

Spring 2021

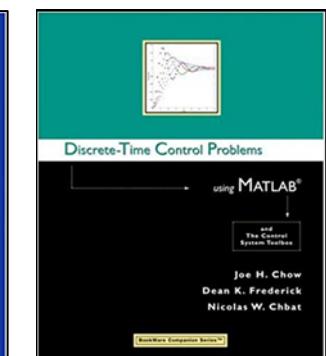
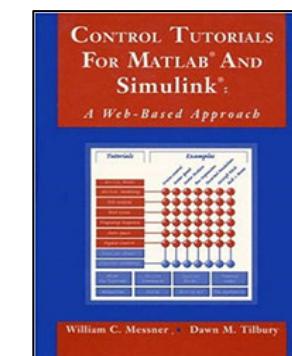
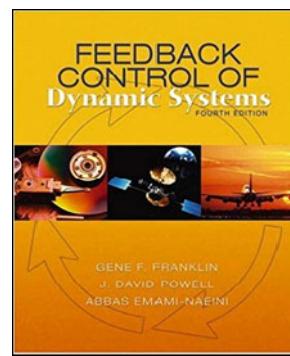
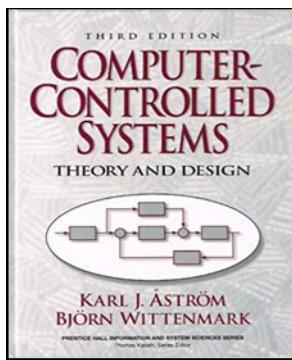
數位控制系統
Digital Control Systems

DCS-10
Discrete-Time Systems

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NTU-EE

Feb – Jun, 2021



Papers on Digital Control and Discret-Time Control

- Go go [IEEE Xplore](#)
at <https://ieeexplore.ieee.org/>

- Use Advanced Search to find paper with the keywords:

- Document Title: [Digital Control](#)
- Document Title: [Discrete Time Control](#)
- Publication Title: [Control Systems Technology](#)
- Publication Year: [2001 - 2021](#)

The screenshot shows the IEEE Xplore search interface with the following parameters applied:

- Show: All Results
- Sort By: Relevance
- Filters Applied: Journals, 2001 - 2021
- Year: Single Year (2006 to 2018)
- Author, Affiliation, Publication Title, Publisher, Publication Topics dropdown menus

Search results:

- Performance Analysis of Digital Flight Control Systems With Rollback Error Recovery Subject to Simulated Neutron-Induced Upsets (Hong Zhang; W. Steven Gray; Oscar R. Gonzalez, 2008)
- Digital Control Strategies With Attractiveness and Invariance Specifications (Mingxuan Sun; Lingwei Wu; Yi Hu; Wenwei Zhou, 2018)
- Enhanced-Performance Control of an Electromagnetic Solenoid System Using a Digital Controller (Chong Li; Haoyue Yang; Luke L. Jenkins; Robert N. Dean; George T. Flowers; John Y. Hung, 2016)
- Digital Sliding Mode Prediction Control of Piezoelectric Micro/Nanopositioning System (Qingsong Xu, 2015)

The screenshot shows the IEEE Xplore search interface with the following parameters applied:

- Show: All Results
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- Filters Applied: Journals, 2001 - 2021
- Conferences (5) checkbox
- Year: Single Year (2001 to 2019)
- Author, Affiliation, Publication Title, Publisher, Publication Topics dropdown menus

Search results:

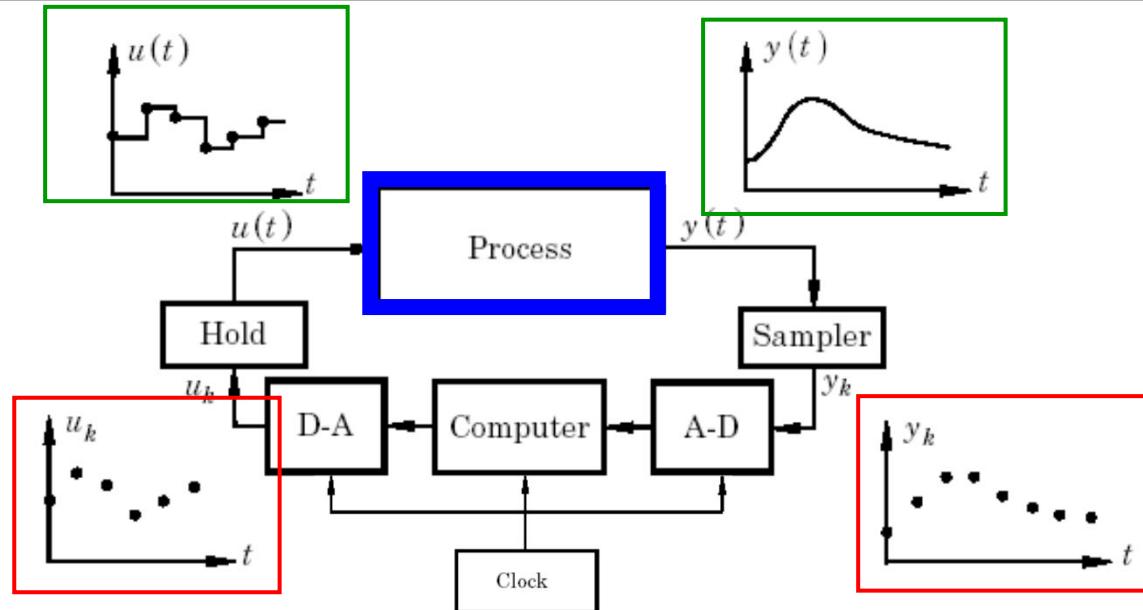
- Discrete-Time Sliding Mode Control With Time-Varying Surface for Hard Disk Drives (Qinglei Hu; Chunling Du; Lihua Xie; Youyi Wang, 2009)
- Discrete-Time Sliding-Mode Congestion Control in Multisource Communication Networks With Time-Varying Delay (Przemyslaw Ignacuk; Andrzej Bartoszewicz, 2011)
- Adaptive Tracking Control of a Common Rail Injection System for Gasoline Engines: A Discrete-Time Integral Minimal Control Synthesis Approach (Umberto Montanaro; Alessandro di Gaeta; Veniero Giglio, 2013)
- Discrete-Time Sliding Mode Spatial Control of Advanced Heavy Water Reactor (Ravindra K. Munje; Balasheb M. Patre; Akhilanand P. Tiwari, 2016)

- Discrete-time composite nonlinear feedback control with an application in design of a hard disk drive servo system
 - V. Venkataraman;Kemao Peng;B.M. Chen;T.H. Lee
 - IEEE Transactions on Control Systems Technology
 - Year: 2003 | Volume: 11, Issue: 1 | Journal Article | Publisher: IEEE
- Discrete-Time Sliding Mode Control With Time-Varying Surface for Hard Disk Drives
 - Qinglei Hu;Chunling Du;Lihua Xie;Youyi Wang
 - IEEE Transactions on Control Systems Technology
 - Year: 2009 | Volume: 17, Issue: 1 | Journal Article | Publisher: IEEE
- Optimal Discrete-Time Design of Three-Axis Magnetic Attitude Control Laws
 - Tiziano Pulecchi;Marco Lovera;Andras Varga
 - IEEE Transactions on Control Systems Technology
 - Year: 2010 | Volume: 18, Issue: 3 | Journal Article | Publisher: IEEE
- Observer-based discrete-time sliding mode throttle control for drive-by-wire operation of a racing motorcycle engine
 - A. Beghi;L. Nardo;M. Stevanato
 - IEEE Transactions on Control Systems Technology
 - Year: 2006 | Volume: 14, Issue: 4 | Journal Article | Publisher: IEEE

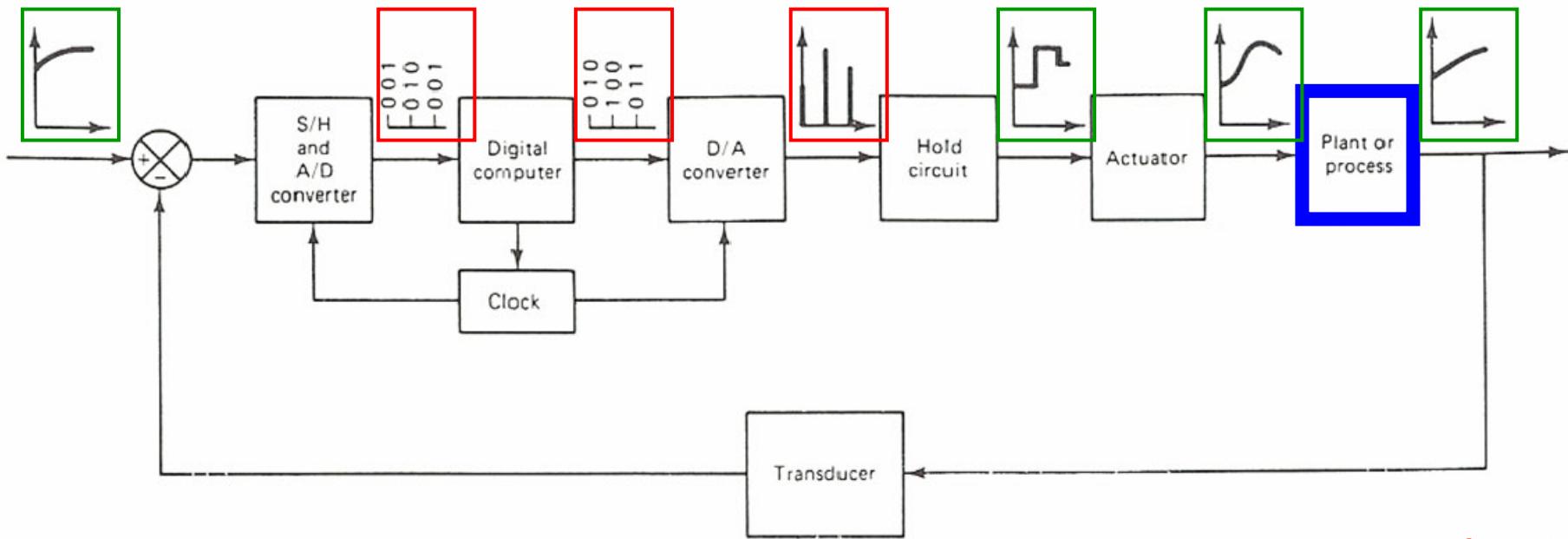
- System Modeling at Control Tutorials for Matlab & Simulink:
 - <http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction§ion=SystemModeling>
- System Models in LS:
 - <http://cc.ee.ntu.edu.tw/~fengli/Teaching/LinearSystems/971lsNotePhoto080924.pdf>
- Solving 1st-order Differential Equations:
 - https://case.ntu.edu.tw/CASTUDIO/Files/speech/Ref/CS0101S1B02_03.pdf
- Systems of Linear 1st-order Differential Equations:
 - http://case.ntu.edu.tw/CASTUDIO/Files/speech/Ref/CS0101S1B02_16.pdf
- Solving State-Space Equations:
 - <http://cc.ee.ntu.edu.tw/~fengli/Teaching/LinearSystems/971lsNotePhoto081001.pdf>

Introduction: Digital Signals and Systems

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DCS10-DTModel-5

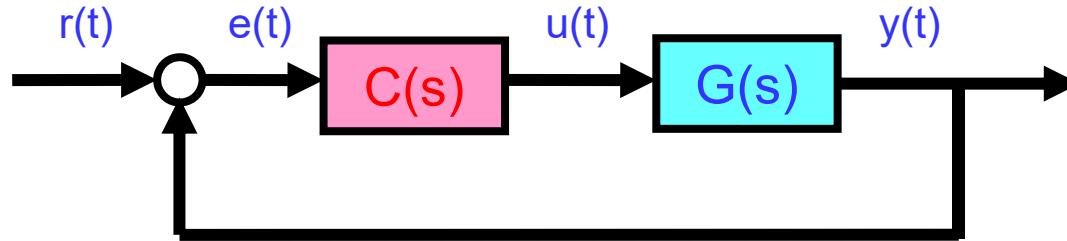


Astrom & Wittenmark 1997

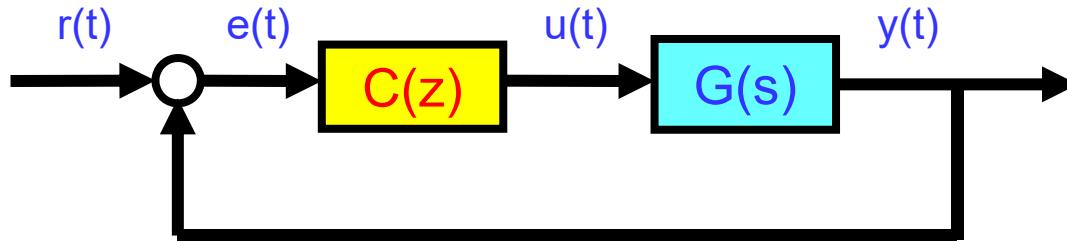


Ogata 1995

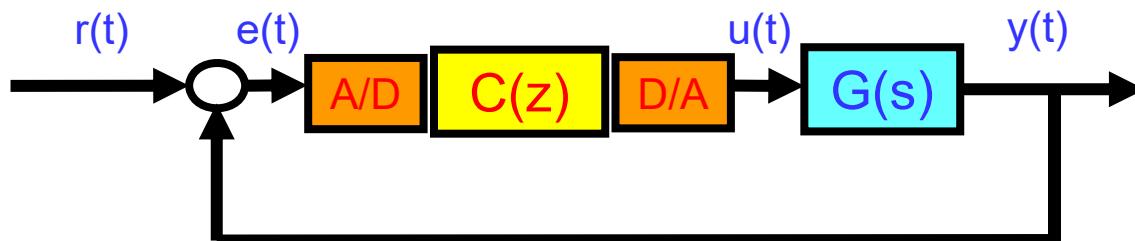
Introduction: Analog Control and Digital Control



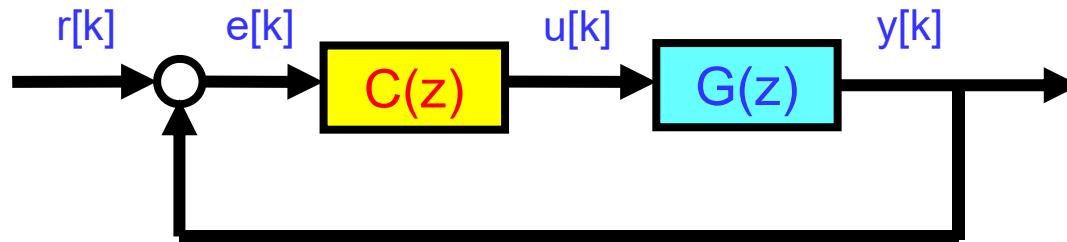
$$G(s) \rightarrow C(s)$$



$$G(s) \rightarrow C(z) ???$$

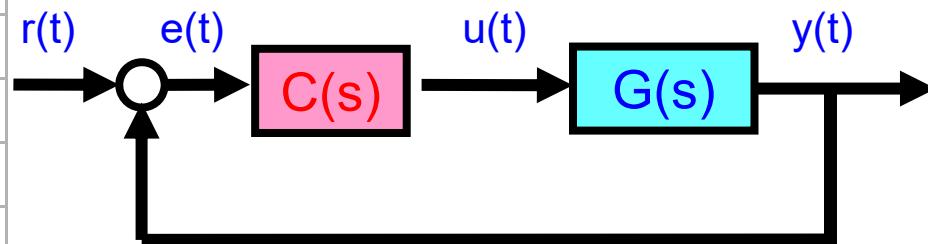


$$G(s) \rightarrow C(s)$$



$$G(z) \downarrow \rightarrow C(z)$$

Introduction: CT and DT Plant-Controller

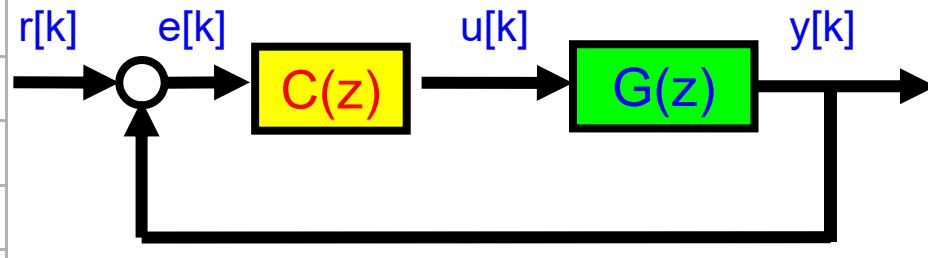


- Transform CT plant into DT plant
- By DT plant, design DT controller

$$G(s) \rightarrow C(s)$$

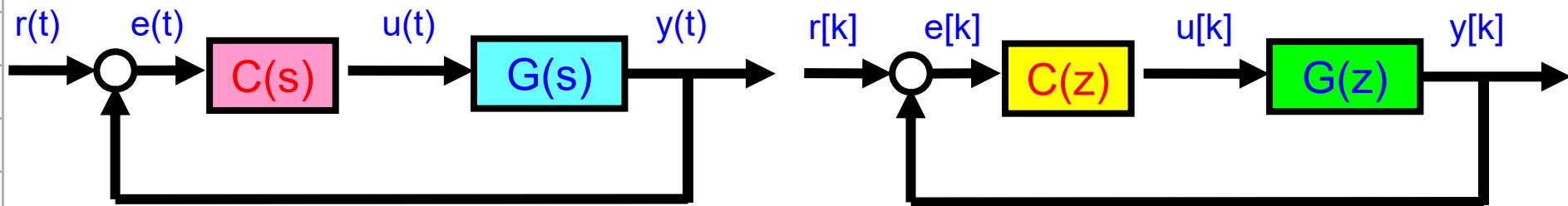
- By CT plant, design CT controller
- Transform CT controller into DT controller

$$\begin{array}{ccc} G(s) & \rightarrow & C(s) \\ \downarrow & & \downarrow \\ G(z) & \rightarrow & C(z) \end{array}$$



$$G(z) \rightarrow C(z)$$

Introduction: From CT Plant to DT Plant

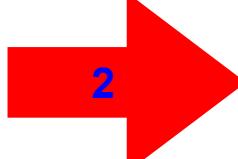


- Plant (CT):

- Input-Output Model:

$$u(t) \quad \ddot{y} + 2\dot{y} - 3y = 5u(t)$$

$$y(t)$$

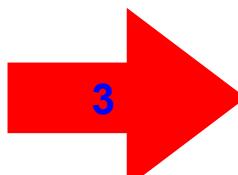


- Plant (DT):

- Input-Output Model:

$$u[k] \quad y[k+2] + 4y[k+1] - 5y[k] \\ y[k]$$

$$G(s) = \frac{Y(s)}{U(s)}$$

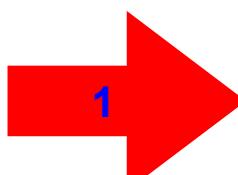


$$G(z) = \frac{Y(z)}{U(z)}$$

- State-Space Model:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

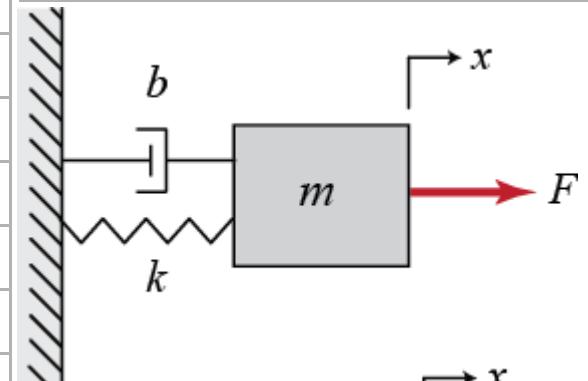


- State-Space Model:

$$x[k+1] = Fx[k] + Hu[k]$$

$$y[k] = Cx[k] + Du[k]$$

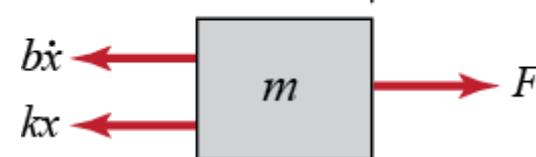
Introduction: Example - Mass-Spring-Damper System



$$F(t) - b\dot{x}(t) - kx(t) = m\ddot{x}(t)$$

$$\Rightarrow F(s) - bsX(s) - kX(s) = ms^2X(s)$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} = \frac{Y(s)}{U(s)} = G(s)$$



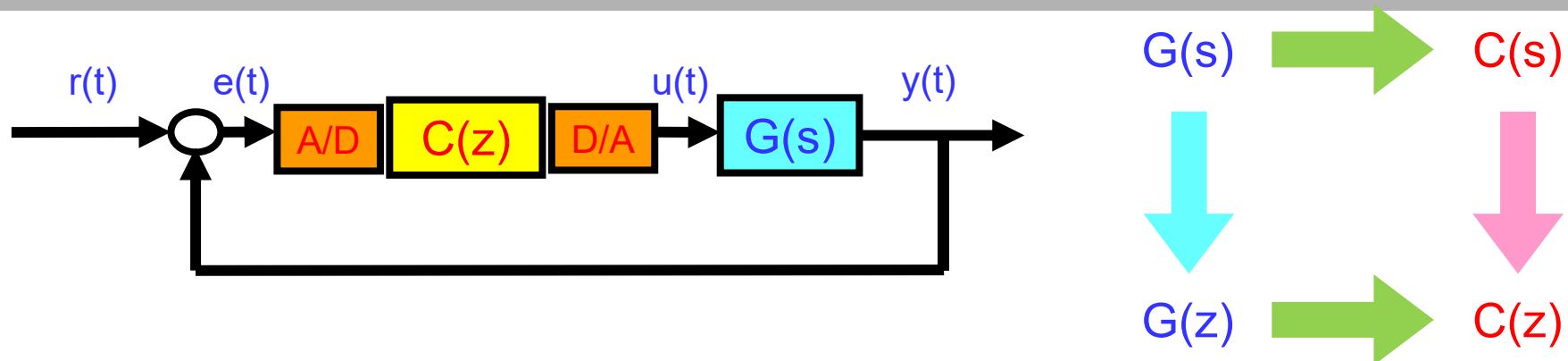
$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$y = x = [1 \ 0] \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + [0] u$$

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -\frac{k}{m}x - \frac{b}{m}\dot{x} + \frac{1}{m}F(t) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t) \end{aligned}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

Introduction: From CT Models to DT Models



- $G(s) \rightarrow (A, B, C, D) \rightarrow (F, H, C, D) \rightarrow G(z)$
 - Piecewise Constant Input $F(h) = e^{Ah}$
 - D.E. Solution $H(h) = \left(\int_0^h e^{A\eta} d\eta \right) B$
 - $G(z) = C(zI - F)^{-1}H + D$
- $G(s) \rightarrow G(s) 1/s$
 - $\rightarrow ZT\{ ILT\{ G(s)/s \} \} \rightarrow ZT\{ ILT\{ G(s)/s \} \} / (z/(z-1)) = G(z)$
 - Step Input to the CT model and DT model
- $y[n] + a_{-1} y[n-1] + \dots = b_{-1} u[n] + b_{-2} u[n-1] + \dots \rightarrow ZT \rightarrow G(z)$
 - ZT on dynamic model of difference equation

$$y[k] = \sum_{m=0}^k \bar{g}[k-m]u[m]$$

Introduction: From CT Models to DT Models

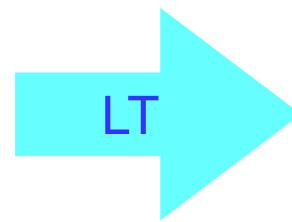
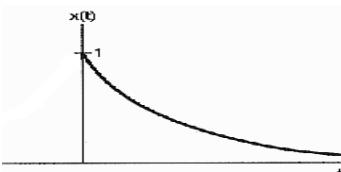
- $\mathbf{F} = \exp(\mathbf{A}h)$

$$\Rightarrow z = e^{sT}$$

$$\Rightarrow z = e^{sh}$$

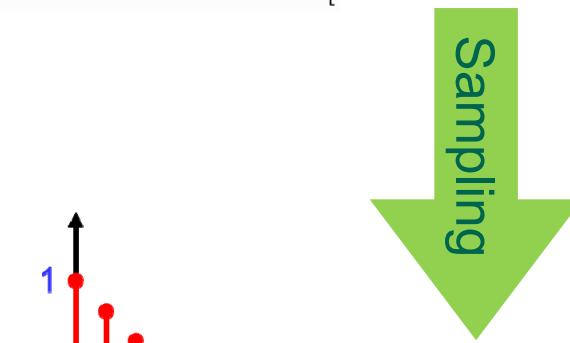
- T, h: sampling period

$$f(t) = e^{-at}, \quad t > 0$$

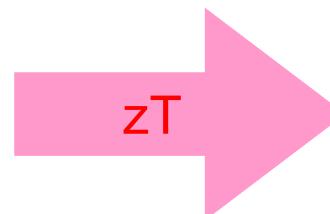


$$\Rightarrow F(s) = \frac{1}{s + a}$$

$$\Rightarrow \text{Pole: } s = -a$$



$$f[kT] = e^{-akT}, \quad k \in N$$



$$\Rightarrow F(z) = z \left\{ e^{-akT} \right\}$$

$$= \frac{z}{z - e^{-aT}}$$

$$\Rightarrow \text{Pole: } z = e^{-aT}$$

Introduction: From CT Models to DT Models

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DCS10-DTModel-12

Laplace Transforms and z -Transforms of Simple Discrete Time Functions

$F(s)$ is the Laplace transform of $f(t)$, and $F(z)$ is the z -transform of $f(kT)$. Note: $f(t) = 0$ for $t = 0$.

No.	$F(s)$	$f(kT)$	$F(z)$
1		$1, k = 0; 0, k \neq 0$	1
2		$1, k = k_o; 0, k \neq k_o$	z^{-k_o}
3	$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
4	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
5	$\frac{1}{s^3}$	$\frac{1}{2!}(kT)^2$	$\frac{T^2}{2} \left[\frac{z(z+1)}{(z-1)^3} \right]$
6	$\frac{1}{s^4}$	$\frac{1}{3!}(kT)^3$	$\frac{T^3}{6} \left[\frac{z(z^2+4z+1)}{(z-1)^4} \right]$
7	$\frac{1}{s^m}$	$\lim_{a \rightarrow 0} \frac{(-1)^{m-1}}{(m-1)!} \left(\frac{\partial^{m-1}}{\partial a^{m-1}} e^{-akT} \right)$	$\lim_{a \rightarrow 0} \frac{(-1)^{m-1}}{(m-1)!} \left(\frac{\partial^{m-1}}{\partial a^{m-1}} \frac{z}{z - e^{-aT}} \right)$
8	$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z - e^{-aT}}$
9	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$
10	$\frac{1}{(s+a)^3}$	$\frac{1}{2}(kT)^2 e^{-akT}$	$\frac{T^2}{2} e^{-aT} z \frac{(z + e^{-aT})}{(z - e^{-aT})^3}$
11	$\frac{1}{(s+a)^m}$	$\frac{(-1)^{m-1}}{(m-1)!} \left(\frac{\partial^{m-1}}{\partial a^{m-1}} e^{-akT} \right)$	$\frac{(-1)^{m-1}}{(m-1)!} \left(\frac{\partial^{m-1}}{\partial a^{m-1}} \frac{z}{z - e^{-aT}} \right)$

12	$\frac{a}{s(s+a)}$	$1 - e^{-akT}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-aT})}$
13	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(akT - 1 + e^{-akT})$	$\frac{z[(aT - 1 + e^{-aT})z + (1 - e^{-aT} - aTe^{-aT})]}{a(z-1)^2(z - e^{-aT})}$
14	$\frac{b-a}{(s+a)(s+b)}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z}{(z - e^{-aT})(z - e^{-bT})}$
15	$\frac{s}{(s+a)^2}$	$(1 - akT)e^{-akT}$	$\frac{z[z - e^{-aT}(1 + aT)]}{(z - e^{-aT})^2}$
16	$\frac{c^2}{s(s+a)^2}$	$1 - e^{-akT}(1 + akT)$	$\frac{z[z(1 - e^{-aT} - aTe^{-aT}) + e^{-2aT} - e^{-aT} + aTe^{-aT}]}{(z-1)(z - e^{-aT})^2}$
17	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bkT} - ae^{-akT}$	$\frac{z[z(b-a) - (be^{-aT} - ae^{-bT})]}{(z - e^{-aT})(z - e^{-bT})}$
18	$\frac{a}{s^2 + a^2}$	$\sin akT$	$\frac{z \sin aT}{z^2 - (2 \cos aT)z + 1}$
19	$\frac{s}{s^2 + a^2}$	$\cos akT$	$\frac{z(z - \cos aT)}{z^2 - (2 \cos aT)z + 1}$
20	$\frac{s+a}{(s+a)^2 + b^2}$	$e^{-akT} \cos bkT$	$\frac{z(z - e^{-aT} \cos bT)}{z^2 - 2e^{-aT}(\cos bT)z + e^{-2aT}}$
21	$\frac{b}{(s+a)^2 + b^2}$	$e^{-akT} \sin bkT$	$\frac{ze^{-aT} \sin bT}{z^2 - 2e^{-aT}(\cos bT)z + e^{-2aT}}$
22	$\frac{a^2 + b^2}{s[(s+a)^2 + b^2]}$	$1 - e^{-akT} (\cos bkT + \frac{a}{b} \sin bkT)$	$\frac{z(Az + B)}{(z-1)[z^2 - 2e^{-aT}(\cos bT)z + e^{-2aT}]}$

$$A = 1 - e^{-aT} \cos bT - \frac{a}{b} e^{-aT} \sin bT$$

$$B = e^{-2aT} + \frac{a}{b} e^{-aT} \sin bT - e^{-aT} \cos bT$$

Introduction: From CT Models to DT Models

- $F = \exp(Ah)$

$$\Rightarrow z = e^{sh}$$

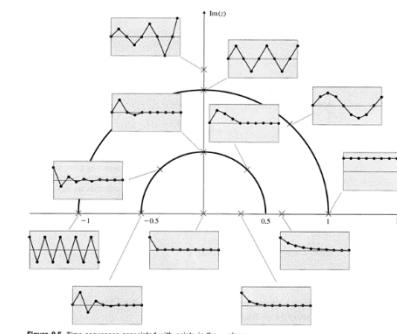
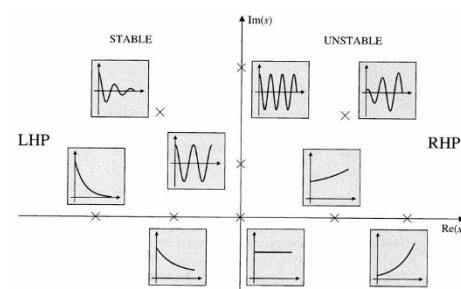
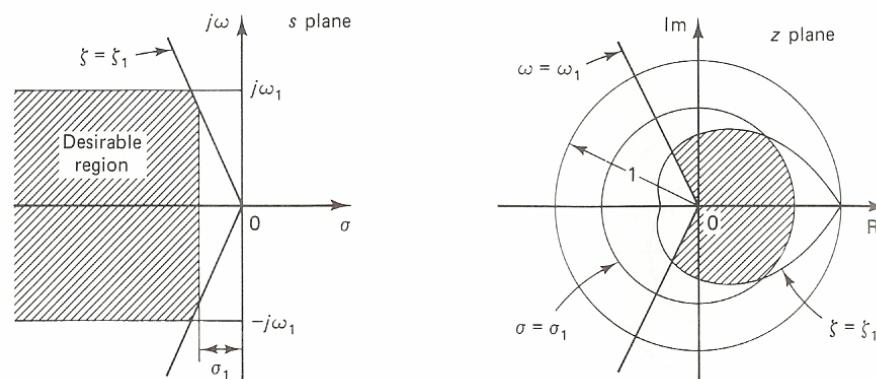
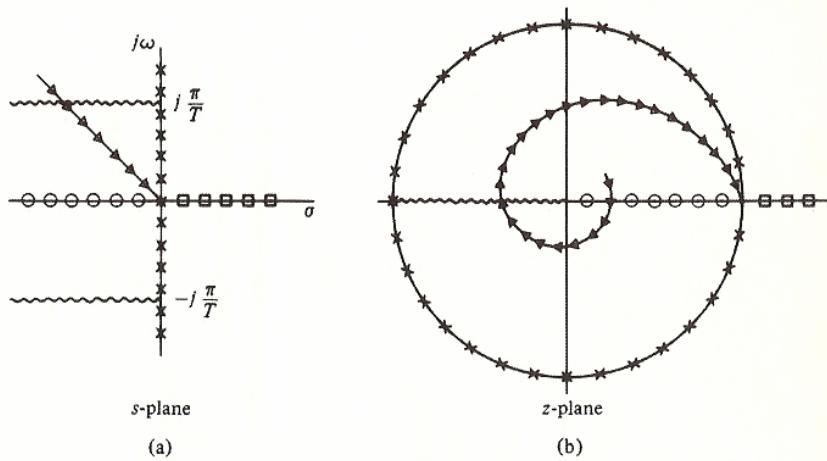


Figure 8.5 Time sequences associated with points in the z-plane