Spring 2019

數位控制系統

Digital Control Systems

# DCS-34
# Discretized Controller – Design by Emulation


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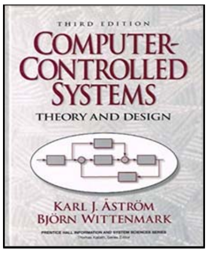
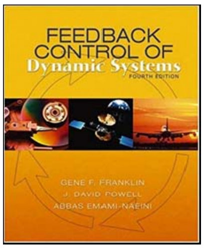
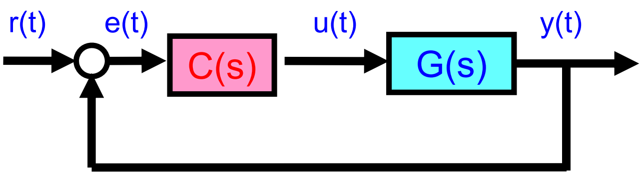
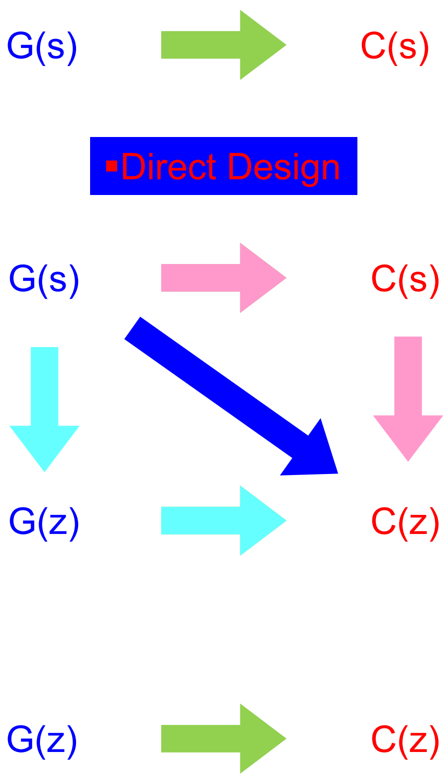


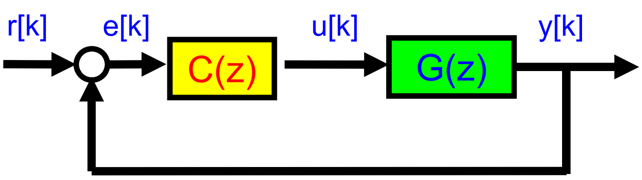

## Introduction: CT and DT Plant-Controller






- Discrete Design
  - Transform CT plant into DT plant
  - By DT plant, design DT controller
- Emulation
  - By CT plant, design CT controller
  - Transform CT controller into DT controller



## Basic Design Concept

- Basic principles of low-order controller design

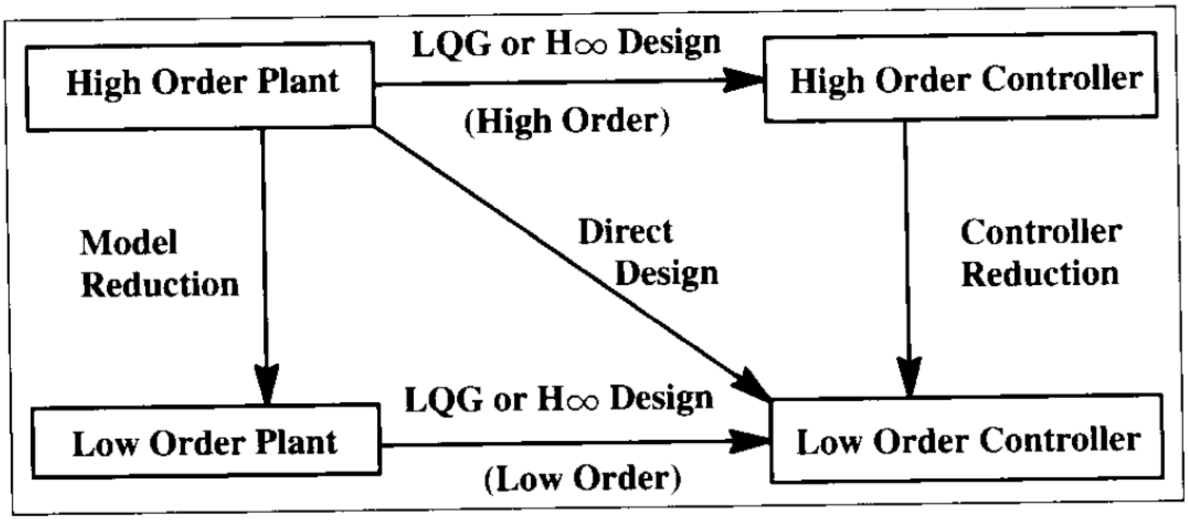



*Fig. 1. Basic principles of low order controller design.*

B.D.O. Anderson, "Controller Design: Moving from Theory to Practice," IEEE Control Systems Magazine, 13(4), pp. 16-25, Aug. 1993

Anderson 1993



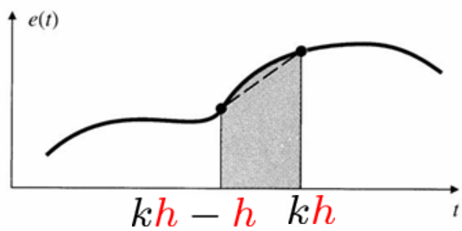
## Introduction: CT and DT Plant-Controller

- Study in Digital Control Systems
  - Controller Design of Digital Control Systems
    - **Design Process**
      - > Discrete Design:
        - » CT plant -> DT plant -> DT controller
      - > Emulation:
        - » CT plant -> CT controller -> DT controlle
      - > Direct Design: (B.D.O. Anderson, 1992 Bode Prize Lecture)
        - » CT plant -> DT controller

- Discrete Design
  - By Transfer Function
  - By State Space
- Design by Emulation
  - Tustin's Method or bilinear approximation
  - Matched Pole-Zero method (MPZ)
  - Modified Matched Pole-Zero method (MMPZ)
  - Digital PID-Controllers
- Techniques for Enhancing the Performance



- Tustin's method:




$$\frac{U(s)}{E(s)} = D(s) = \frac{1}{s}$$

$$\Rightarrow \quad u[kT] = \int_0^{kh-h} e(t)dt + \int_{kh-h}^{kh} e(t)dt$$

$$= u[kh-h] + \text{ area under } e(t) \text{ over last } h$$

$$\Rightarrow \quad u[k] = u[k-1] + \frac{h}{2}\left[e[k-1] + e[k]\right]$$

$$\Rightarrow \quad \frac{U(z)}{E(z)} = D(z) = \frac{h}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) = \frac{1}{\frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Franklin et al. 2002

- Tustin's method:

$$\frac{U(s)}{E(s)} = D(s) = \frac{a}{s+a}$$

$$\Rightarrow \frac{U(z)}{E(z)} = D(z) = \frac{a}{\frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+a}$$

$$\Rightarrow s = \frac{2}{h}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

for every occurance of $s$ in any $D(s)$
yields a $D(z)$ on the trapezoidal integration

Franklin et al. 2002



- Tustin's method:

$$D(s) = 10\,\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$$w_s = 25 \times w_{\text{BW}} = 25 \times 10 = 250 \text{ rad/sec}$$

$$f_s = w_s/(2\pi) \approx 40 \text{ Hz}$$

$$h = \frac{1}{f_s} = \frac{1}{40} = 0.025 \text{ sec}$$

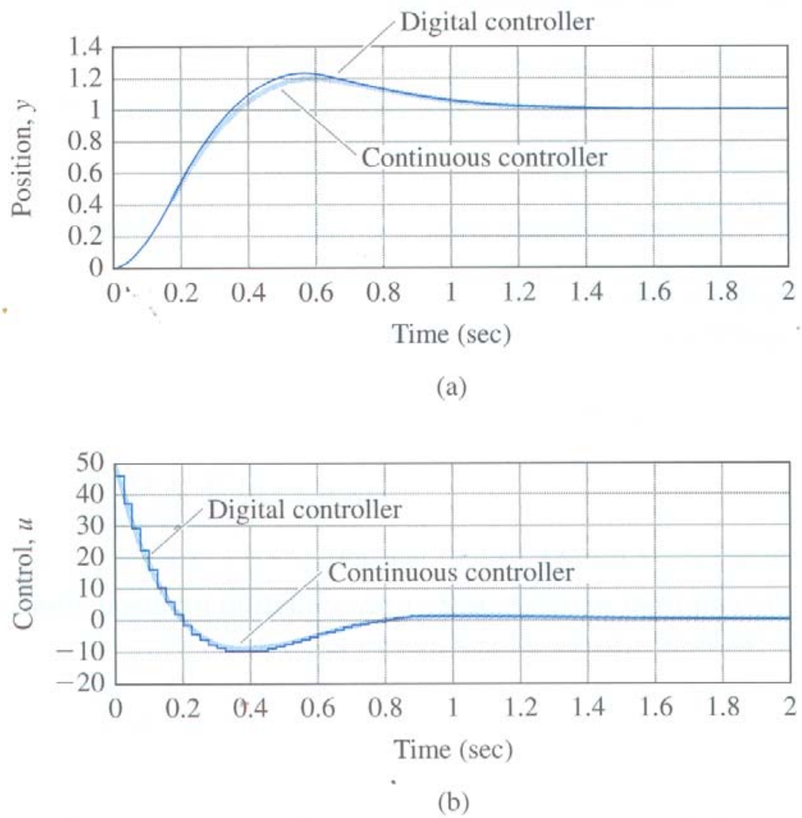
```
sysDs = tf( 10*[0.5 1], [0.1 1] );
sysDz = c2d( sysDs, 0.025, 'tustin');
```

$$\Rightarrow D(z) = \frac{45.56 - 43.33z^{-1}}{1-0.7778z^{-1}}$$

$$\Rightarrow u[k] = 0.7778u[k-1] + 45.56e[k] - 43.33e[k-1]$$

Franklin et al. 2002

- Tustin's method:

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- MATLAB's command: c2d

C2D Conversion of continuous-time systems to discrete time.

SYSD = C2D( SYSC, TS, METHOD )
converts the continuous system SYSC to a discrete-time system SYSD
with sample time TS.
The string METHOD selects the discretization method among the
following:

| | |
|---|---|
| 'zoh' | Zero-order hold on the inputs. |
| 'foh' | Linear interpolation of inputs (triangle appx.) |
| 'tustin' | Bilinear (Tustin) approximation. |
| 'prewarp' | Tustin approximation with frequency prewarping. The critical frequency Wc is specified last as in C2D( SysC, Ts, 'prewarp', Wc) |
| 'matched' | Matched pole-zero method (for SISO systems only). |

- MathWorks: c2d: https://www.mathworks.com/help/control/ref/c2d.html

- Differentiation & Tustin Approximation:
  - Forward difference: (Euler's method)

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t+h) - x(t)}{h} = \frac{q-1}{h} x(t)$$

  - Backward difference:

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t) - x(t-h)}{h} = \frac{q-1}{qh} x(t)$$

Astrom & Wittenmark 1997



- Differentiation & Tustin Approximation:
  - Trapezoidal method: (Tustin, bilinear)

$$\frac{\dot{x}(t+h) + \dot{x}(t)}{2} \approx \frac{x(t+h) - x(t)}{h}$$

$$\frac{q+1}{2} p\, x(t) \approx \frac{q-1}{h} x(t)$$

$$p\, x(t) \approx \frac{2}{h} \cdot \frac{q-1}{q+1} x(t)$$

Astrom & Wittenmark 1997

## Properties of Approximations:

$$G(z) = G(s) \qquad p\,x(t) \approx \begin{cases} \dfrac{q-1}{h}\,x(t) & \text{Forward} \\ \dfrac{q-1}{qh}\,x(t) & \text{Backward} \\ \dfrac{2}{h}\cdot\dfrac{q-1}{q+1}\,x(t) & \text{Trapezoidal} \end{cases}$$

$$s = \frac{z-1}{h} \quad \text{(Forward difference or Euler's method)}$$

$$s = \frac{z-1}{zh} \quad \text{(Backward difference)}$$

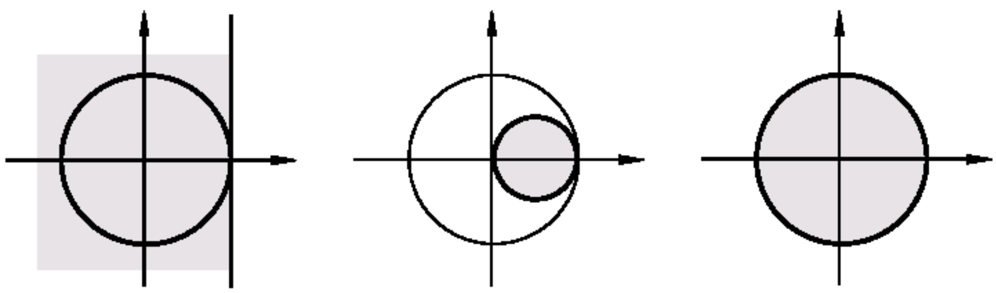
$$s = \frac{2}{h}\frac{z-1}{z+1} \quad \text{(Tustin's or bilinear approximation)}$$

- What is the difference between the above approximations and $z = e^{sh}$?

Astrom & Wittenmark 1997



## Stability of Approximations:



Forward differences  Backward differences  Tustin

$$s = \frac{z-1}{h} \qquad s = \frac{z-1}{zh} \qquad s = \frac{2}{h}\frac{z-1}{z+1}$$

$$z = e^{sh} \quad \approx 1 + sh \quad \approx \frac{1}{1-sh} \quad \approx \frac{1+sh/2}{1-sh/2}$$

Astrom & Wittenmark 1997

- Approximations introduce frequency distortion

$$G(z) = G(s)|_{z=e^{sh}} \qquad s = \frac{2}{h}\,\frac{z-1}{z+1}$$

$$\Rightarrow\ G(e^{iwh}) = \frac{1}{iwh}(1-e^{-iwh})G\left(\frac{2}{h}\cdot\frac{e^{iwh}-1}{e^{iwh}+1}\right)$$

$$\Rightarrow\ \frac{2}{h}\cdot\frac{e^{iwh}-1}{e^{iwh}+1} = \frac{2}{h}\cdot\frac{e^{iwh/2}-e^{-iwh/2}}{e^{iwh/2}+e^{-iwh/2}}$$

$$= i\cdot\left(\frac{2}{h}\tan\left(\frac{wh}{2}\right)\right)$$

$$= i\cdot w'$$

$$G(z=e^{iwh}) = G(s=iw) \qquad G\left(i\cdot\left(\frac{2}{h}\tan\left(\frac{wh}{2}\right)\right)\right) = G\left(i\cdot w'\right)$$

Astrom & Wittenmark 1997





- Approximations introduce frequency distortion

$$G(z) = G(s)|_{z=e^{sh}}$$

$$z = e^{iw} \iff s = iw'$$

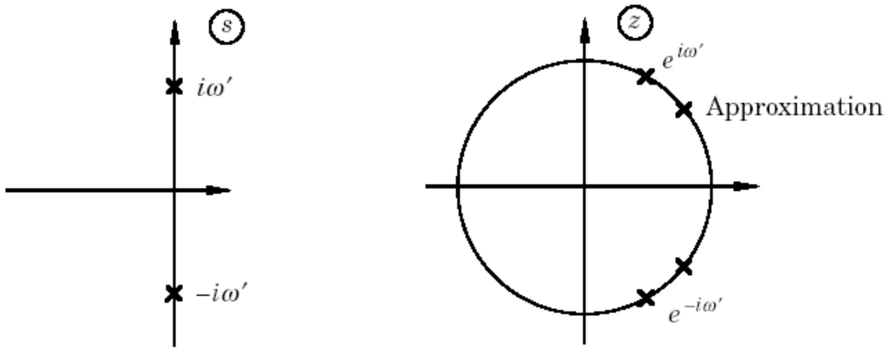
$$\Rightarrow\ w' = \frac{2}{h}\tan\left(\frac{wh}{2}\right)$$

$$\Rightarrow\ w = \frac{h}{2}\tan^{-1}\left(\frac{w'h}{2}\right)$$

$$\approx w'\left(1-\frac{(w'h)^2}{12}\right)$$

Astrom & Wittenmark 1997

- Approximations introduce frequency distortion




Astrom & Wittenmark 1997



- Tustin with Prewarping:

$$s' = \frac{w_1}{\tan(w_1 h/2)} \cdot \frac{z-1}{z+1} \qquad s = \frac{2}{h}\,\frac{z-1}{z+1}$$

$$\Rightarrow \; G(e^{iw_1 h}) = G(iw_1)$$

- No distortion 0 (always true) and at $w_1$
- Still distortion at other frequencies

Astrom & Wittenmark 1997

- Matched Pole-Zero (MPZ) method:

$$z = e^{sh}$$

1. Map poles and zeros according to the relation

2. If the numerator is of lower order than the denominator, add powers of (z+1) to the numerator until numerator and denominator are of equal order

3. Set the DC or low-frequency gain of D(z) = that of D(s)

Franklin et al. 2002



- Matched Pole-Zero (MPZ) method:
  - Case 1:

$$D(s) = K_c \frac{s+a}{s+b}$$

$$\Rightarrow D(z) = K_d \frac{z-e^{-ah}}{z-e^{-bh}}$$

- By the Final Value Theorem:

$$K_c \frac{a}{b} = K_d \frac{1-e^{-ah}}{1-e^{-bh}}$$

$$\text{or} \quad K_d = K_c \frac{a}{b} \left( \frac{1-e^{-bh}}{1-e^{-ah}} \right)$$

Franklin et al. 2002

- Matched Pole-Zero (MPZ) method:
  - Case 2:

$$D(s) = K_c \frac{s+a}{s(s+b)}$$

$$\Rightarrow \quad D(z) = K_d \frac{(z+1)(z-e^{-ah})}{(z-1)(z-e^{-bh})}$$

- By the Final Value Theorem:

$$\Rightarrow \quad K_d = K_c \frac{a}{2b} \left( \frac{1-e^{-bh}}{1-e^{-ah}} \right)$$

Franklin et al. 2002



- Matched Pole-Zero (MPZ) method:
  - The same power of z in the num & den of D(z):

$$\frac{U(z)}{E(z)} = D(z) = K_d \frac{1-\alpha z^{-1}}{1-\beta z^{-1}}$$

$$\alpha = e^{-ah} \ \ \& \ \ \beta = e^{-bh}$$

$$\Rightarrow \quad u[k+1] = \beta u[k] + K_d \left[ e[k+1] - \alpha e[k] \right]$$

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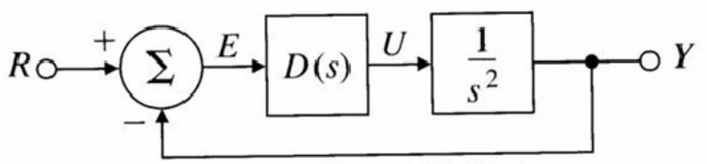
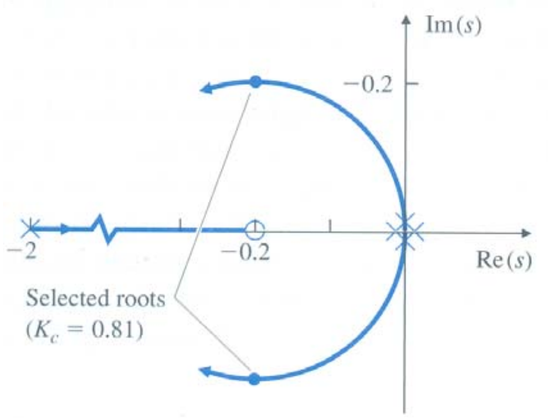
- Matched Pole-Zero (MPZ) method:
  - Space station attitude digital controller




$$w_n \approx 0.3 \text{ rad/sec}$$
$$\zeta = 0.7$$

$$\Rightarrow \quad D(s) = 0.81 \frac{s+0.2}{s+2}$$




Franklin et al. 2002



- Matched Pole-Zero (MPZ) method:
  - Space station attitude digital controller

$$w_s = 0.3 \times 20 = 6 \text{ rad/sec}$$

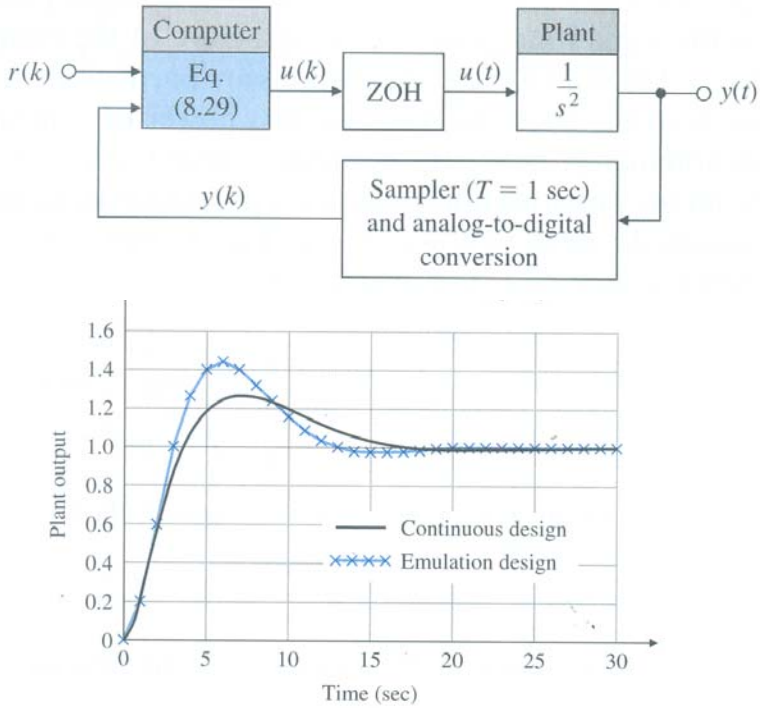
$$\Rightarrow \quad h \approx 1 \text{ sec}$$

$$\Rightarrow \quad D(z) = 0.389 \frac{z-0.82}{z-0.135} = \frac{0.389 - 0.319z^{-1}}{1-0.135z^{-1}}$$

$$\Rightarrow \quad u[k+1] = 0.135u[k] + 0.389e[k+1] - 0.319e[k]$$

Franklin et al. 2002

- Matched Pole-Zero (MPZ) method:
  - Space station attitude digital controller

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- Modified Matched Pole-Zero (MMPZ) method:
  - u[k+1] depends only on e[k], but not e[k+1]

$$D(s) = K_c \frac{s+a}{s(s+b)}$$

$$\Rightarrow \quad D(z) = K_d \frac{(z - e^{-ah})}{(z-1)(z-e^{-bh})}$$

$$\Rightarrow \quad K_d = K_c \frac{a}{b} \left( \frac{1-e^{-bh}}{1-e^{-ah}} \right)$$

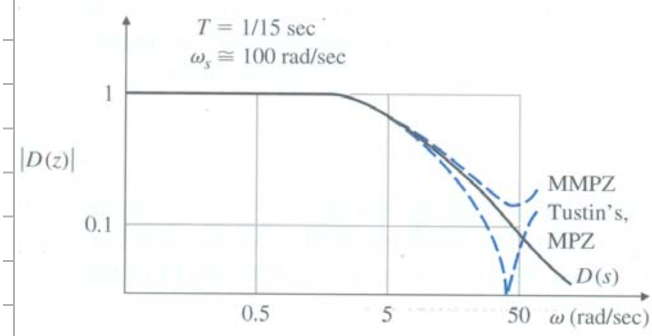
$$\Rightarrow \quad u[k+1] = (1+e^{-bh})u[k] - e^{-bh}u[k-1] + K_d \left[ e[k] - e^{-ah}e[k-1] \right]$$

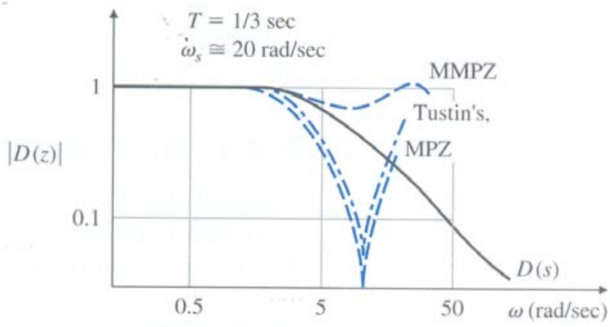
Franklin et al. 2002

## ▪ Comparison of Digital Approximation Methods:

$$D(s) = \frac{5}{s+5}$$

| | $w_s$ | $w_s$ |
|---|---|---|
| Method | 100 rad/sec | 20 rad/sec |
| MPZ | $0.143\frac{z+1}{z-0.715}$ | $0.405\frac{z+1}{z-0.189}$ |
| MMPZ | $0.285\frac{1}{z-0.715}$ | $0.811\frac{1}{z-0.189}$ |
| Tustin's | $0.143\frac{z+1}{z-0.713}$ | $0.454\frac{z+1}{z-0.0914}$ |

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## ▪ The "textbook" version of the PID-controller:

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int^t e(s)\,ds + T_d\frac{de(t)}{dt}\right) \qquad e = u_c - y$$

- $e = u_c - y$:
  difference between command and output
- $K$: gain or proportional gain
- $T_i$: integration time or reset time
- $T_d$: derivative time

Astrom & Wittenmark 1997

- Modification of Linear Response:
  - A pure derivative cannot be,
    and should not be, implemented:
    - Because amplification of measurement noise
    - Derivative gain should be limited

$$sT_d \approx \frac{sT_d}{1 + sT_d/N}$$

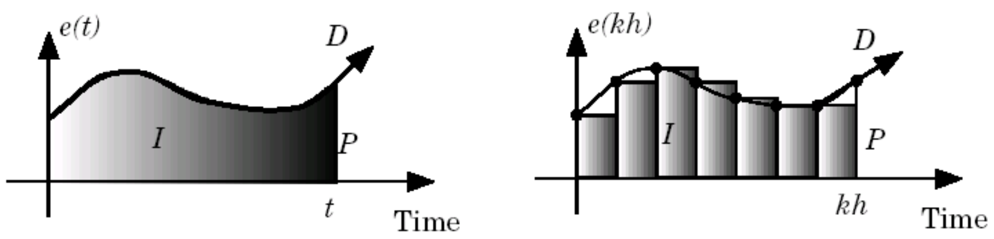
  - Two-input-&-one-output controller:

$$U(s) = K\left(bU_c(s) - Y(s) + \frac{1}{sT_i}\left(U_c(s) - Y(s)\right) - \frac{sT_d}{1 + sT_d/N}\,Y(s)\right)$$

Astrom & Wittenmark 1997



- Discrete-time PID:




  - One popular approximation:

P-part: $P(t) = K\left(bu_c(t) - y(t)\right)$ (No Approximation)

I-part: $I(kh + h) = I(kh) + \frac{Kh}{T_i}\,e(kh)$ (Forward)

D-part: $D(kh) = \frac{T_d}{T_d + Nh}\,D(kh - h) - \frac{KT_dN}{T_d + Nh}\left(y(kh) - y(kh - h)\right)$ (Backward)

Astrom & Wittenmark 1997

- Discrete-time PID:

$$u(kh) = P(kh) + I(kh) + D(kh)$$

  - Can be written as:

$$R(q)u(kh) = T(q)u_c(kh) - S(q)y(kh)$$

with $R(q) = (q-1)(q-a_d)$,
$S(q)$, and $T(q)$ of 2nd order

Astrom & Wittenmark 1997



- Coefficients in different approximations:

| | Special | Tustin | Ramp Equivalence |
|---|---|---|---|
| $s_0$ | $K(1+b_d)$ | $K(1+b_i+b_d)$ | |
| $s_1$ | $-K(1+a_d+2b_d-b_i)$ | $-K\left(1+a_d+2b_d-b_i(1-a_d)\right)$ | |
| $s_2$ | $K(a_d+b_d-b_ia_d)$ | $K(a_d+b_d-b_ia_d)$ | |
| $t_0$ | $Kb$ | $K(b+b_i)$ | |
| $t_1$ | $-K(b(1+a_d)-b_i)$ | $-K\left(b(1+a_d)-b_i(1-a_d)\right)$ | |
| $t_2$ | $Ka_d(b-b_i)$ | $Ka_d(b-b_i)$ | |
| $a_d$ | $\dfrac{T_d}{Nh+T_d}$ | $\dfrac{2T_d-Nh}{2T_d+Nh}$ | $\exp\left(-\dfrac{Nh}{T_d}\right)$ |
| $b_d$ | $Na_d$ | $\dfrac{2NT_d}{2T_d+Nh}$ | $\dfrac{T_d}{h}(1-a_d)$ |
| $b_i$ | $\dfrac{h}{T_i}$ | $\dfrac{h}{2T_i}$ | $\dfrac{h}{2T_i}$ |

Astrom & Wittenmark 1997

## Outline



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