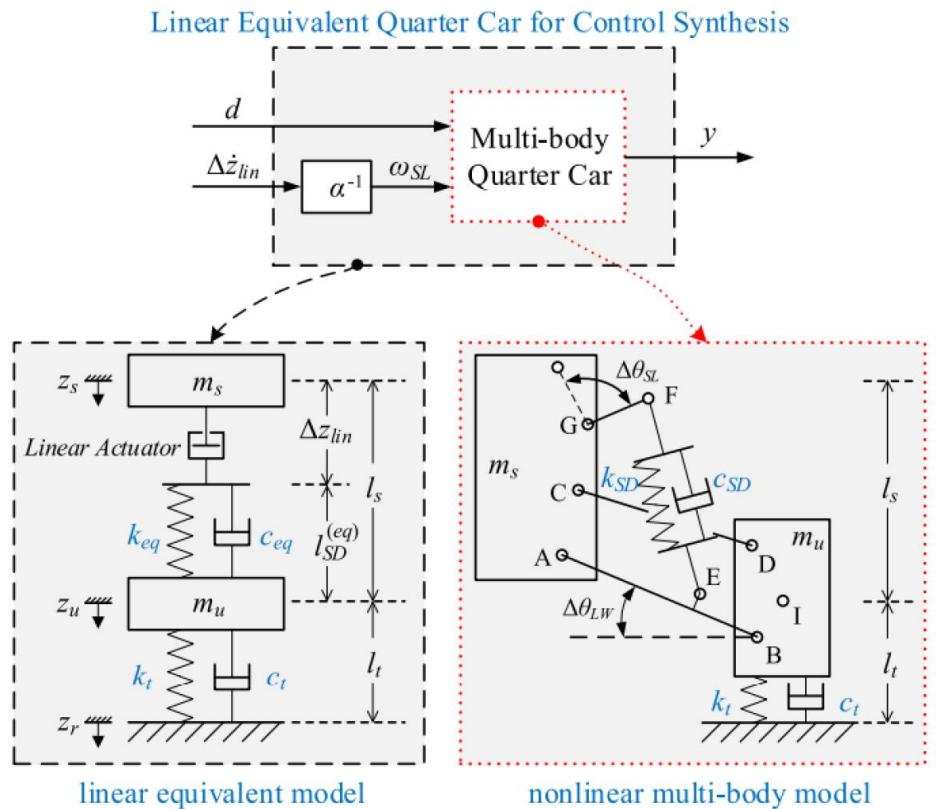
Feng-Li Lian

NTU-EE

Feb – Jun, 2021



Problem, Model, Analysis, and Design

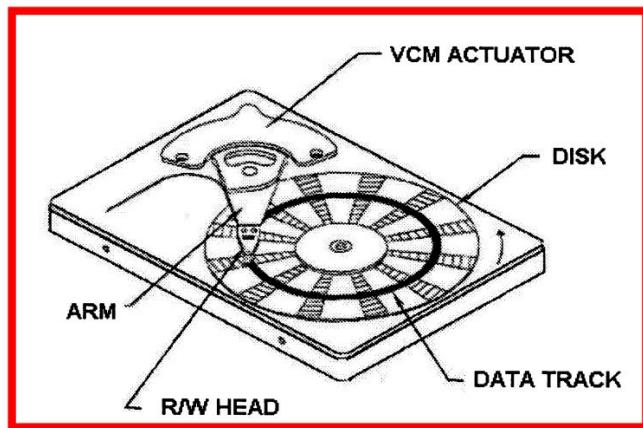


$$m_s \ddot{z}_s = k_{eq}(\Delta l_s - \Delta z_{lin}) + c_{eq}(\dot{l}_s - \dot{z}_{lin})$$

$$m_u \ddot{z}_u = -k_{eq}(\Delta l_s - \Delta z_{lin}) - c_{eq}(\dot{l}_s - \dot{z}_{lin}) + k_t \Delta l_t + c_t \dot{l}_t.$$

- Min Yu , Carlos Arana, Simos A. Evangelou, and Daniele Dini
- Quarter-Car Experimental Study for Series Active Variable Geometry Suspension
- IEEE T. on Control Systems Technology, 27(2):743-759, Mar. 2019

- Discrete-time composite nonlinear feedback control with an application in design of a hard disk drive servo system
 - V. Venkataraman;Kemao Peng;B.M. Chen;T.H. Lee
 - IEEE Transactions on Control Systems Technology
 - Year: 2003 | Volume: 11, Issue: 1 | Journal Article | Publisher: IEEE



$$\begin{pmatrix} \dot{y} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ a \end{pmatrix} u \quad (1)$$

$$G_{v1}(s) = \frac{a}{s^2}.$$

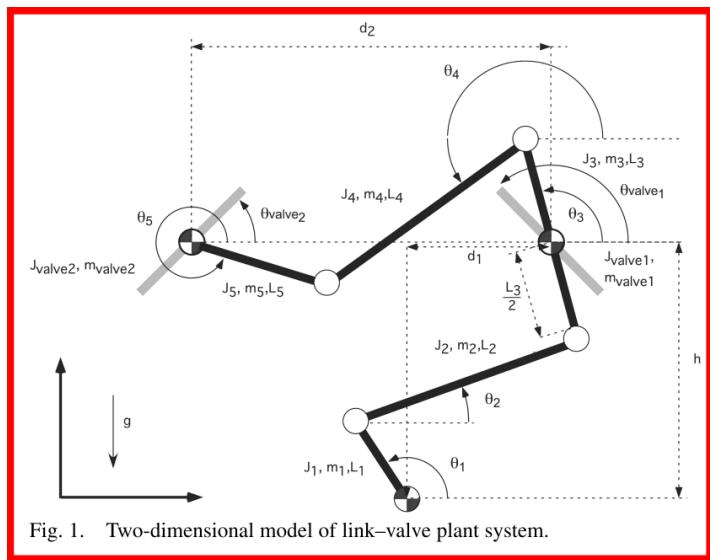
$$G_v(s) = \frac{a}{s^2} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

$$G_v(s) = \frac{6.4013 \times 2.467 \times 10^{15}}{s^2(s^2 + 2.513 \times 10^3 s + 2.467 \times 10^8)}. \quad (4)$$

$$\begin{cases} x(k+1) = A x(k) + B \text{ sat}[u(k)], \\ y(k) = C_2 x(k) \end{cases} \quad x(0) = x_0 \quad (5)$$

- Observer-based discrete-time sliding mode throttle control for drive-by-wire operation of a racing motorcycle engine

- A. Beghi;L. Nardo;M. Stevanato
- IEEE Transactions on Control Systems Technology
- Year: 2006 | Volume: 14, Issue: 4 | Journal Article | Publisher: IEEE



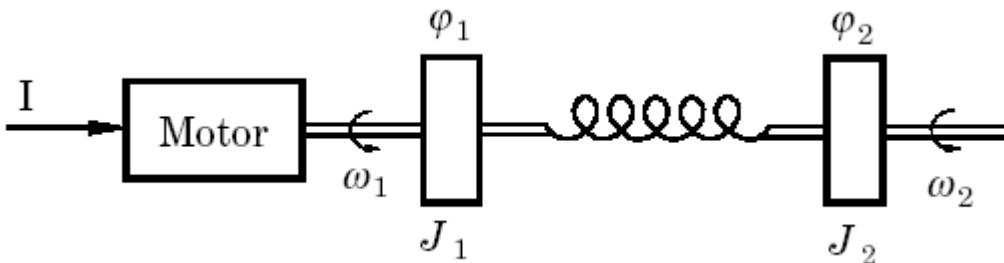
$$Ri_m + L \frac{di_m}{dt} = v_m - K_i \dot{\theta}_m \quad (1)$$

$$\begin{aligned} J_{\text{Tot}_{\text{eq}}}(\theta_m) \ddot{\theta}_m + a(\theta_m) \dot{\theta}_m^2 + G(\theta_m) + F_{\text{Tot}_{\text{eq}}}(\theta_m, \dot{\theta}_m) \\ + K_{\text{Tot}_{\text{eq}}}(\theta_m) \theta_m + \tau_{\text{Tot}_{\text{prel}}} = K_i i_m \end{aligned} \quad (2)$$

$$x_{k+1} = Ax_k + Bu_k + d_k. \quad (5)$$

$$\begin{cases} x_{k+1}^d = Ax_k^d + Bu_k^d \\ u_k^d = -Kx_k^d + \gamma r_k \\ y_k^d = Cx_k^d. \end{cases} \quad (6)$$

■ A Flexible Robot Arm:



■ CT Input-Output Model:

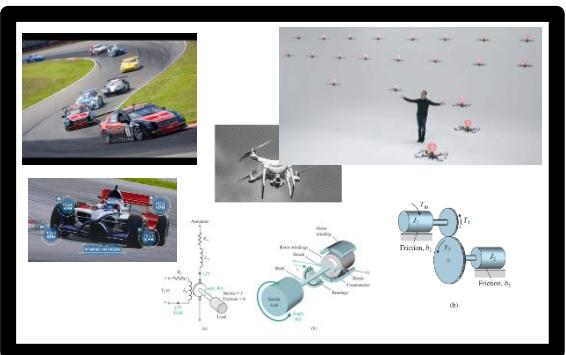
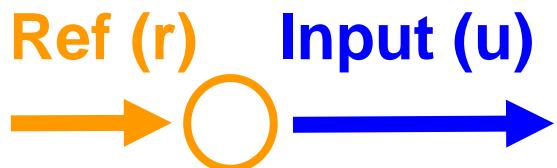
$$\frac{\text{Output}}{\text{Input}} = \frac{w_2}{I} = \frac{B(s)}{A(s)} = G(s)$$

■ CT State-Space Model:

$$\begin{array}{lcl} x_1 & = & \dot{\varphi}_1 - \dot{\varphi}_2 \\ x_2 & = & w_1 \\ x_3 & = & w_2 \end{array} \quad \begin{array}{lcl} \frac{dx}{dt} & = & \mathbf{A} \mathbf{x} + \mathbf{B} u \\ y & = & \mathbf{C} \mathbf{x} \end{array}$$

- Stability:
- Plant Poles & Zeros: $G(s) = \frac{B(s)}{A(s)}$ $\Rightarrow p_{p1}, p_{p2}, \dots$
- Plant Eigenvalues: $\frac{d\mathbf{x}}{dt} = \mathbf{A} \mathbf{x} + \mathbf{B} u$
 $y = \mathbf{C} \mathbf{x}$ $\Rightarrow \lambda_{p1}, \lambda_{p2}, \dots$
- Characteristics:
 - \Rightarrow Damping Ratio: ζ_p
 - \Rightarrow Natural Frequency: w_p
- Root Locus & Bode Plot
- Impulse Response & Step Response:

Signals & Systems

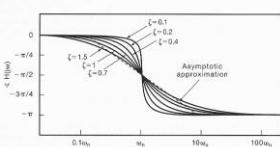
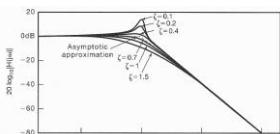
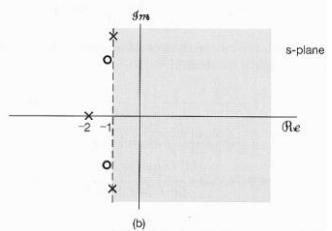


Output (y)

→

$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3 y(t) = 5 u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s - 3}$$



- Design Specifications:
 - ζ_d, w_d

- Sampling Time:
$$\Rightarrow w_N > (10 \sim 20)w_d$$

$$\Rightarrow h = \frac{2\pi}{w_s} \quad w_s = 2w_N$$

- DT Models:

$$G(z) = \frac{B(z)}{A(z)}$$

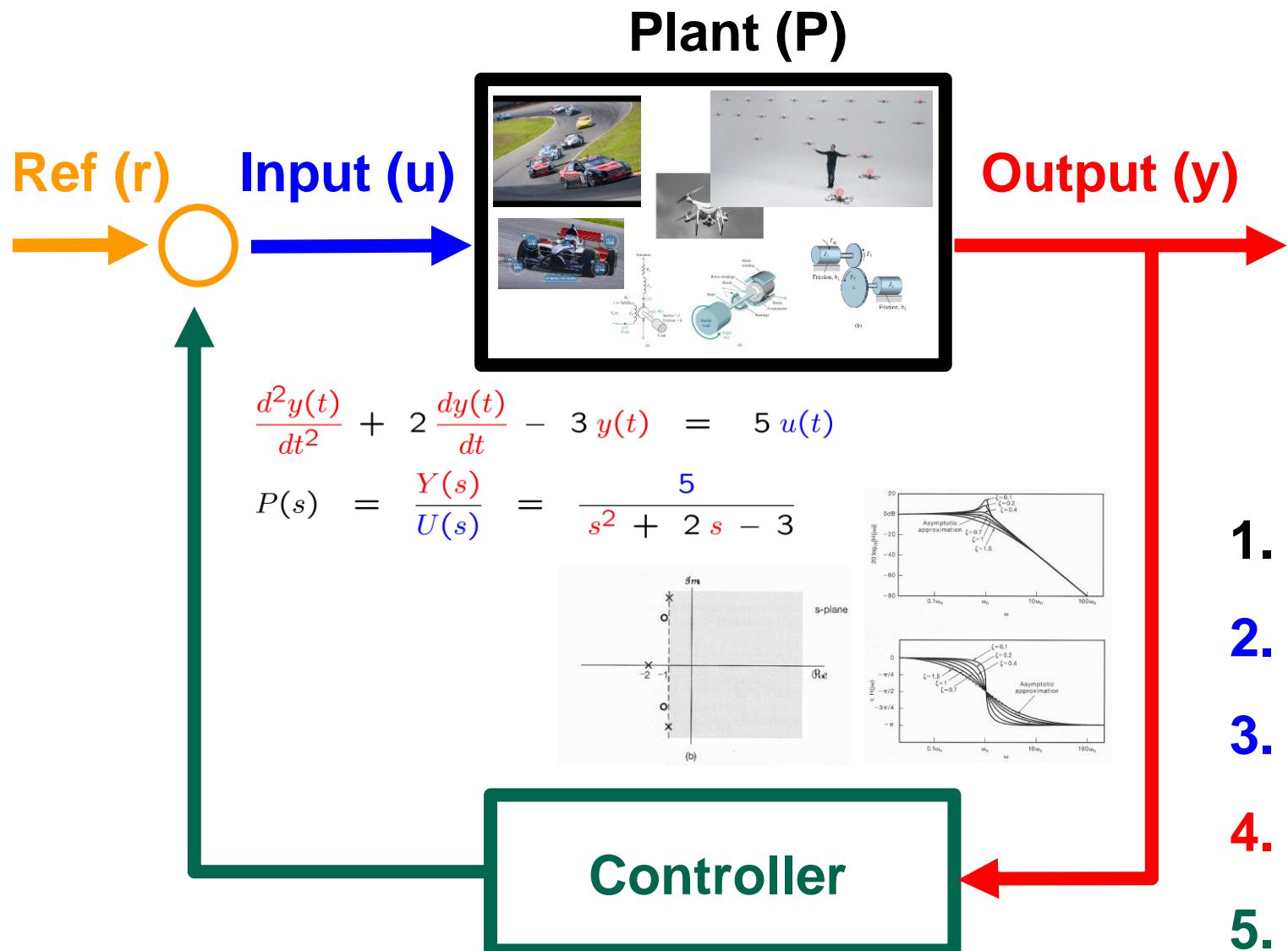
$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{F} \mathbf{x}(k) + \mathbf{H} u(k) \\ y(k) &= \mathbf{C} \mathbf{x}(k) \end{aligned}$$

- Desired Poles & Zeros:
$$\Rightarrow p_{d1}, p_{d2}, \dots$$

- Desired Eigenvalues:
 - ζ_d, w_d

$$\Rightarrow \lambda_{d1}, \lambda_{d2}, \dots$$

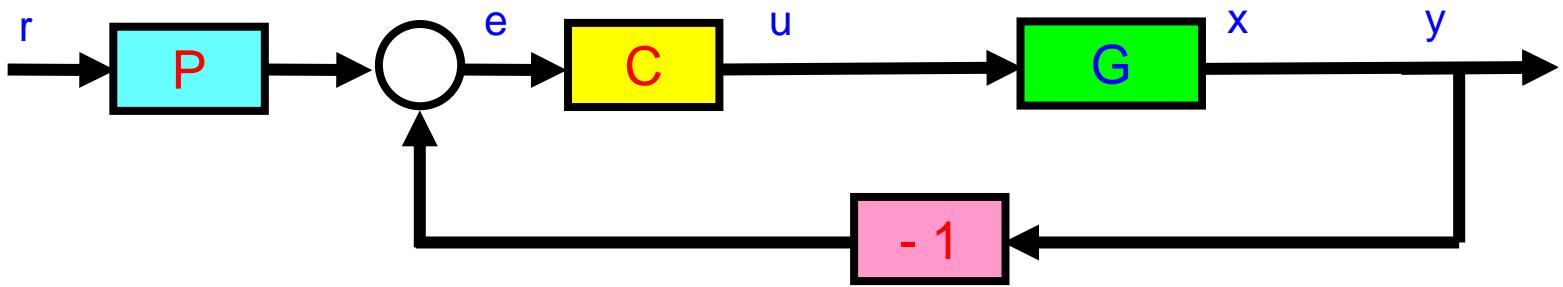
Signals & Systems



Control Systems

1. Model
2. Response
3. Analysis
4. Feedback
5. Control

■ Block Diagram of a Typical Control System:



$$G(z) = \frac{B(z)}{A(z)}$$

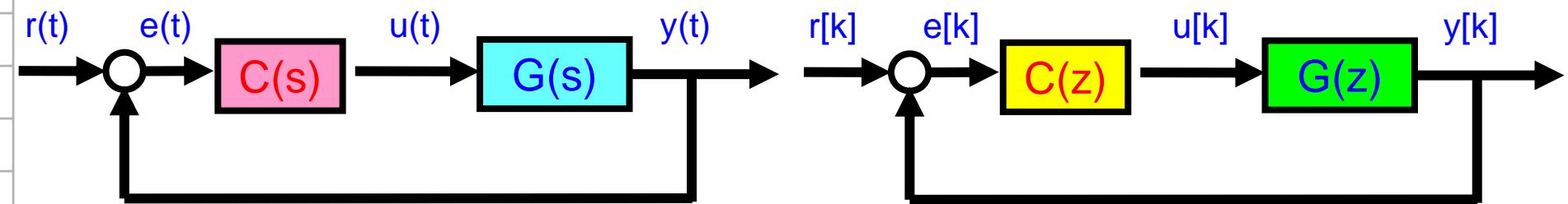
$$p_{p1}, p_{p2}, \dots \Rightarrow p_{d1}, p_{d2}, \dots$$

$$\mathbf{x}(k+1) = \mathbf{F} \mathbf{x}(k) + \mathbf{H} u(k)$$

$$y(k) = \mathbf{C} \mathbf{x}(k)$$

$$\lambda_{p1}, \lambda_{p2}, \dots \Rightarrow \lambda_{d1}, \lambda_{d2}, \dots$$

Issues: Sampling Times



$$G(s) = \frac{B(s)}{A(s)}$$

$$\begin{aligned} \frac{dx}{dt} &= \mathbf{A} x + \mathbf{B} u \\ y &= \mathbf{C} x \end{aligned}$$

$h_1 \Rightarrow$

$$G_1(z) = \frac{B_1(z)}{A_1(z)}$$

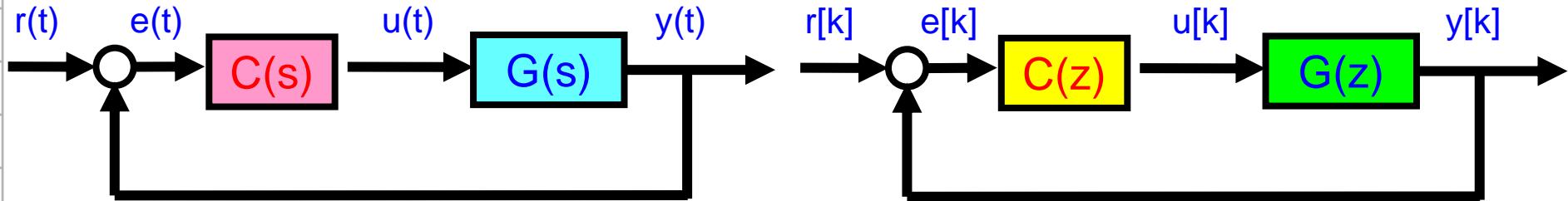
$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{F}_1 \mathbf{x}(k) + \mathbf{H}_1 u(k) \\ y(k) &= \mathbf{C}_1 \mathbf{x}(k) \end{aligned}$$

$h_2 \Rightarrow$

$$G_2(z) = \frac{B_2(z)}{A_2(z)}$$

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{F}_2 \mathbf{x}(k) + \mathbf{H}_2 u(k) \\ y(k) &= \mathbf{C}_2 \mathbf{x}(k) \end{aligned}$$

Issues: Sampling Times



$$G_1(s) = \frac{B_1(s)}{A_1(s)}$$

$h_1 \Rightarrow$

$$\begin{aligned} \frac{dx}{dt} &= \mathbf{A}_1 x + \mathbf{B}_1 u \\ y &= \mathbf{C}_1 x \end{aligned}$$

$$G(z) = \frac{B(z)}{A(z)}$$

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{F} \mathbf{x}(k) + \mathbf{H} u(k) \\ y(k) &= \mathbf{C} \mathbf{x}(k) \end{aligned}$$

$$G_2(s) = \frac{B_2(s)}{A_2(s)}$$

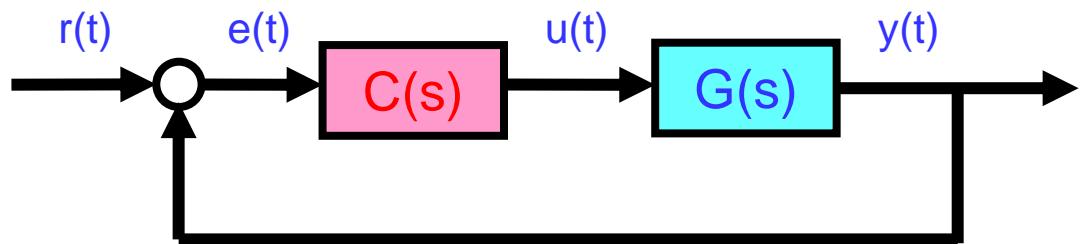
$h_2 \Rightarrow$

$$\begin{aligned} \frac{dx}{dt} &= \mathbf{A}_2 x + \mathbf{B}_2 u \\ y &= \mathbf{C}_2 x \end{aligned}$$

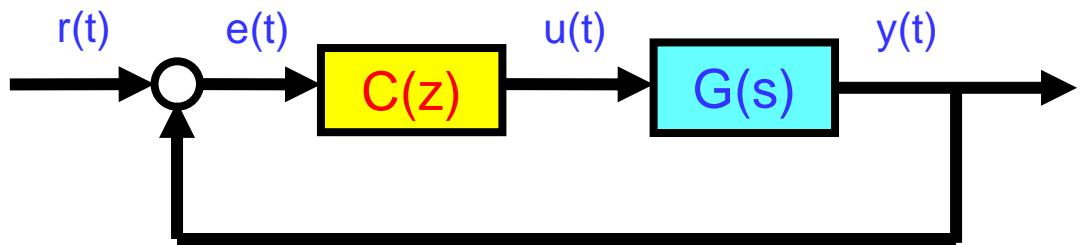
$$G(z) = \frac{B(z)}{A(z)}$$

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{F} \mathbf{x}(k) + \mathbf{H} u(k) \\ y(k) &= \mathbf{C} \mathbf{x}(k) \end{aligned}$$

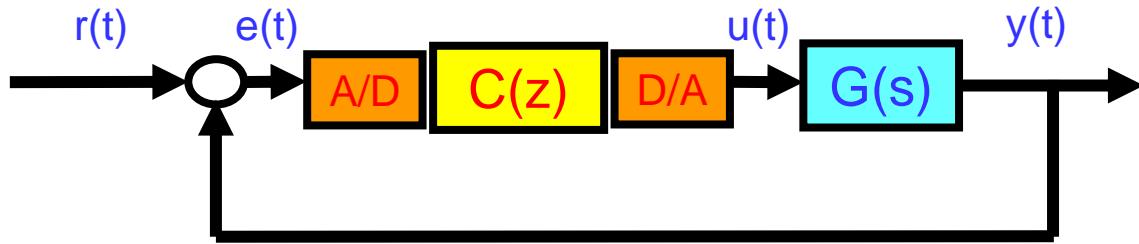
Issues: Analog Control and Digital Control



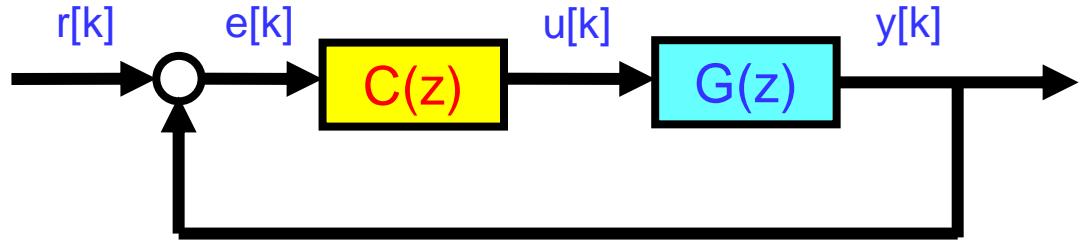
$$G(s) \rightarrow C(s)$$



$$G(s) \rightarrow C(z) ???$$



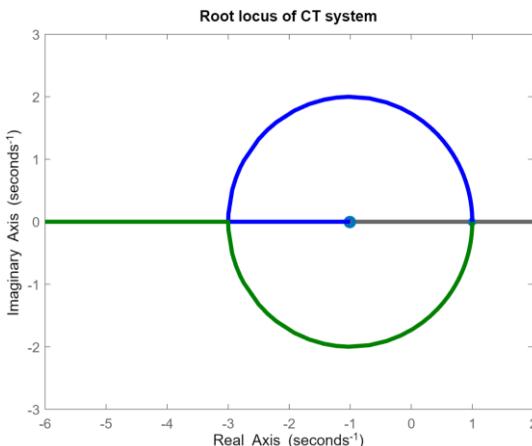
$$G(s) \rightarrow C(s)$$



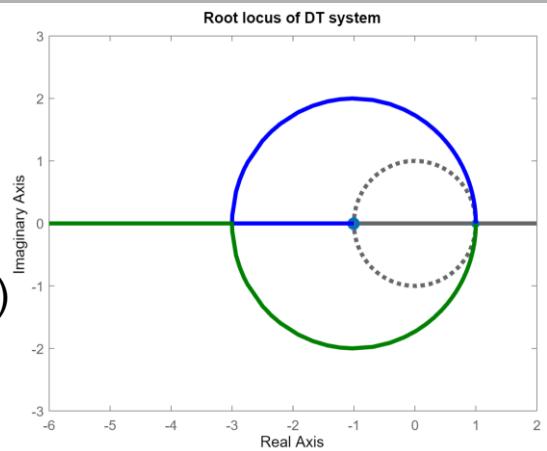
$$\begin{array}{c} G(s) \\ \downarrow \\ G(z) \end{array} \rightarrow \begin{array}{c} C(s) \\ \downarrow \\ C(z) \end{array}$$

Simulation Study: Root Locus

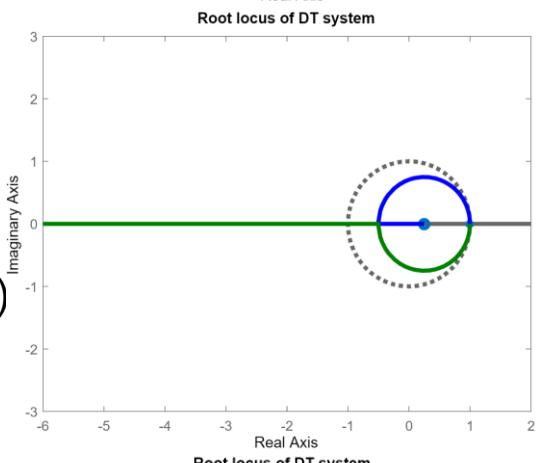
- $\text{den} = [1 \ -2 \ 1];$
- $\text{num} = [0 \ 0.5 \ 0.5];$
- $\text{sysC} = \text{tf}(\text{num}, \text{den})$



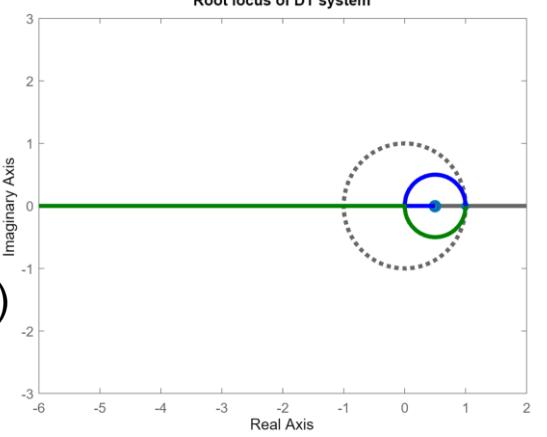
- $\text{den} = [1 \ -2 \ 1];$
- $\text{num} = [0 \ 0.5 \ 0.5];$
- $\text{sysD} = \text{tf}(\text{num}, \text{den}, 1)$



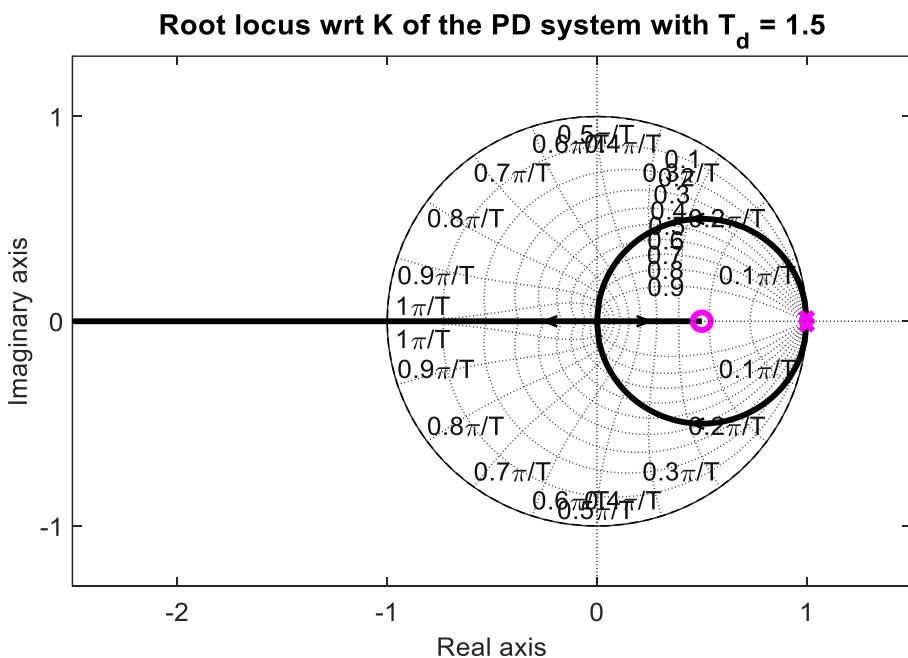
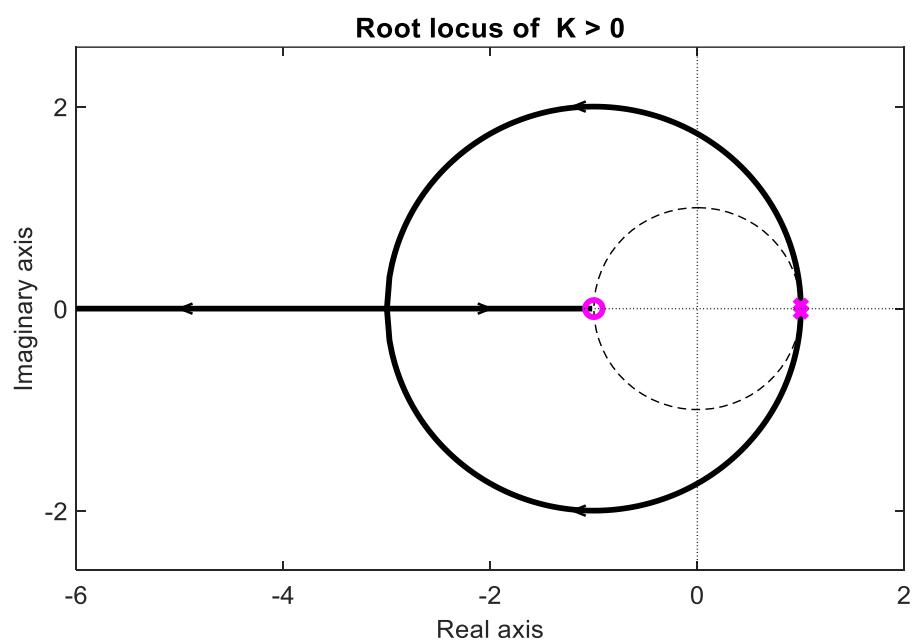
- $\text{den} = [1 \ -2 \ 1];$
- $\text{num} = [0 \ 1 \ -0.25];$
- $\text{sysD} = \text{tf}(\text{num}, \text{den}, 1)$



- $\text{den} = [1 \ -2 \ 1];$
- $\text{num} = [0 \ 2 \ -1];$
- $\text{sysD} = \text{tf}(\text{num}, \text{den}, 1)$



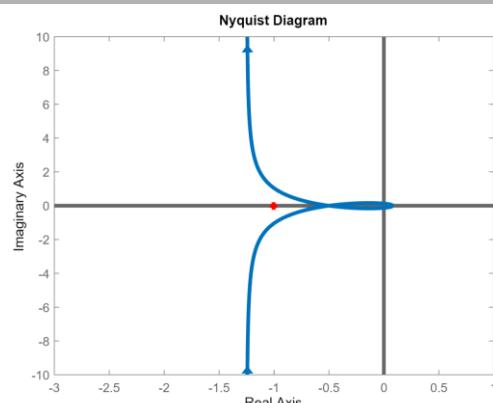
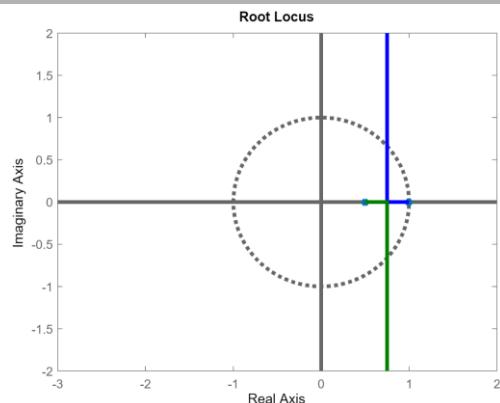
■ Root Locus:



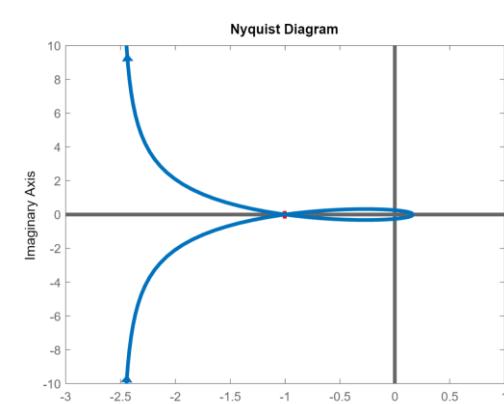
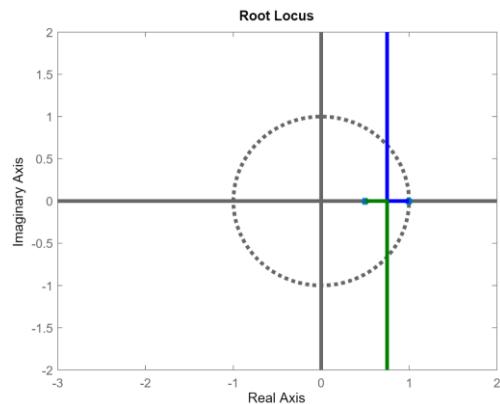
Simulation Study: Root Locus vs Nyquist Plot

- $\text{den} = [1 -1.5 0.5];$
- $\text{num} = [0.25*K];$
- $\text{sysD} = \text{tf}(\text{num}, \text{den}, h)$

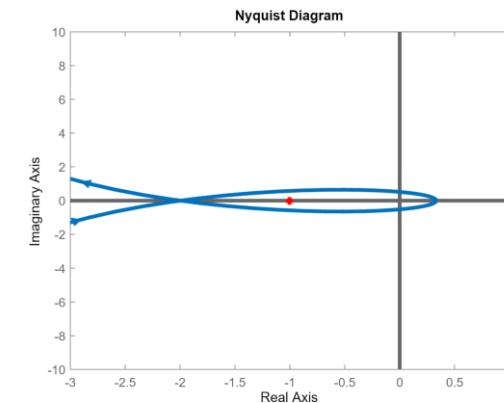
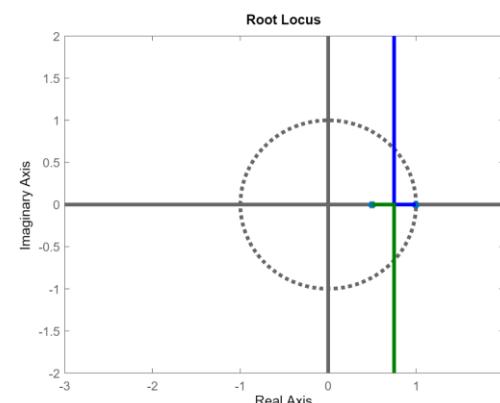
■ $K = 1$



■ $K = 2$



■ $K = 4$



Simulation Study: Root Locus vs Bode Plot vs Nyquist Plot

- den = [1 1.4 1];
- num = [1];
- sysC = tf(num, den)
- sysD = c2d(sysC, h)

sysC =

1

$$\frac{1}{s^2 + 1.4 s + 1}$$

- CT
- DT, h = 4
- DT, h = 1
- DT, h = 0.4
- DT, h = 0.2
- DT, h = 0.1
- DT, h = 0.01

sysD =	1.042 z + 0.07881

	z^2 + 0.1167 z + 0.003698
	Sample time: 4 seconds
	0.3059 z + 0.1902

	z^2 - 0.7505 z + 0.2466
	Sample time: 1 seconds
	0.06609 z + 0.05481

	z^2 - 1.45 z + 0.5712
	Sample time: 0.4 seconds
	0.0182 z + 0.01657

	z^2 - 1.721 z + 0.7558
	Sample time: 0.2 seconds
	0.004771 z + 0.004553

	z^2 - 1.86 z + 0.8694
	Sample time: 0.1 seconds
	4.977e-05 z + 4.954e-05

	z^2 - 1.986 z + 0.9861
	Sample time: 0.01 seconds

Simulation Study: Root Locus vs Bode Plot vs Nyquist Plot

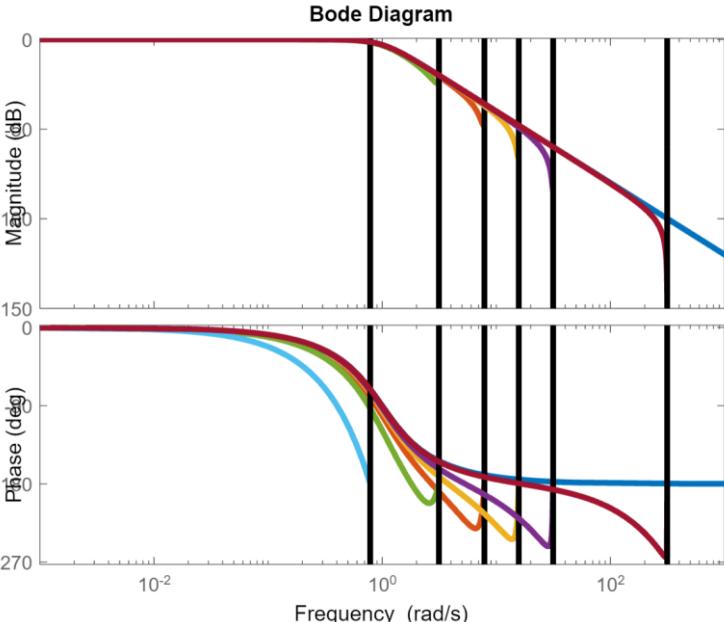
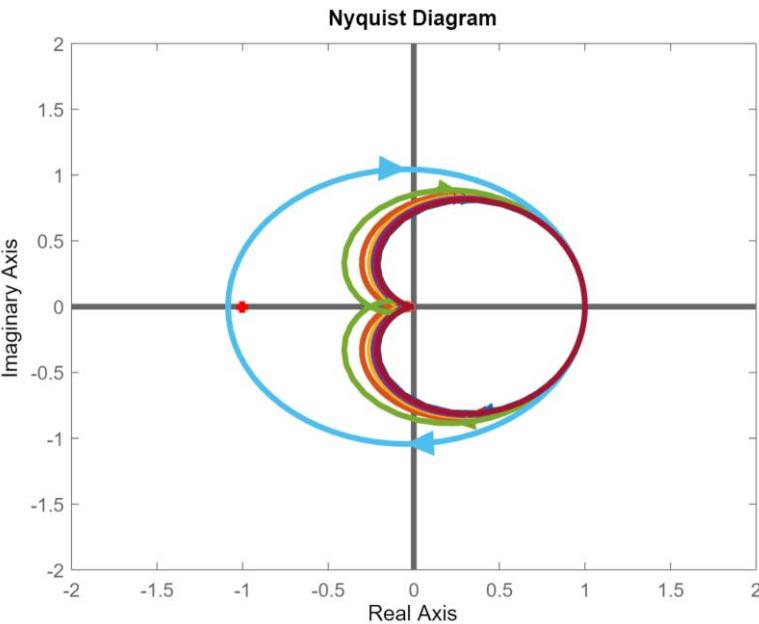
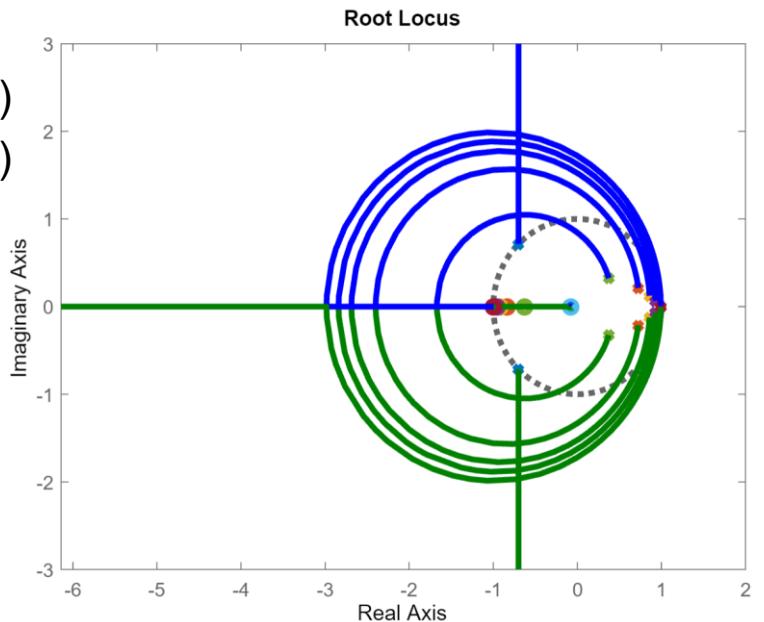
- den = [1 1.4 1];
- num = [1];
- sysC = tf(num, den)
- sysD = c2d(sysC, h)

- rlocus(sysC)
- rlocus(sysD)

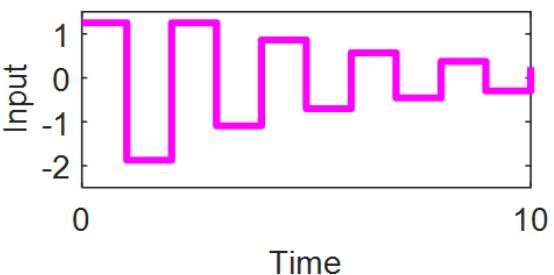
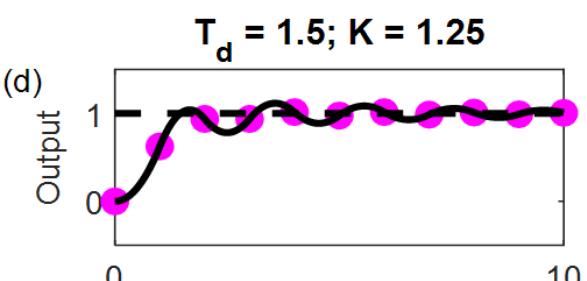
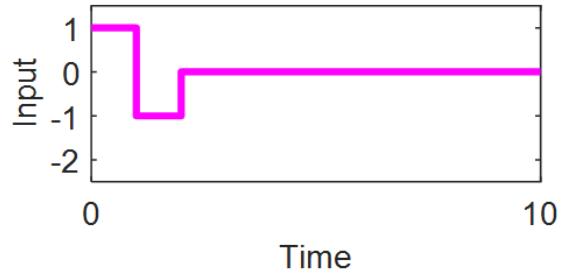
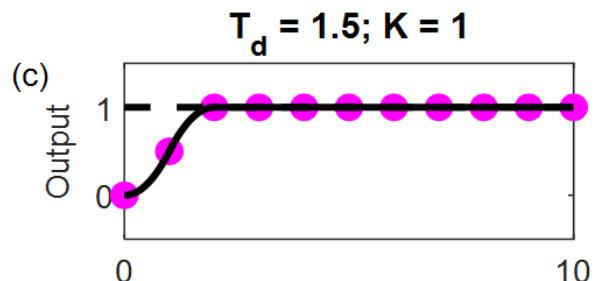
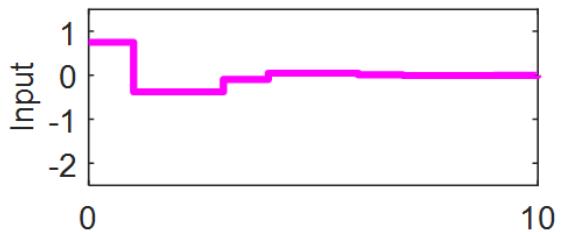
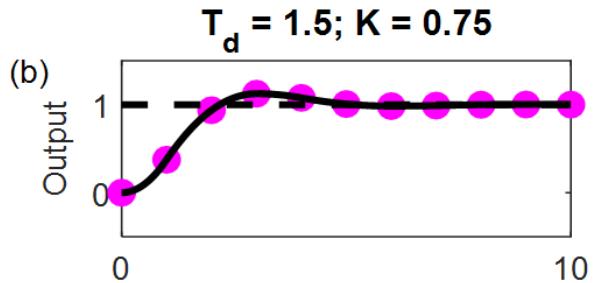
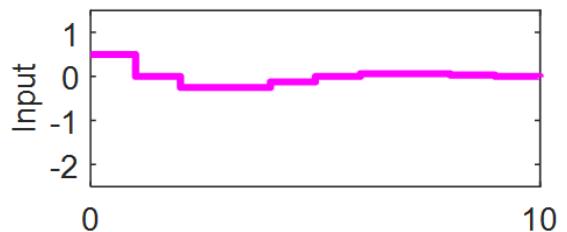
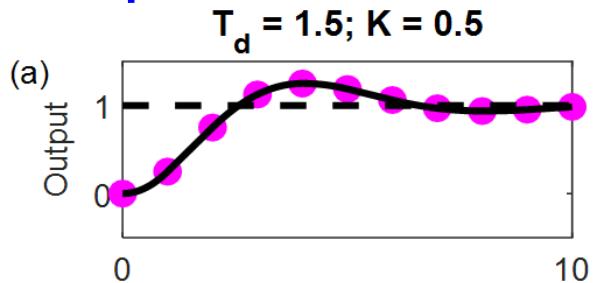
- bode(sysC)
- bode(sysD)

- nyquist(sysC)
- nyquist(sysD)

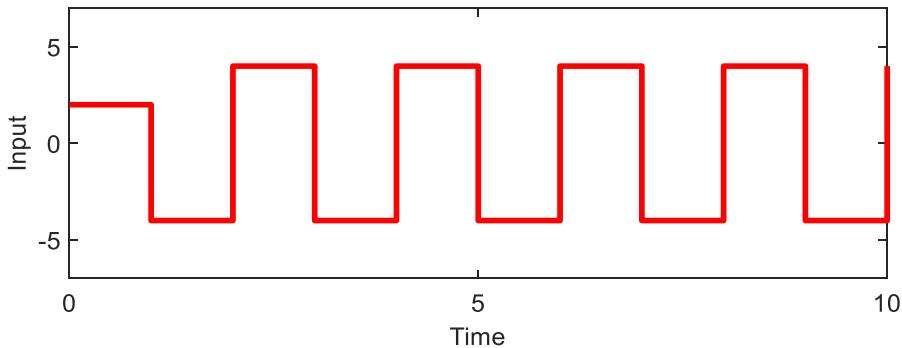
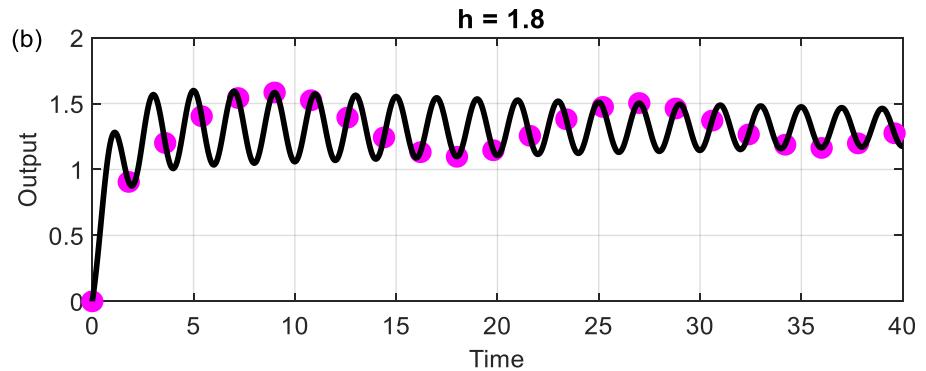
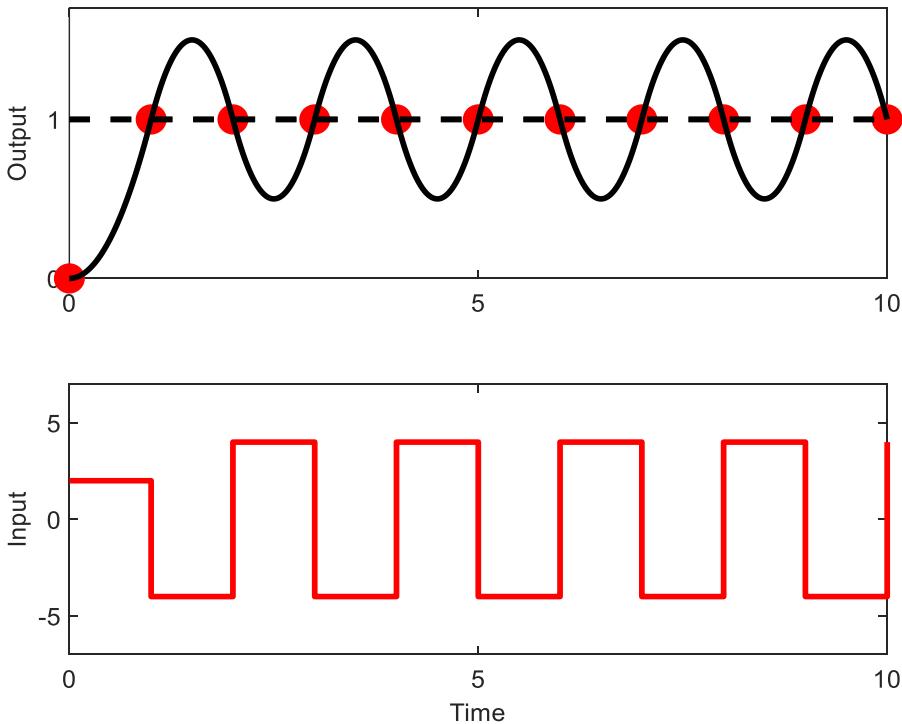
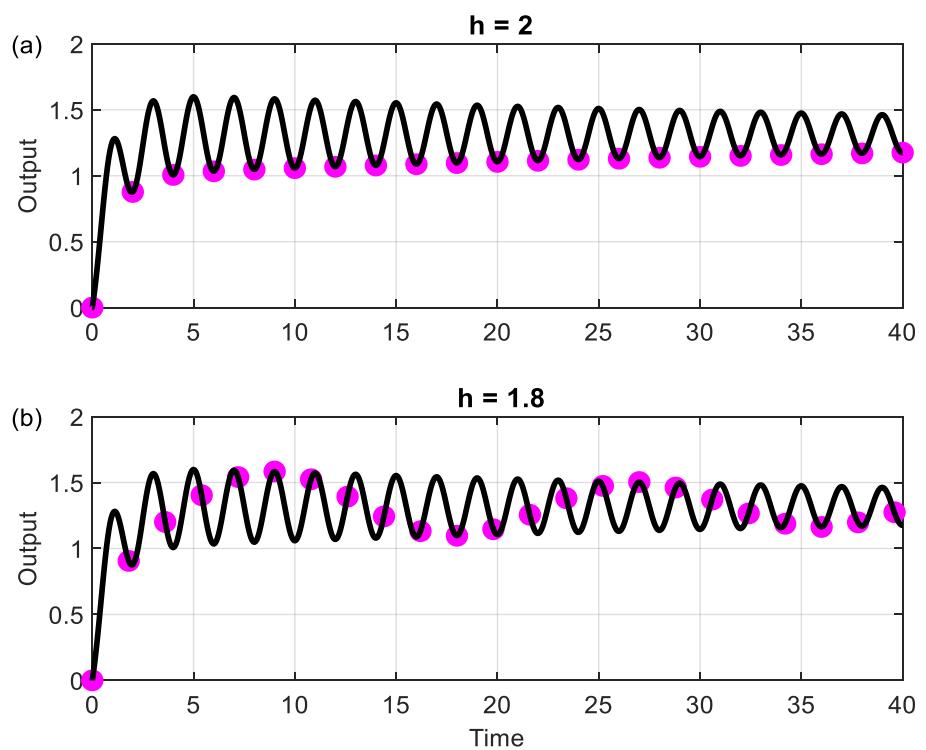
- CT
- DT, h = 4
- DT, h = 1
- DT, h = 0.4
- DT, h = 0.2
- DT, h = 0.1
- DT, h = 0.01



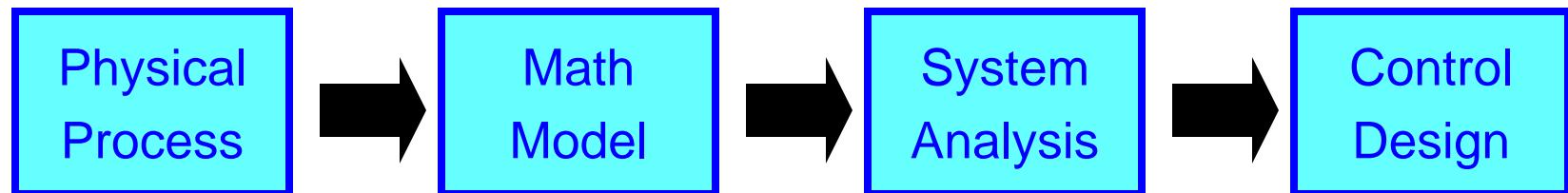
■ Step Responses of Different Gains:



■ Hidden Oscillation of Different Sampling Periods:



■ The Research Procedure in Control Science



- Plant
- Sensor
- Actuator
- Computer
- Communication
- Noise
- Disturbance

