Spring 2021

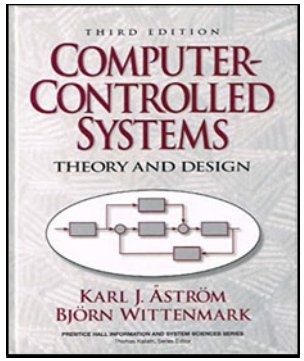
數位控制系統
Digital Control Systems

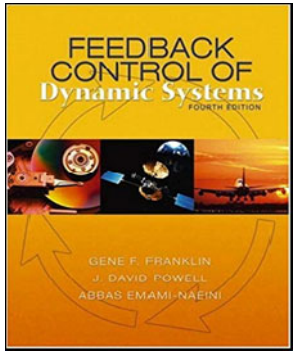
DCS-10
Discrete-Time Systems

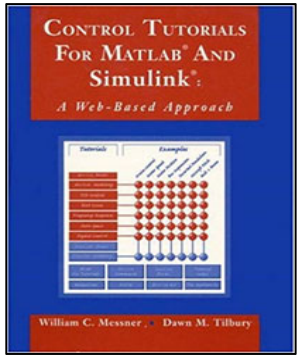
Feng-Li Lian

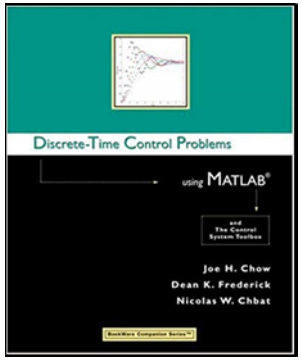
NTU-EE

Feb – Jun, 2021





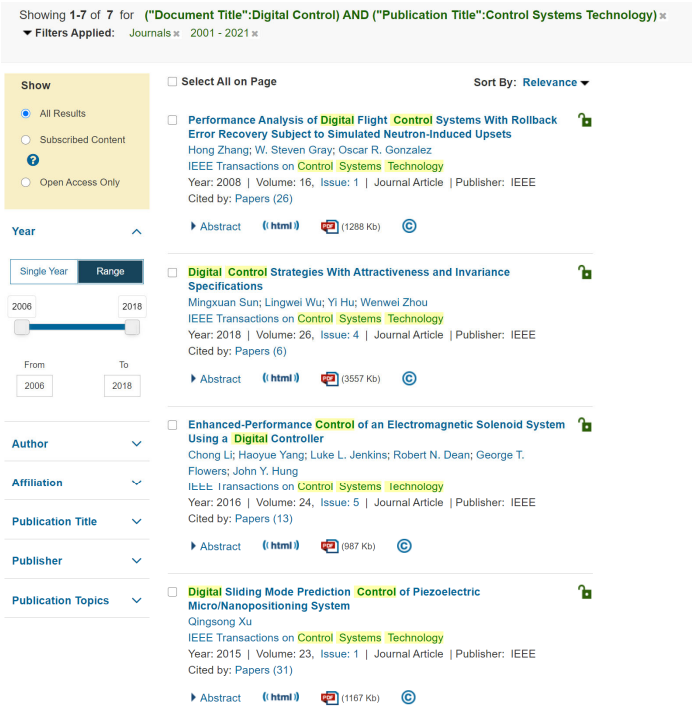


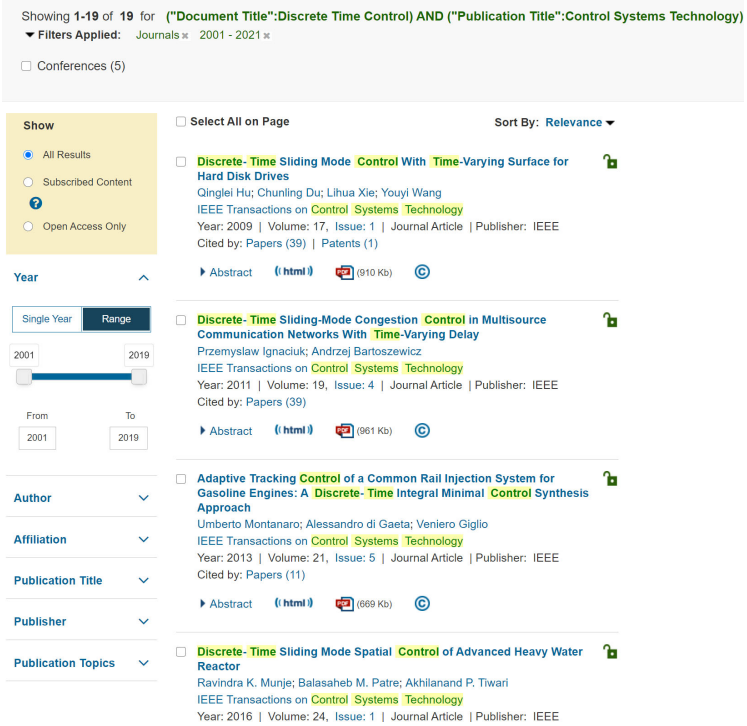


Papers on Digital Control and Discret-Time Control

- Go go IEEE Xplore
  at https://ieeexplore.ieee.org/

- Use Advanced Search to find paper with the keywords:
  - Document Title: Digital Control
  - Document Title: Discrete Time Control
  - Publication Title: Control Systems Technology
  - Publication Year: 2001 - 2021





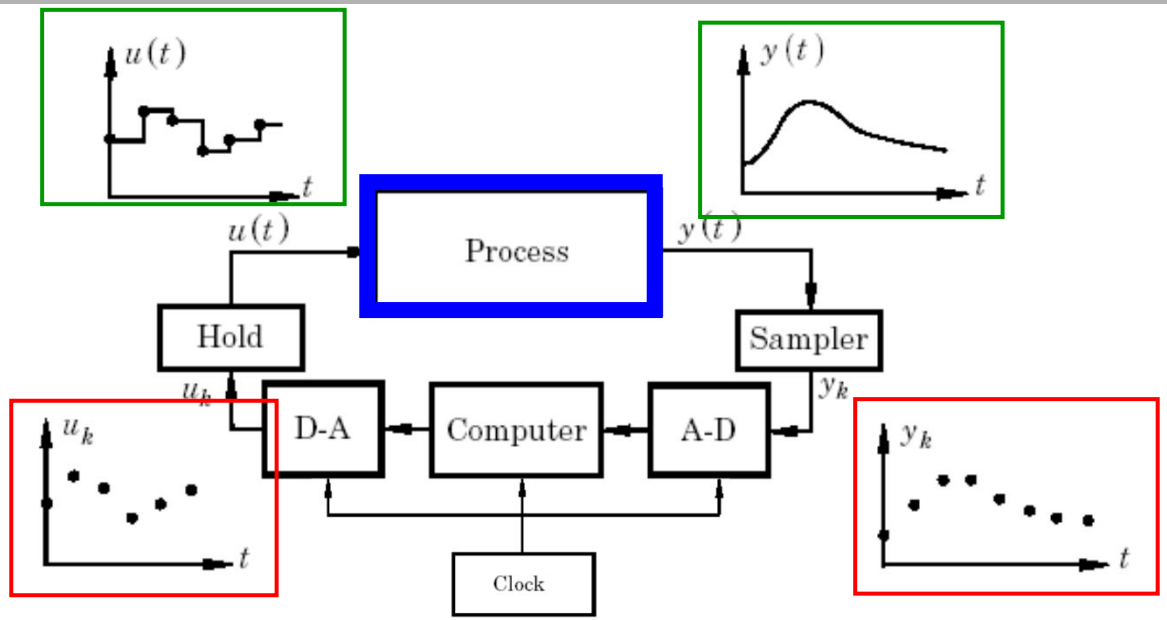
Papers on Digital Control and Discret-Time Control

- Discrete-time composite nonlinear feedback control with an application in design of a hard disk drive servo system
  - V. Venkataramanan;Kemao Peng;B.M. Chen;T.H. Lee
  - IEEE Transactions on Control Systems Technology
  - Year: 2003 | Volume: 11, Issue: 1 | Journal Article | Publisher: IEEE
- Discrete-Time Sliding Mode Control With Time-Varying Surface for Hard Disk Drives
  - Qinglei Hu;Chunling Du;Lihua Xie;Youyi Wang
  - IEEE Transactions on Control Systems Technology
  - Year: 2009 | Volume: 17, Issue: 1 | Journal Article | Publisher: IEEE
- Optimal Discrete-Time Design of Three-Axis Magnetic Attitude Control Laws
  - Tiziano Pulecchi;Marco Lovera;Andras Varga
  - IEEE Transactions on Control Systems Technology
  - Year: 2010 | Volume: 18, Issue: 3 | Journal Article | Publisher: IEEE
- Observer-based discrete-time sliding mode throttle control for drive-by-wire operation of a racing motorcycle engine
  - A. Beghi;L. Nardo;M. Stevanato
  - IEEE Transactions on Control Systems Technology
  - Year: 2006 | Volume: 14, Issue: 4 | Journal Article | Publisher: IEEE

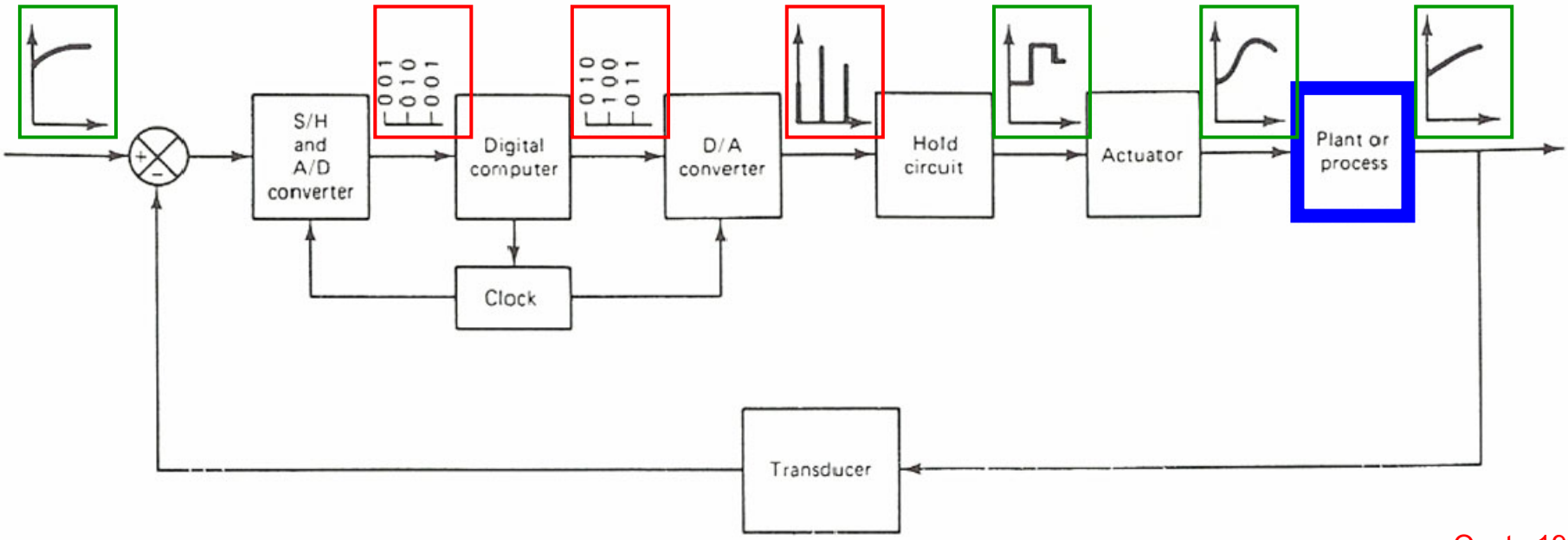
Review:

- System Modeling at Control Tutorials for Matlab & Simulink:
  - http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=SystemModeling
- System Models in LS:
  - http://cc.ee.ntu.edu.tw/~fengli/Teaching/LinearSystems/971lsNotePhoto080924.pdf
- Solving 1st-order Differential Equations:
  - https://case.ntu.edu.tw/CASTUDIO/Files/speech/Ref/CS0101S1B02_03.pdf
- Systems of Linear 1st-order Differential Equations:
  - http://case.ntu.edu.tw/CASTUDIO/Files/speech/Ref/CS0101S1B02_16.pdf
- Solving State-Space Equations:
  - http://cc.ee.ntu.edu.tw/~fengli/Teaching/LinearSystems/971lsNotePhoto081001.pdf

Introduction: Digital Signals and Systems

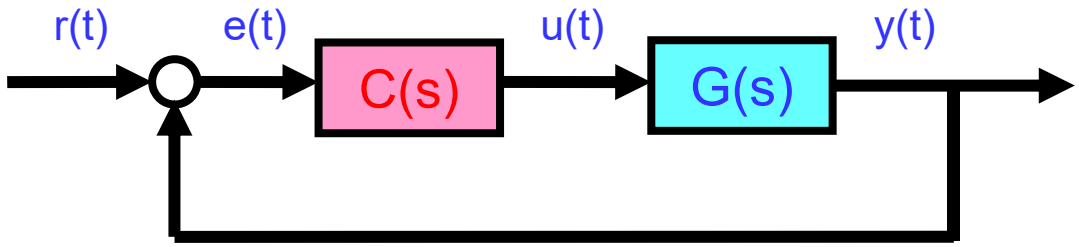


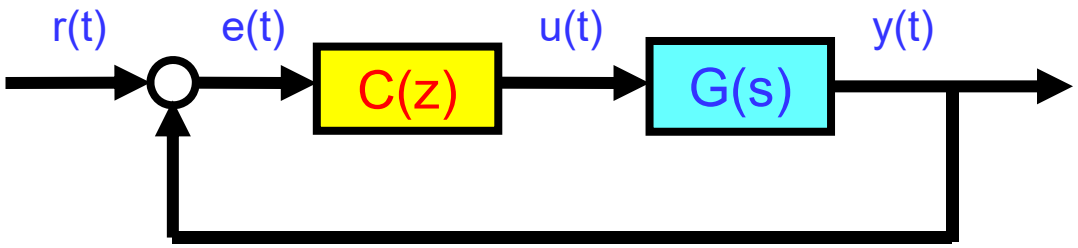
Astrom & Wittenmark 1997

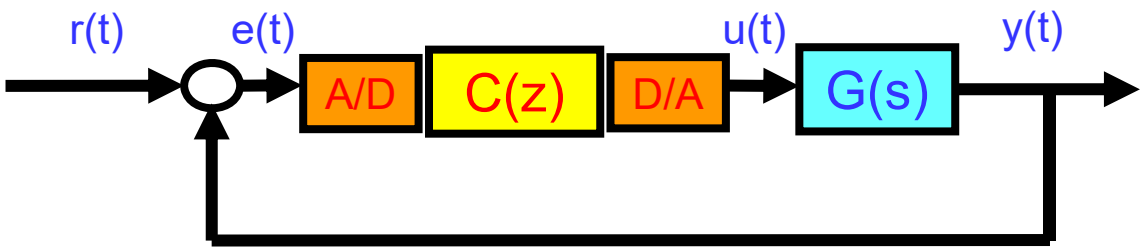


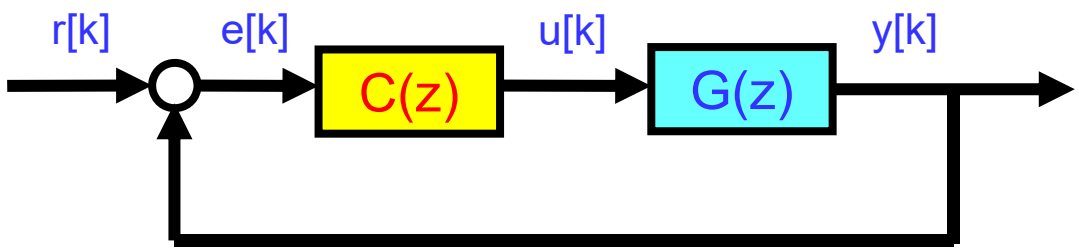
Ogata 1995

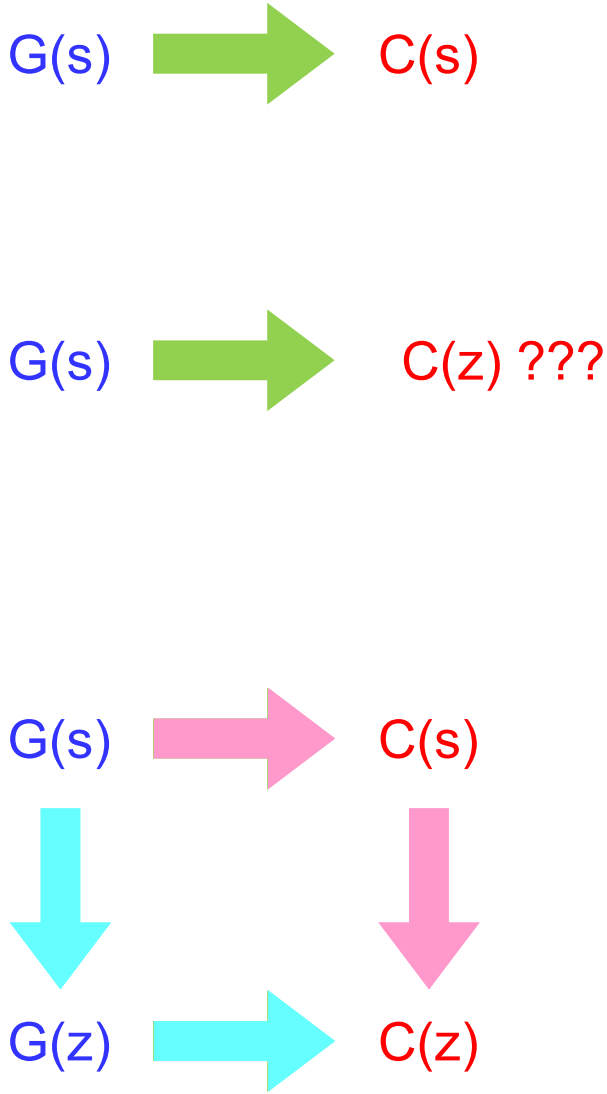
Introduction: Analog Control and Digital Control



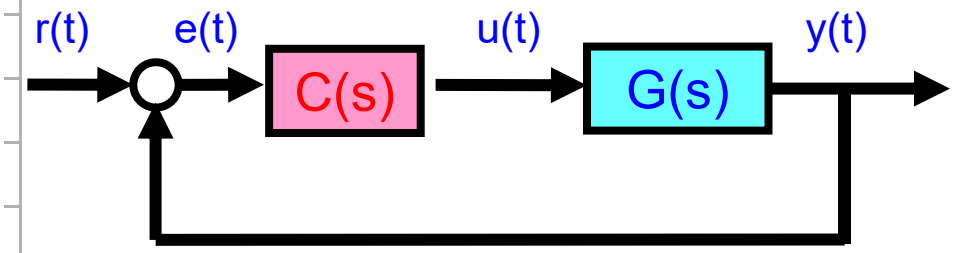




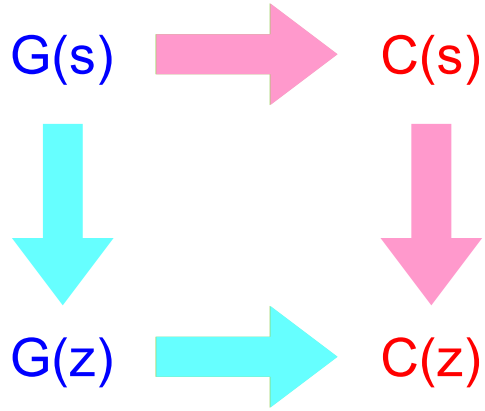




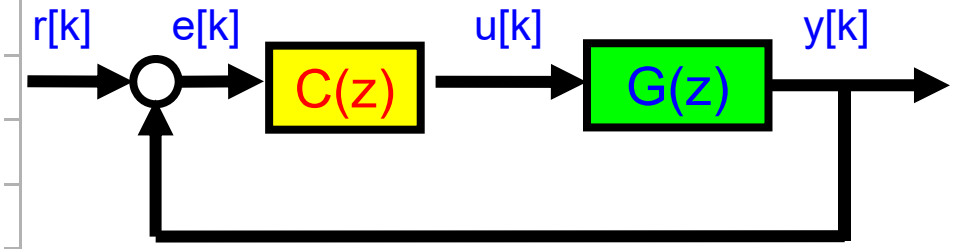
Introduction: CT and DT Plant-Controller



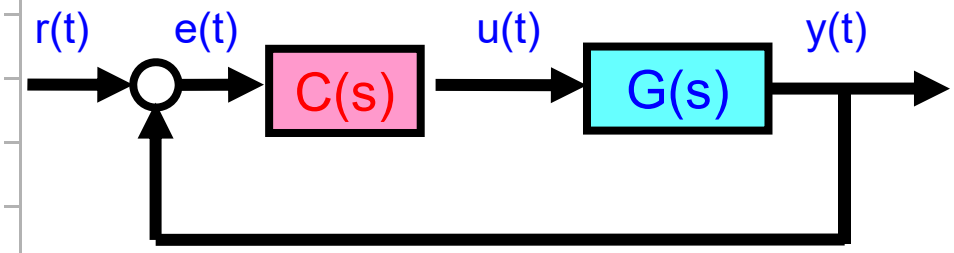
- Transform CT plant into DT plant
- By DT plant, design DT controller

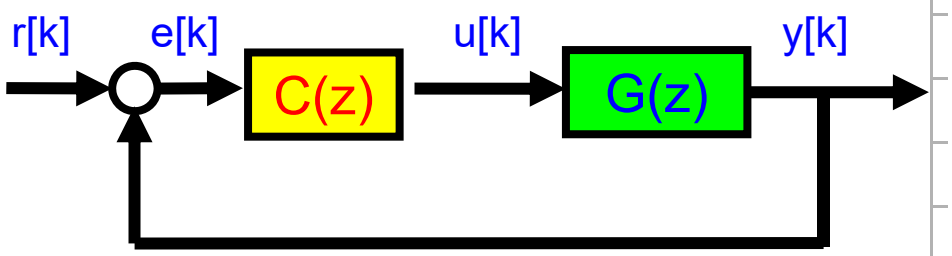


- By CT plant, design CT controller
- Transform CT controller into DT controller



Introduction: From CT Plant to DT Plant





- Plant (CT):
  - Input-Output Model:

$$u(t) \qquad y(t) \qquad \ddot{y} + 2\dot{y} - 3y = 5u(t)$$

$$G(s) = \frac{Y(s)}{U(s)}$$

  - State-Space Model:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{aligned}$$

- Plant (DT):
  - Input-Output Model:

$$u[k] \qquad y[k] \qquad \begin{aligned} y[k+2] + 4y[k+1] - 5y[k] \\ = 3u[k] \end{aligned}$$

$$G(z) = \frac{Y(z)}{U(z)}$$

  - State-Space Model:

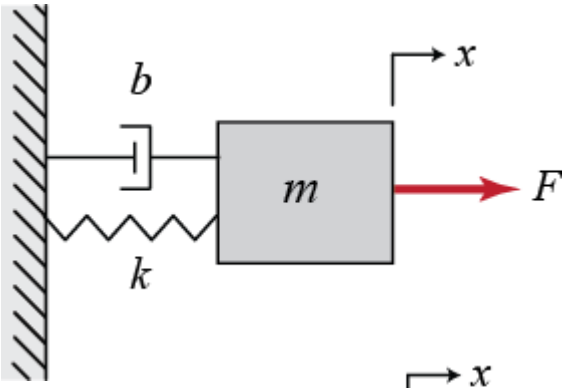
$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] + \mathbf{D}u[k] \end{aligned}$$

2

3

1

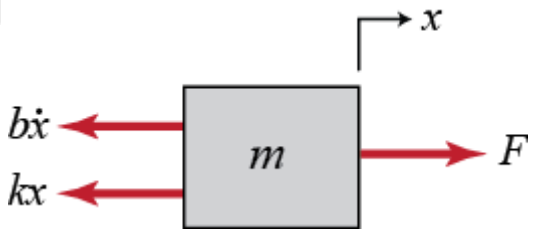
Introduction: Example - Mass-Spring-Damper System



$$F(t) - b\dot{x}(t) - kx(t) = m\ddot{x}(t)$$

$$\Rightarrow \quad F(s) - bsX(s) - kX(s) = ms^2X(s)$$

$$\Rightarrow \quad \boxed{\frac{X(s)}{F(s)} = \frac{1}{ms^2+bs+k} = \frac{Y(s)}{U(s)} = G(s)}$$



$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

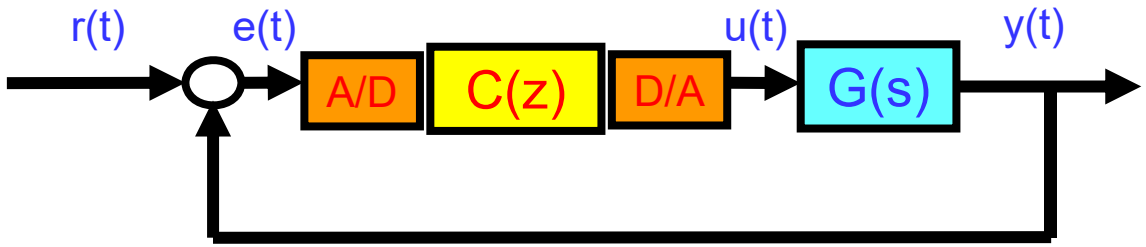
$$\boxed{y = x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u}$$

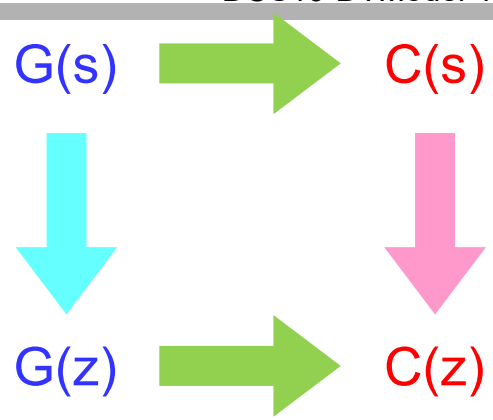
$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -\frac{k}{m}x - \frac{b}{m}\dot{x} + \frac{1}{m}F(t) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t)$$

$$\boxed{\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u}$$

Introduction: From CT Models to DT Models





- G(s) → (A, B, C, D) → (F, H, C, D) → G(z)
  - Piecewise Constant Input $\mathbf{F}(h) = e^{\mathbf{A}h}$
  - D.E. Solution $\mathbf{H}(h) = \left(\int_0^h e^{\mathbf{A}\eta} d\eta\right)\mathbf{B}$

$$G(z) = \mathbf{C}(z\mathbf{I} - \mathbf{F})^{-1}\mathbf{H} + \mathbf{D}$$

- G(s) → G(s) 1/s
  → ZT{ ILT{ G(s)/s } } → ZT{ ILT{ G(s)/s } } / (z/(z-1)) = G(z)
  - Step Input to the CT model and DT model

- y[n] + a_1 y[n-1] + … = b_0 u[n] + b_1 u[n-1] + … → ZT → G(z)
  - ZT on dynamic model of difference equation

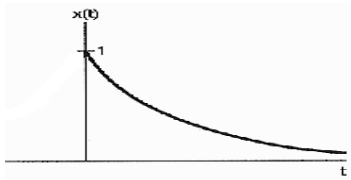
$$y[k] = \sum_{m=0}^{k} \bar{g}[k-m]u[m]$$

Introduction: From CT Models to DT Models

- $\mathbf{F} = \exp(\mathbf{A}h)$ $\Rightarrow$ $z = e^{sT}$
  $\Rightarrow$ $z = e^{sh}$

▪T, h: sampling period

$$f(t) = e^{-at}, \quad t > 0$$

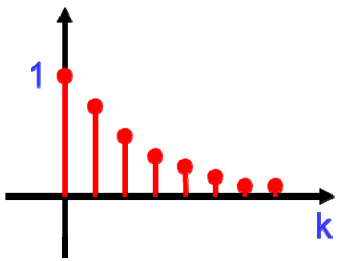


LT

$$\Rightarrow F(s) = \frac{1}{s+a}$$

$$\Rightarrow \text{Pole: } s = -a$$

Sampling



zT

$$\Rightarrow F(z) = \mathcal{Z}\left\{e^{-akT}\right\}$$

$$= \frac{z}{z - e^{-aT}}$$

$$f[kT] = e^{-akT}, \quad k \in N$$

$$\Rightarrow \text{Pole: } z = e^{-aT}$$

Introduction: From CT Models to DT Models

Laplace Transforms and z-Transforms of Simple Discrete Time Functions

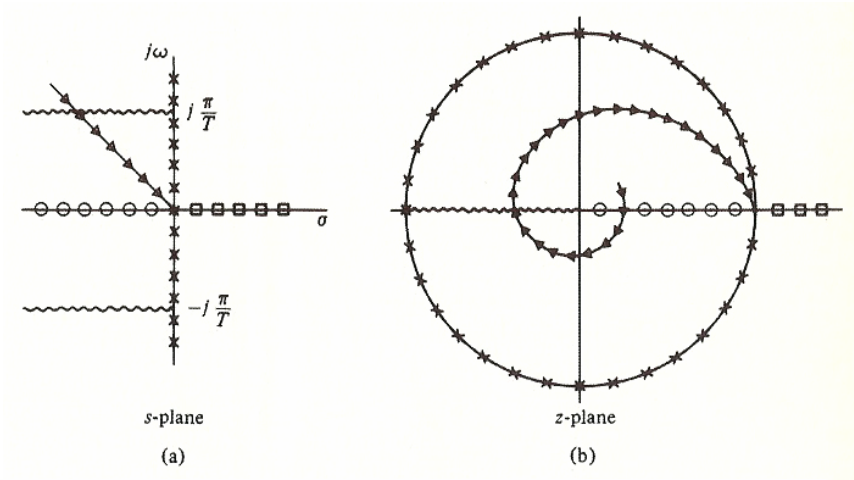
$F(s)$ is the Laplace transform of $f(t)$, and $F(z)$ is the z-transform of $f(kT)$. *Note:* $f(t) = 0$ for $t = 0$.

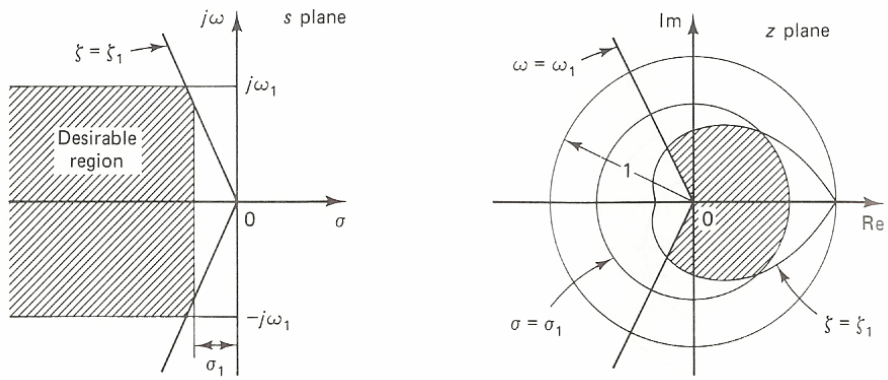
No.	$F(s)$	$f(kT)$	$F(z)$
1		$1, k = 0; 0, k \neq 0$	1
2		$1, k = k_o; 0, k \neq k_o$	z^{-k_o}
3	$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
4	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
5	$\frac{1}{s^3}$	$\frac{1}{2!}(kT)^2$	$\frac{T^2}{2}\left[\frac{z(z+1)}{(z-1)^3}\right]$
6	$\frac{1}{s^4}$	$\frac{1}{3!}(kT)^3$	$\frac{T^3}{6}\left[\frac{z(z^2+4z+1)}{(z-1)^4}\right]$
7	$\frac{1}{s^m}$	$\lim_{a\to 0}\frac{(-1)^{m-1}}{(m-1)!}\left(\frac{\partial^{m-1}}{\partial a^{m-1}}e^{-akT}\right)$	$\lim_{a\to 0}\frac{(-1)^{m-1}}{(m-1)!}\left(\frac{\partial^{m-1}}{\partial a^{m-1}}\frac{z}{z-e^{-aT}}\right)$
8	$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z-e^{-aT}}$
9	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
10	$\frac{1}{(s+a)^3}$	$\frac{1}{2}(kT)^2e^{-akT}$	$\frac{T^2}{2}e^{-aT}z\frac{(z+e^{-aT})}{(z-e^{-aT})^3}$
11	$\frac{1}{(s+a)^m}$	$\frac{(-1)^{m-1}}{(m-1)!}\left(\frac{\partial^{m-1}}{\partial a^{m-1}}e^{-akT}\right)$	$\frac{(-1)^{m-1}}{(m-1)!}\left(\frac{\partial^{m-1}}{\partial a^{m-1}}\frac{z}{z-e^{-aT}}\right)$
12	$\frac{a}{s(s+a)}$	$1-e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
13	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(akT-1+e^{-akT})$	$\frac{z[(aT-1+e^{-aT})z+(1-e^{-aT}-aTe^{-aT})]}{a(z-1)^2(z-e^{-aT})}$
14	$\frac{b-a}{(s+a)(s+b)}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$
15	$\frac{s}{(s+a)^2}$	$(1-akT)e^{-akT}$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$
16	$\frac{a^2}{s(s+a)^2}$	$1-e^{-akT}(1+akT)$	$\frac{z[z(1-e^{-aT}-aTe^{-aT})+e^{-2aT}-e^{-aT}+aTe^{-aT}]}{(z-1)(z-e^{-aT})^2}$
17	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bkT}-ae^{-akT}$	$\frac{z[z(b-a)-(be^{-aT}-ae^{-bT})]}{(z-e^{-aT})(z-e^{-bT})}$
18	$\frac{a}{s^2+a^2}$	$\sin akT$	$\frac{z\sin aT}{z^2-(2\cos aT)z+1}$
19	$\frac{s}{s^2+a^2}$	$\cos akT$	$\frac{z(z-\cos aT)}{z^2-(2\cos aT)z+1}$
20	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-akT}\cos bkT$	$\frac{z(z-e^{-aT}\cos bT)}{z^2-2e^{-aT}(\cos bT)z+e^{-2aT}}$
21	$\frac{b}{(s+a)^2+b^2}$	$e^{-akT}\sin bkT$	$\frac{ze^{-aT}\sin bT}{z^2-2e^{-aT}(\cos bT)z+e^{-2aT}}$
22	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-akT}\left(\cos bkT+\frac{a}{b}\sin bkT\right)$	$\frac{z(Az+B)}{(z-1)[z^2-2e^{-aT}(\cos bT)z+e^{-2aT}]}$ $A = 1-e^{-aT}\cos bT-\frac{a}{b}e^{-aT}\sin bT$ $B = e^{-2aT}+\frac{a}{b}e^{-aT}\sin bT-e^{-aT}\cos bT$

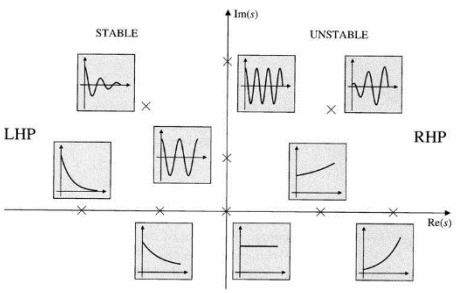
Introduction: From CT Models to DT Models

- $\mathbf{F} = \exp(\mathbf{A}h)$

$\Rightarrow \quad z = e^{sh}$







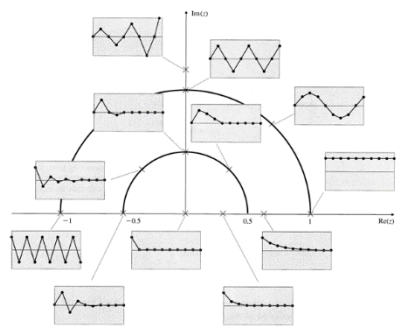


Figure 8.5 Time sequences associated with points in the z-plane