Spring 2019

數位控制系統
Digital Control Systems

DCS-35
Discretized Controller – Techniques for Enhancing Performance (T/2-Delay Compensation)

Feng-Li Lian
NTU-EE

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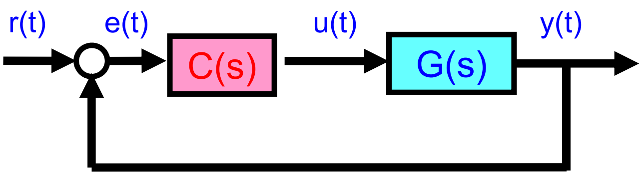
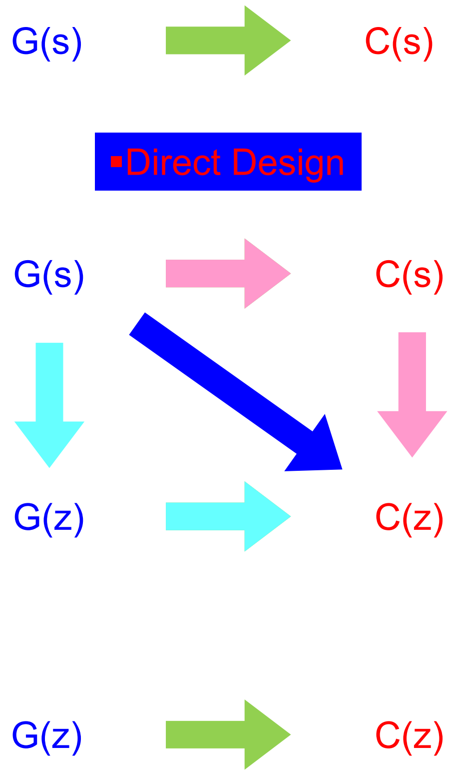
Figures and images used in these lecture notes are adopted from

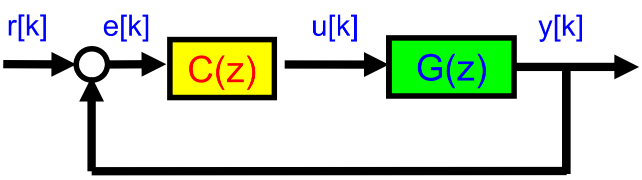
1: B.D.O. Anderson, "Controller Design: Moving from Theory to Practice," IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993
2: D. Raviv & E.W. Djaja, "Technique for Enhancing the Performance of Discretized Controllers," IEEE Control Systems Magazine, 19(3), pp. 52-57, June 1999

Introduction: CT and DT Plant-Controller



- Discrete Design
  - Transform CT plant into DT plant
  - By DT plant, design DT controller
- Emulation
  - By CT plant, design CT controller
  - Transform CT controller into DT controller





Basic Design Concept

- Basic principles of low-order controller design

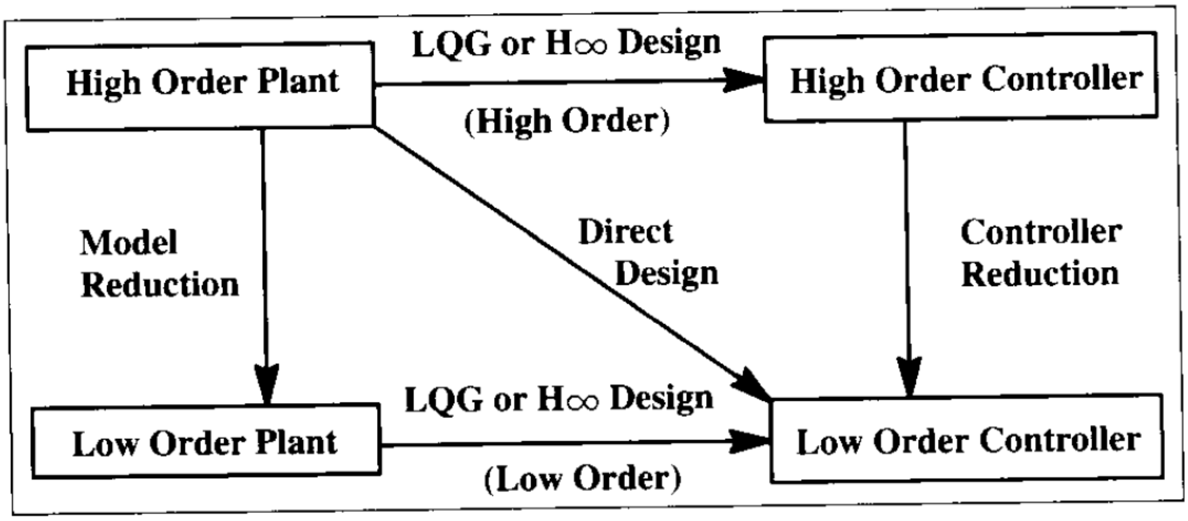


Fig. 1. Basic principles of low order controller design.

B.D.O. Anderson, "Controller Design: Moving from Theory to Practice," IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993

Anderson 1993

Introduction: CT and DT Plant-Controller

- Study in Digital Control Systems
  - Controller Design of Digital Control Systems
    - **Design Process**
      - > Discrete Design:
        - » CT plant -> DT plant -> DT controller
      - > Emulation:
        - » CT plant -> CT controller -> DT controlle
      - > Direct Design: (B.D.O. Anderson, 1992 Bode Prize Lecture)
        - » CT plant -> DT controller

Outline

- Discrete Design
  - By Transfer Function
  - By State Space
- Design by Emulation
  - Tustin's Method or bilinear approximation
  - Matched Pole-Zero method (MPZ)
  - Modified Matched Pole-Zero method (MMPZ)
  - Digital PID-Controllers
- Techniques for Enhancing the Performance

Basic Design Concept

- Basic principles of low-order controller design

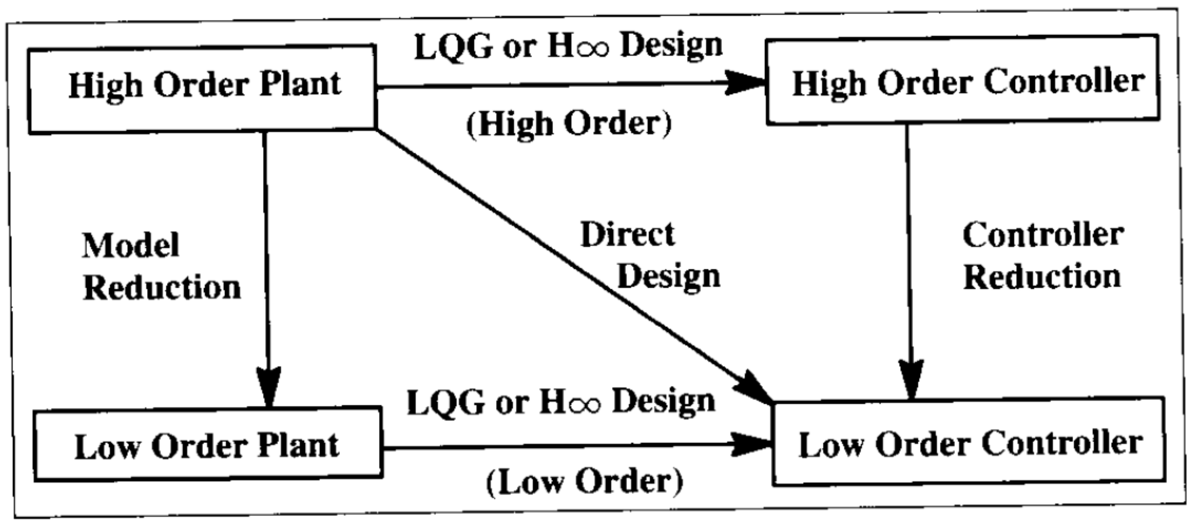


Fig. 1. Basic principles of low order controller design.

Anderson 1993

Basic Design Concept

- Textbook schemes for replacing a CT controller by a DT one

With $C(s)$ continuous time and $C_d(z)$ discrete time,

$C_d(z) = C\left(\frac{z-1}{T}\right)$	Euler or forward difference
$C_d(z) = C\left(\frac{z-1}{zT}\right)$	Balanced difference
$C_d(z) = C\left(\frac{2}{T}\frac{z-1}{z+1}\right)$	Tustin or bilinear
$C_d(z)=C\left(\frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}\frac{z-1}{Z+1}\right)$	Tustin with prewarping
$C_d(z)=\frac{(z-1)}{Tz}\frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\frac{e^{sT}}{z-e^{sT}}\frac{G(s)}{s}ds$	Step-invariance
$C_d(z)=\frac{(z-1)^2}{Tz}\frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\frac{e^{sT}}{z-e^{sT}}\frac{G(s)}{s^2}ds$	Ramp-invariance

Poles and zeros of $C_d(z)$ are images under $z = e^{sT}$ of those of $C(s)$, with $C_d(1) = C(0)$.

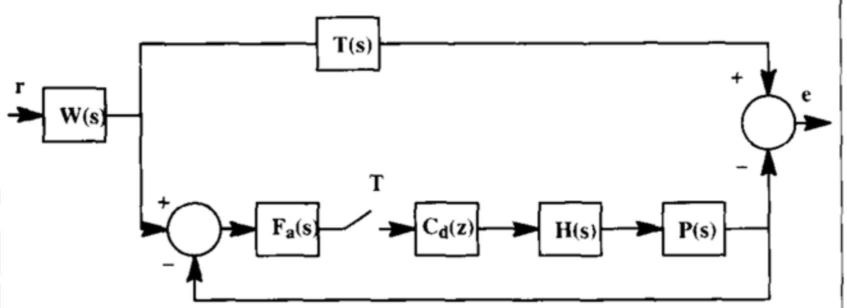
Zero-order hold equivalance.
First-order hold equivalance.
Triangular-hold equivalance.

Anderson 1993

Basic Design Concept

- Measuring the difference
  between using Continuous and Discrete controllers

- **Data**
  Shaping or Weighting Filter W(s), rational, stable and strictly proper
- **Assumption**
  $C_d(z)$ (unknown) is stabilizing



- **Operator from $r \in L_2(0, \infty)$ to $e \in L_2(0, \infty)$ is**
  - **bounded**
  - **dependent on $C_d(z)$**
  - ***not describable via a transfer function***
  - **periodically time–varying**

Fig. 7. Measuring the difference between using continuous and discrete controllers.

Anderson 1993

Problem Formulation

- Digital control design through discretizing an analog controller

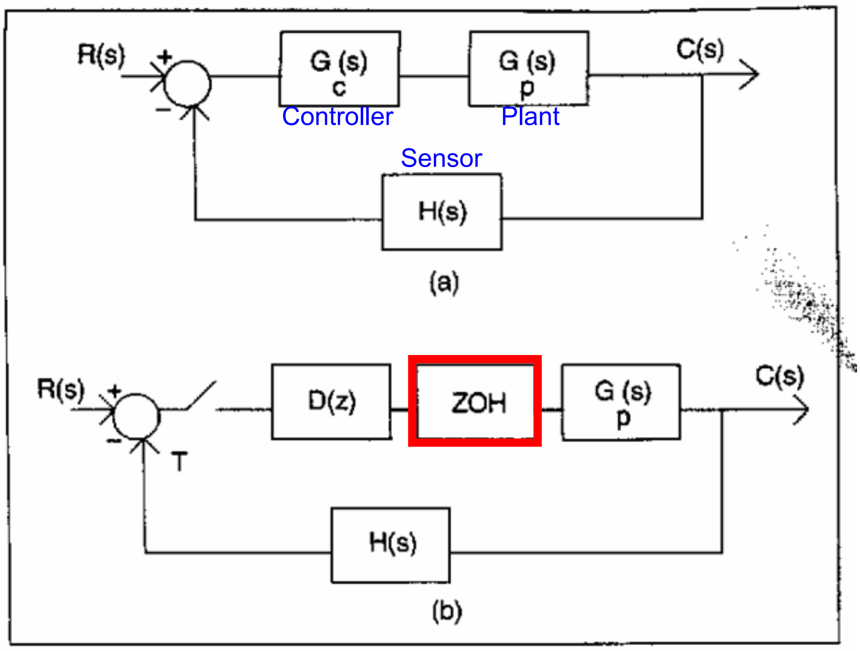


Fig. 1. (a) The analog closed-loop control system, (b) The digital closed-loop control system.

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Problem Formulation

- Given
  - A process $G_p(s)$
  - A sensor $H(s)$
  - A presumably well designed analog controller $G_c(s)$
- Find
  - A digital controller $D(z)$
    which produces closed-loop behavior
    similar to the analog system
    both in the time and frequency domains

Raviv & Djaja 1999

- Solutions:
  - Analog control design followed by controller discretization
    - More convenient
    - Deal with sampling time T at the final phase
  - Direct digital control design
  - To enhance the performance by the first method
    - Add a pole-zero pair in the z-plane
    - To compensate for the low-frequencies and mid-frequencies phase and gain response effects contributed by ZOH

Raviv & Djaja 1999

- Potential problem:
  - The ZOH causes a delay of approximately T/2

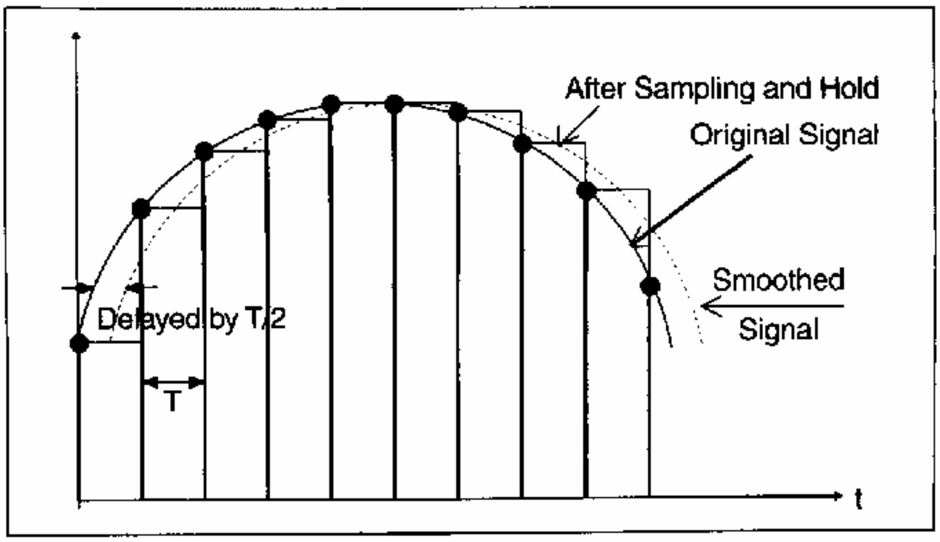

Fig. 2. A reconstructed signal using ZOH and its smoothed approximation.

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▪ A pole-zero compensation for delay:

$$C(z) = \frac{2z}{z+1}$$

– Provides a phase of ($\omega T/2$)

– Which exactly cancels the frequency phase response of the ZOH obtained from

$$\frac{1-e^{-sT}}{s}$$

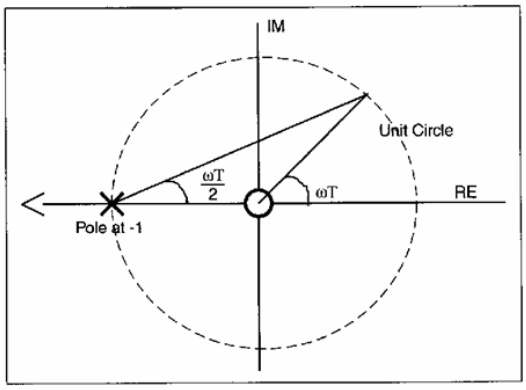

Fig. 3. The location of pole and zero of ZOH compensator in the Z-domain.

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▪ A pole-zero compensation for delay:

- The ZOH transfer function:

$$\frac{1-e^{-sT}}{s} \approx \frac{T}{1+\frac{sT}{2}}$$ 1st-order Pade approximation

$$\Rightarrow \left.\frac{T}{1+\frac{sT}{2}}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{T}{2}\frac{z+1}{z}$$ Tustin transformation

- The characteristic polynomial

$$1 + \left(\frac{2z}{z+1}\right) D'(z)\,(1-z^{-1})\mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = 0$$

$$D'(z): \text{ any discretiezed } D(s)$$

Raviv & Djaja 1999

▪ A pole-zero compensation for delay:

- IF the proposed compensation causes instability
  a modified ZOH compensation

$$C'(z) = \frac{2(z-\varepsilon)}{z+1-2\varepsilon}$$

- The characteristic polynomial

$$1+\left(\frac{2(z-\varepsilon)}{z+1-2\varepsilon}\right) D'(z)\,(1-z^{-1})\mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = 0$$

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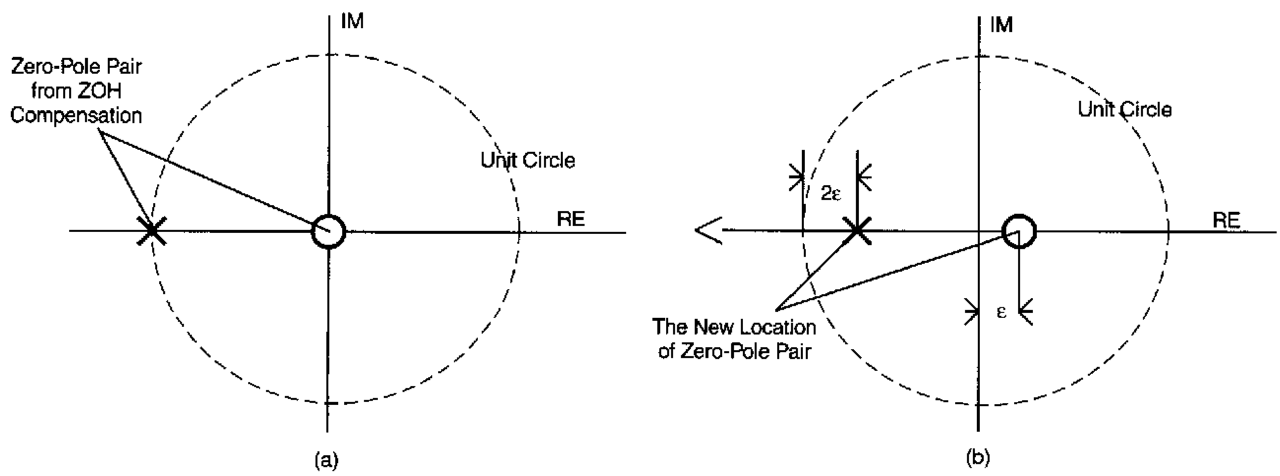
▪ A pole-zero compensation for delay:



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▪ Lag Compensator:

$$G_p(s) = \frac{4 \times 10^6}{s(s+20)(s+200)}$$

$$H(s) = 1$$

- Design specifications:
  1. Velocity error constant K_v at least 1000 s^{-1}
  2. Attenuation of all sinusoidal inputs of frequency above 400 rad/sec by at least 16
  3. Steady-state error of (up to) 1% for sinusoidal inputs for frequencies less than 1 rad/sec

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$$\Rightarrow \quad G_c(s) = \frac{1}{80}\frac{(s+8)}{(s+0.1)}$$

▪ D'(z): by Tustin

Table 1

	D'(z)	Multiplier	D(z)
T = 0.01 s	$\frac{0.0130z-0.0120}{z-0.9990}$	$\frac{2z}{z+1}$	$\frac{0.0260z^2-0.0240z}{z^2+0.0010z-0.9990}$
T = 0.05 s	$\frac{0.0150z-0.0100}{z-0.9950}$	$\frac{2z}{z+1}$	$\frac{0.0299z^2-0.0200z}{z^2+0.0050z-0.9950}$
T = 0.1 s	Unstable $\frac{0.0174z-0.0075}{z-0.9900}$	$\frac{2z}{z+1}$	Unstable $\frac{0.0348z^2-0.0150z}{z^2+0.0100z-0.9900}$
T = 0.1 s	Unstable $\frac{0.0174z-0.0075}{z-0.9900}$	$\frac{2(z-0.2)}{(z+0.6)}$	$\frac{0.0348z^2-0.0219z+0.0030}{z^2-0.3900z-0.5940}$

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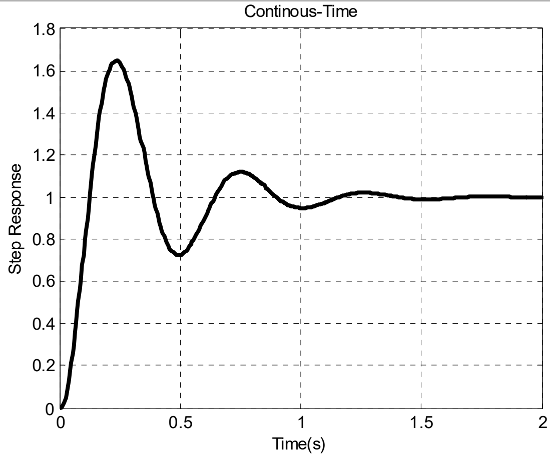
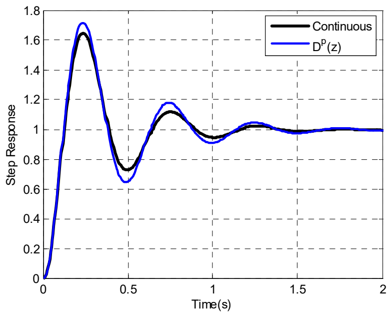
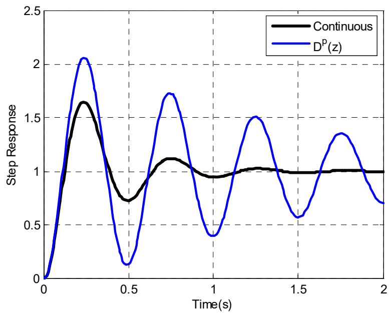
Examples – Lag



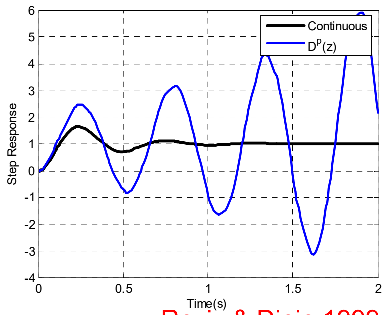
$T = 0.01s$



$T = 0.05s$



$T = 0.1s$



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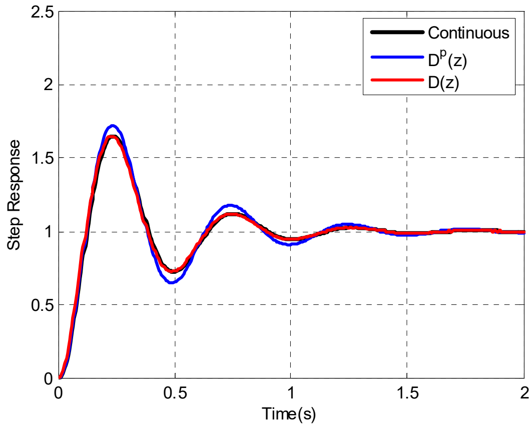
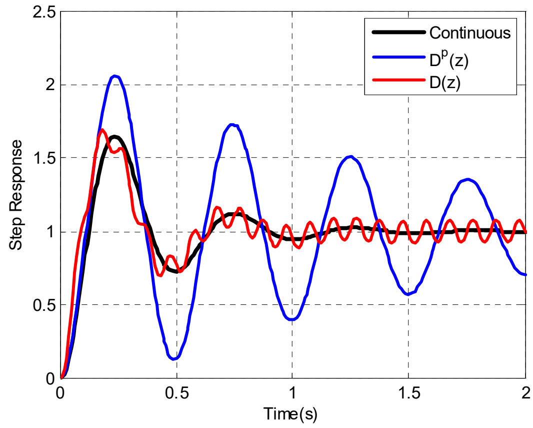
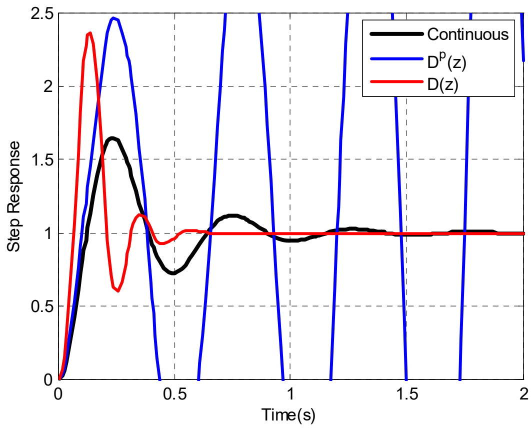
Examples – Lag





$T = 0.01s$

$T = 0.05s$



$T = 0.1s$

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Examples – Lead-Lag

- Lead-Lag Compensator:

$$G_p(s) = \frac{1000}{s(1+\frac{s}{10})(1+\frac{s}{250})}$$

$$H(s) = 1$$

- Design specifications:
  1. Phase margin of at least 50°
  2. Velocity error constant K_v at least 1000 s^{-1}
  3. Attenuation of the input noise at 60 Hz and above by a factor of 100
  4. Steady-state error for frequencies less than 1 rad/sec less than 1%

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Examples – Lead-Lag

$$\Rightarrow \quad G_c(s) = \frac{(1+\frac{s}{4.5})(1+\frac{s}{10})}{(1+\frac{s}{0.1})(1+\frac{s}{110})}$$

?

$$T = 0.01s$$

$$\Rightarrow \quad D'(z) = \frac{0.6597z^2 - 1.2897z + 0.6300}{1.00z^2 - 1.2893z + 0.2900}$$

With the ZOH compensation of $\dfrac{2z}{z+1}$

$$\Rightarrow \quad D(z) = \frac{1.3194z^3 - 2.5793z^2 + 1.26z}{z^3 - 0.2893z^2 - 0.9993z + 0.2900}$$

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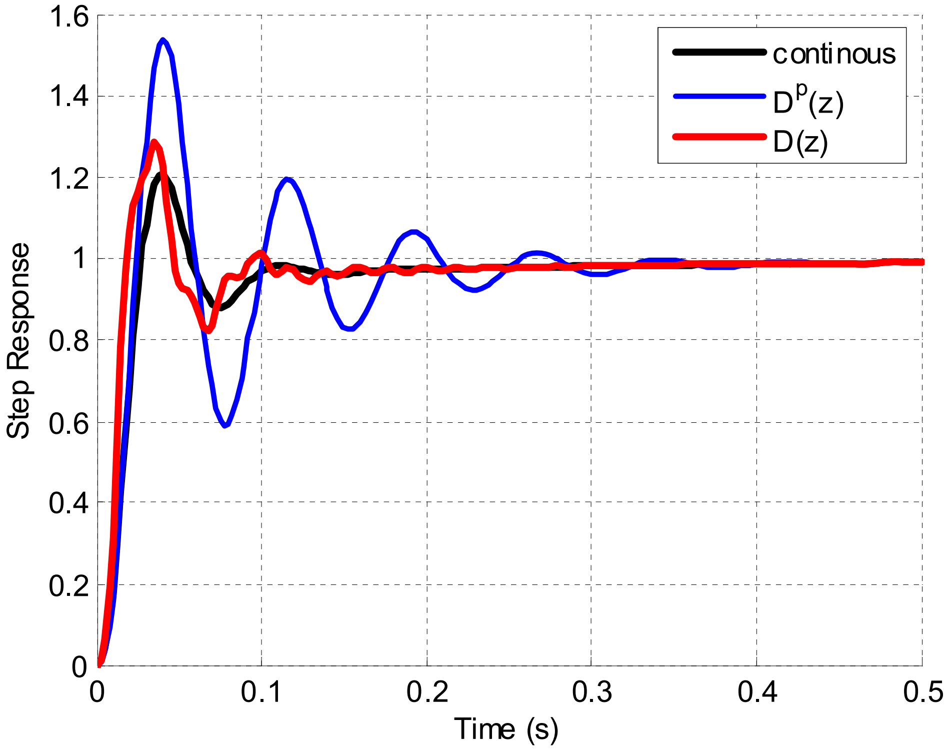
Examples – Lead-Lag



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Examples – Lead-Lag

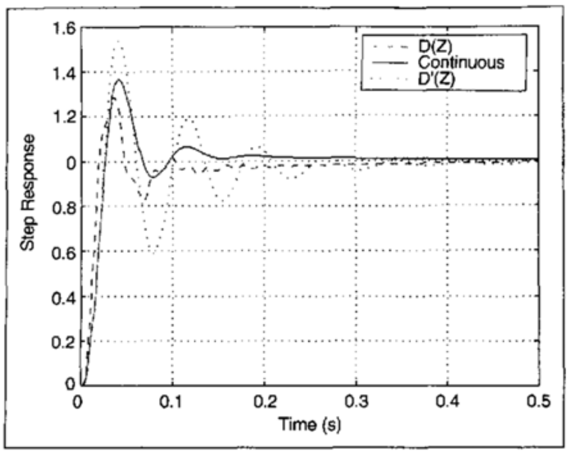
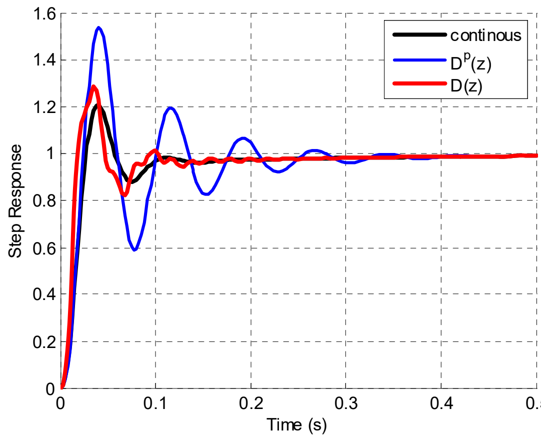
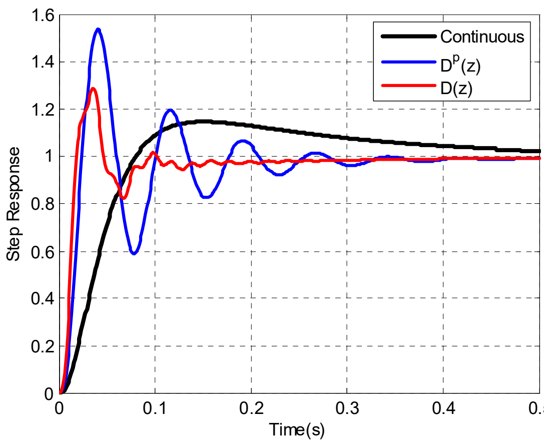


Fig. 6. Closed-loop step response of Example (b). T = 0.01 s.





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- Katz's Example:

$$G_p(s) = \frac{863.3}{s^2}$$

- Design specifications:
  1. Max phase lag at f = 3 Hz should not be more than 13°
  2. At any given frequency the CL gain should not exceed 5 dB beyond the CL dc gain
  3. Max tracking error due to an input disturbance moment of 0.028Nm should not be 0.01 rad

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$$G_c(s) = 2940\frac{(s+29.4)}{(s+294)^2}$$

$$T = 0.03s$$

$$\Rightarrow \quad D'(z) = \frac{1.8958z^2 + 1.1685z - 0.7273}{z^2 + 1.1653z + 0.3395}$$

With the ZOH compensation of $\frac{2z}{z+1}$

$$\Rightarrow \quad D(z) = \frac{3.7916z^3 + 2.3369z^2 - 1.4546z}{z^3 + 2.1653z^2 + 1.5047z + 0.3395}$$

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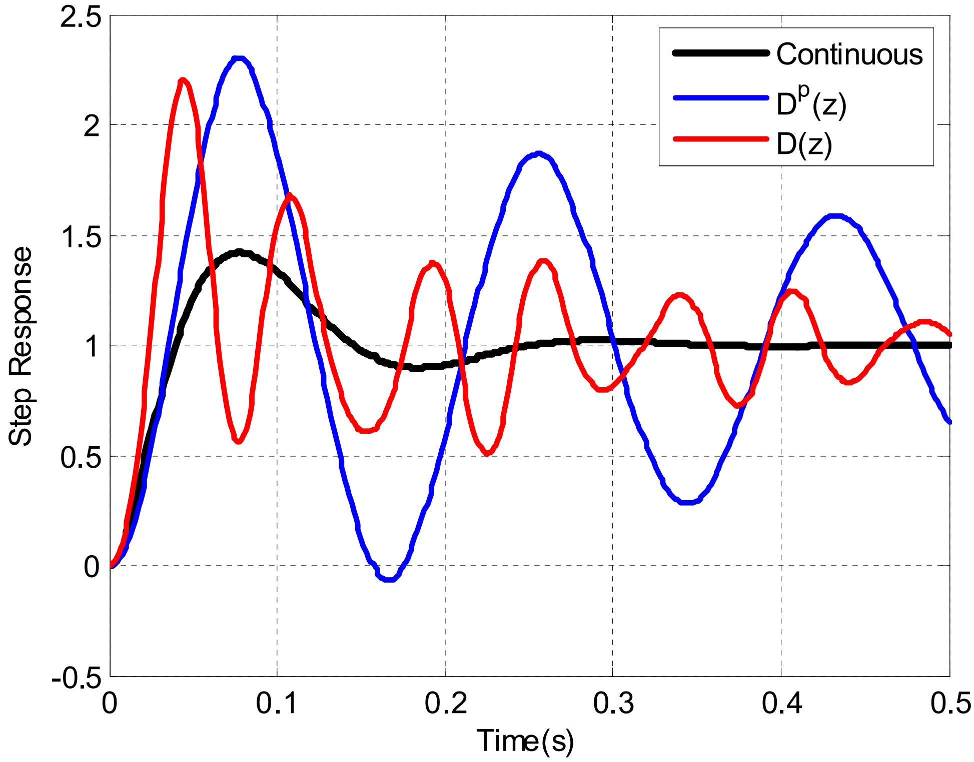
Examples – Katz



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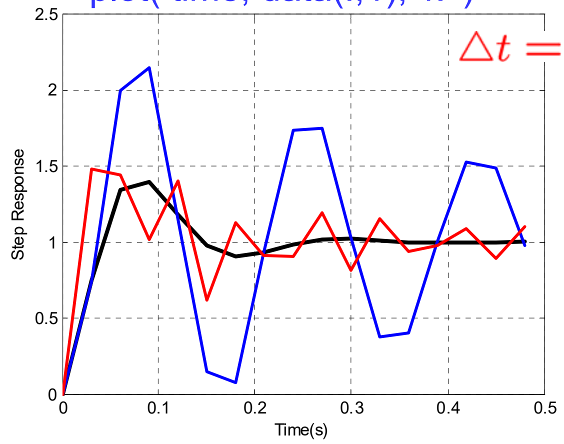
Examples – Katz

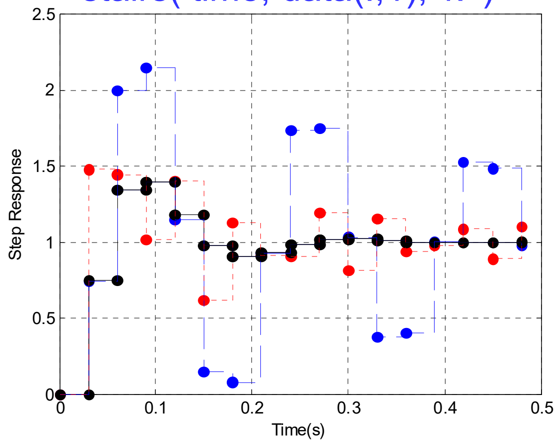
$T = 0.03s$

>> plot(time, data(:,1), 'k-')

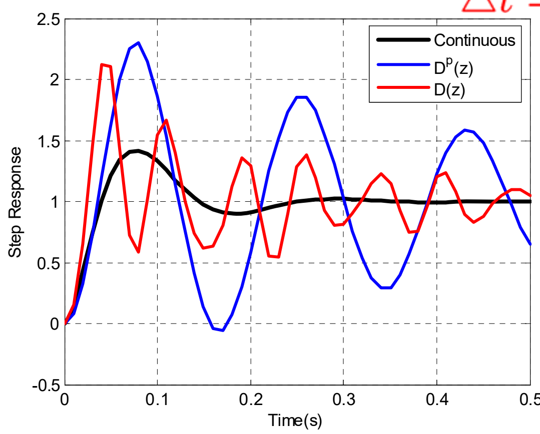
$\triangle t = 0.03s$



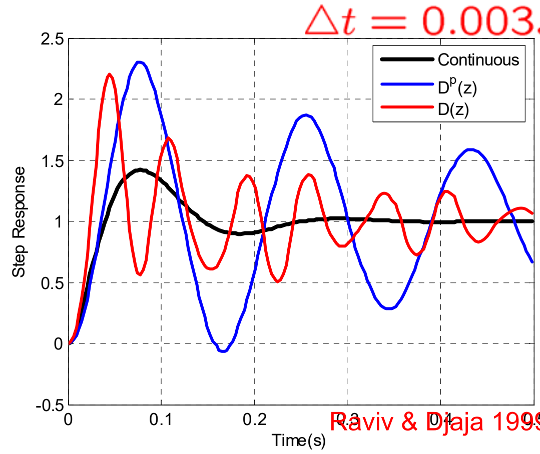
>> stairs(time, data(:,1), 'k-')



$\triangle t = 0.01s$



$\triangle t = 0.003s$



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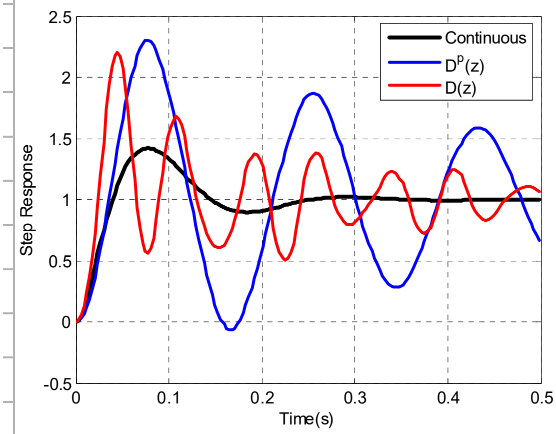
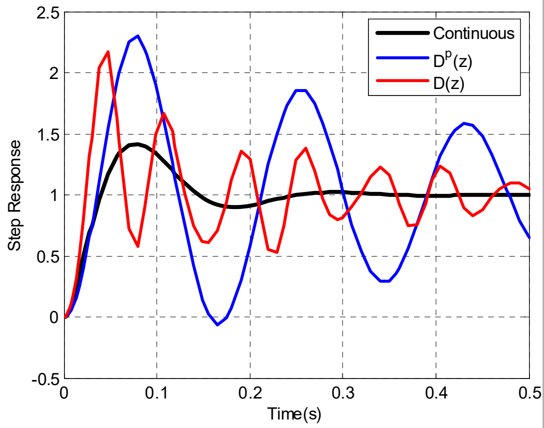
Examples – Katz

$T = 0.03s$

$\triangle t = 0.003s$



Sample time = -1 (inherited)



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Examples – Rattan

- Rattan's Example:

$$G_p(s) = \frac{10}{s(s+1)}$$

$$\Rightarrow \quad G_c(s) = \frac{1+0.416s}{1+0.319s}$$

$T = 0.15s$

$$\Rightarrow \quad D_{\text{Rattan}}(z) = \frac{3.436z - 2.191}{z + 0.2390}$$

$$\Rightarrow \quad D'(z) = \frac{2.294z - 1.5935}{z - 0.2991}$$

Tustin transformation

$$\Rightarrow \quad D(z) = \frac{4.5888z^2 - 3.6459z + 0.3187}{z^2 + 0.5009z - 0.2393}$$

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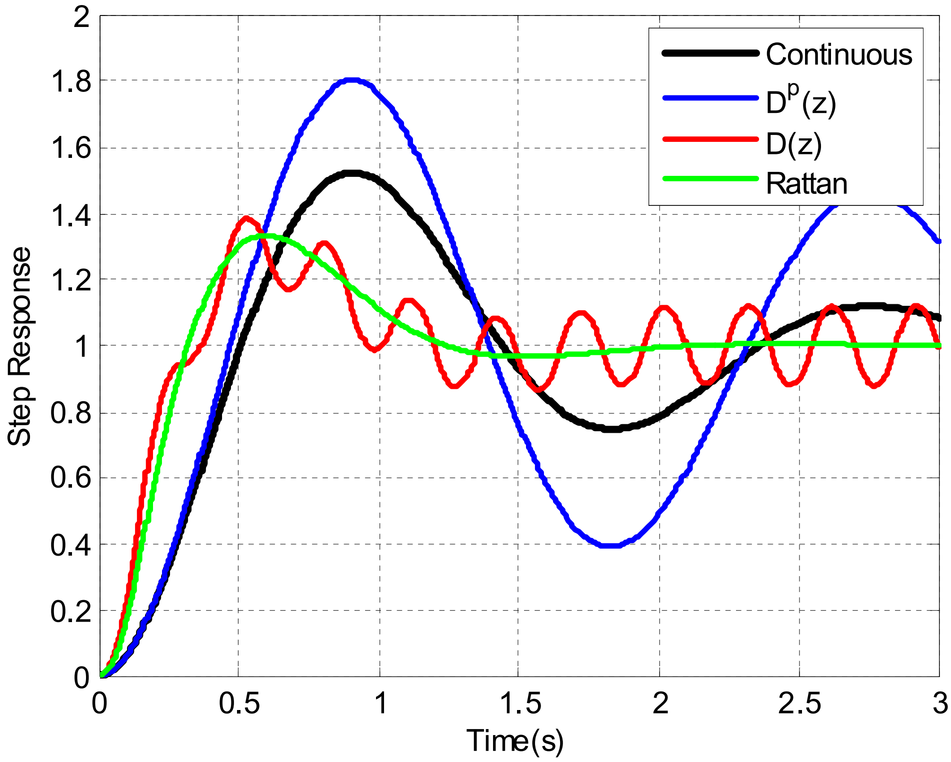
Examples – Rattan



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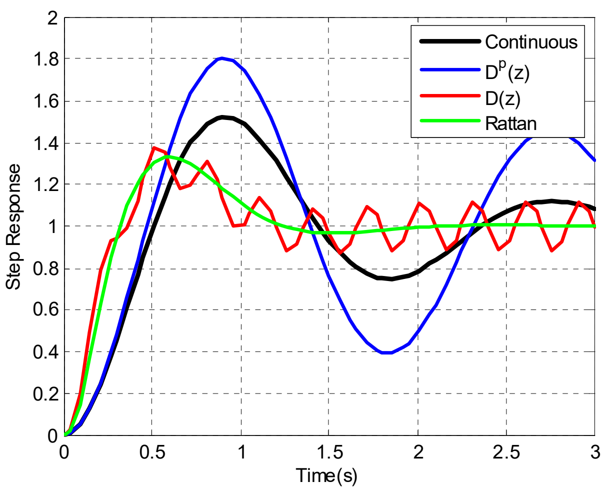
Examples – Rattan



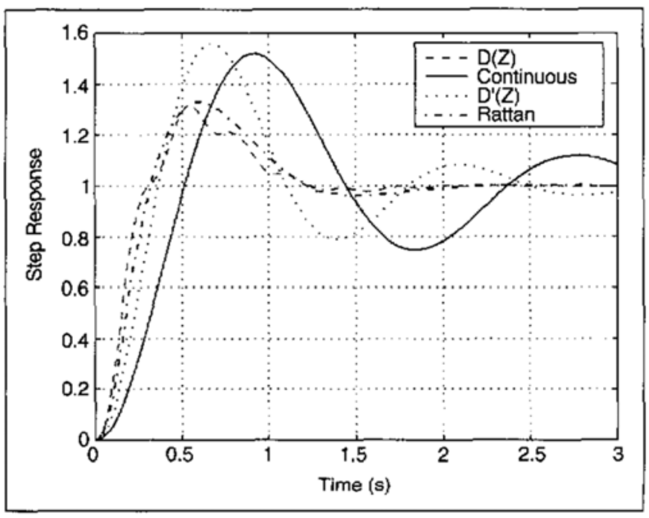


Fig. 8. Closed-loop step response of Example (d). T = 0.15 s.

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