Spring 2019

數位控制系統

Digital Control Systems

DCS-32
Input-Output Design
- By Polynomial Approach

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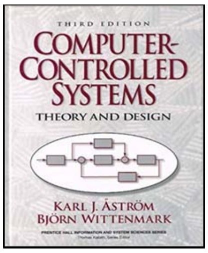
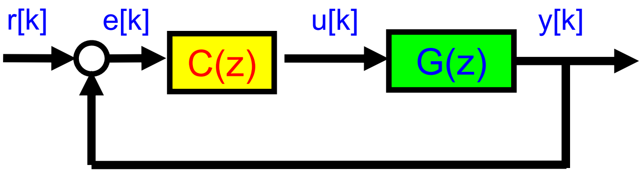
NTU-EE

Feb19 – Jun19



Introduction: Model and Analysis and Design



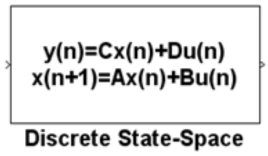
- Plant (DT):
  - Input-Output Model:

$$\frac{Y_d(z)}{\boxed{U_d(z)}} = G_d(z) = \frac{B_d(z)}{A_d(z)}$$

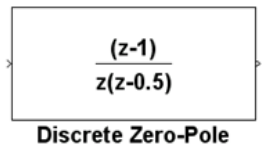
  - State-Space Model:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{F}\mathbf{x}[k] + \mathbf{H}u[k] \\ y[k] &= \mathbf{C}\mathbf{x}[k] + \mathbf{D}u[k] \end{aligned}$$

- System Properties:
  - Stability
  - Controllability and Reachability
  - Observability and Detectability



- Internal model
  - Matrix calculations
- External model
  - Polynomial calculations



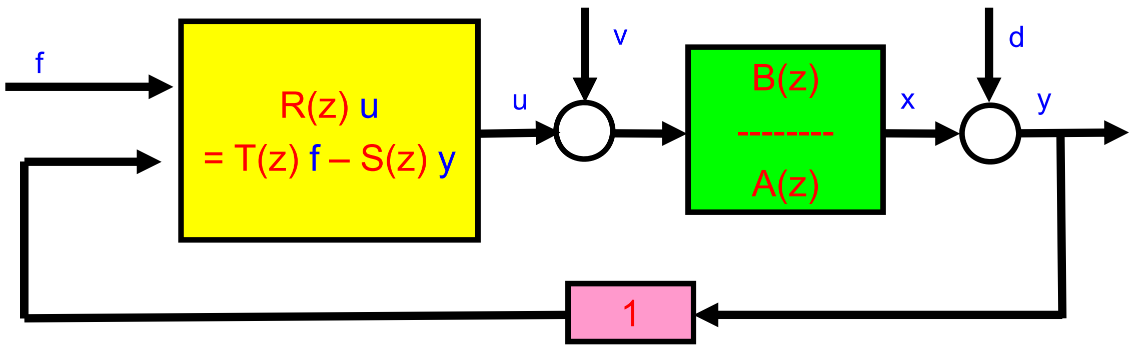
Outline

- Process and Controller Models
  - By Rational Transfer functions
- Poles and Zeros
- Command Signals
- Disturbance Response
- Case Study:
  - Double Integrator

Control Systems Design

- Control System Design:
  - Command signal following (reference)
  - Load disturbance (actuator)
  - Measurement noise (sensor)
  - Process disturbance (un-modeled dynamics)
- Design Parameters:
  - Closed-loop characteristic polynomial
  - Sampling period

Block Diagram of a Typical Control System:



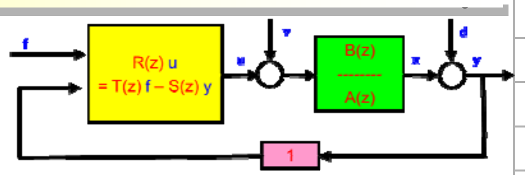
$$(z+0.5)(z-1)u$$

$$= (z+0.3)f \;\; -(z^2-0.2z+1)y$$

$$\frac{(z+1)}{(z^2-0.2z+1)}$$

Simple Design Problem

- The Process Model:



$$A(z)y(k) = B(z)u(k)$$

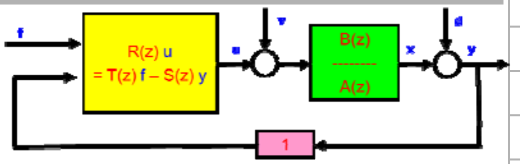
$$\text{Or, } y(k) = \frac{B(z)}{A(z)}u(k)$$

- $A(z) = z^n + a_1 z^{n-1} + \cdots + a_n$
- $B(z) = \qquad b_1 z^{n-1} + \cdots + b_n$

- deg $A(q) >$ deg $B(q)$
- $A(q), B(q)$: no common factors

Simple Design Problem

- The Controller Model:



$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

Or, $u(k) = \frac{T(z)}{R(z)}f(k) - \frac{S(z)}{R(z)}y(k)$

- $G_{\text{ff}}(z) = \frac{T(z)}{R(z)}$
- $G_{\text{fb}}(z) = \frac{S(z)}{R(z)}$
- If causal $\Rightarrow$ deg $R(z)$ $\geq$ deg $S(z)$

$$\Rightarrow \; y(k) = \frac{BT}{AR + BS} f(k)$$

$\Rightarrow$ How to find $T(z), R(z), S(z)$?

Simple Design Problem

- Closed-Loop Characteristic Polynomial:

$$\Rightarrow \; y(k) = \frac{BT}{AR + BS} f(k)$$

$$A_{cl}(z) = A(z)R(z) + B(z)S(z)$$

- $G(z) = \frac{B(z)}{A(z)}$
- $G_{\text{fb}}(z) = \frac{S(z)}{R(z)}$

$\Rightarrow$ Diophantine equation

- Pole-placement design:
  - Find R(z) & S(z),

    such that Diophantine equation is satisfied!

- Closed-Loop Characteristic Polynomial:

$$\begin{aligned} A_{cl}(z) &= A(z)R(z) + B(z)S(z) \\ &= A_c(z)\ \ A_o(z) \end{aligned}$$

$$\Rightarrow\ A_c(z) = \det(zI - \mathbf{F} + \mathbf{HK})$$

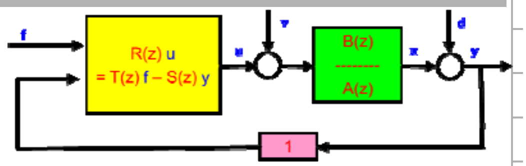
controller polynomial

$$\Rightarrow\ A_o(z) = \det(zI - \mathbf{F} + \mathbf{LC})$$

observer polynomial

- If controllable $\Rightarrow$ any eig $\rightarrow A_c(z)$
- If observable $\Rightarrow$ any eig $\rightarrow A_o(z)$

- Determining T(z) in G_ff(z):



$$Y(z) = \frac{B(z)T(z)}{A_{cl}(z)}F(z)$$

$$= \frac{B(z)T(z)}{A_c(z)A_o(z)}F(z)$$

- Let $T(z) = t_o A_o(z)$

$$\Rightarrow\ Y(z) = \frac{t_o B(z)}{A_c(z)}F(z)$$

- t_o is for desired static gain

Algorithm 5.1: (Simple Pole-Placement Design)

- Data: $\frac{B(z)}{A(z)}$ and $A_{cl}(z)$
  - $A(z), B(z)$ do not have common factors
  - $A_{cl}(z)$ has desired specification
- Step 1: Find $R(z), S(z)$
  - $\deg(S(z)) \leq \deg(R(z))$

  Satisfy

$$A(z)R(z) + B(z)S(z) = A_{cl}(z)$$

Algorithm 5.1: (Simple Pole-Placement Design)

- Step 2: Write $A_{cl}(z) = A_c(z)A_o(z)$
  - $\deg(A_o(z)) \leq \deg(R(z))$

  Select $T(z) = t_o A_o(z)$
  - $t_o = \frac{A_c(1)}{B(1)}$
- Controller Law:

$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

- Response to command signals:

$$A_c(z)y(k) = t_o B(z)f(k)$$

▪ Example 5.1: (Control of a double integrator)

$$\frac{1}{s^2} \quad \Longleftrightarrow \quad \frac{h^2(z+1)}{2(z-1)^2}$$

$$\Rightarrow \quad A(z) = (z-1)^2$$
$$B(z) = \frac{h^2}{2}(z+1)$$

- Diophantine equation

$$A_{cl}(z) = (z-1)^2 R(z) + \frac{h^2}{2}(z+1)S(z)$$

▪ Example 5.1: (Control of a double integrator)

- Try $R(z) = 1, S(z) = s_0$

$\Rightarrow$ This is P controller, b/c $G_{\text{fb}} = \frac{S(z)}{R(z)}$

$$\Rightarrow A_{cl}(z) = (z^2 - 2z + 1) + \frac{s_0 h^2}{2}(z+1)$$

$\Rightarrow$ Impossible for any $A_{cl}(z)$ of 2nd order

Example 5.1: (Control of a double integrator)

- Try $R(z) = z + r_1, S(z) = s_0 z + s_1$

$\Rightarrow$ This is 1st-order controller,

$$\Rightarrow A_{cl}(z) = (z^2 - 2z + 1)(z + r_1) + \frac{h^2}{2}(z+1)(s_0 z + s_1)$$

$$\Rightarrow z^3 + \left(r_1 + \frac{h^2}{2} s_0 - 2 \right) z^2 + \left(1 - 2r_1 + \frac{h^2}{2}(s_0 + s_1) \right) z + r_1 + s_1 \frac{h^2}{2}$$

$\Rightarrow r_1, s_0, s_1$ for any $A_{cl}(z)$ of 3rd order

Example 5.1: (Control of a double integrator)

- Try $R(z) = z + r_1, S(z) = s_0 z + s_1$

$\Rightarrow$ If $A_{cl}(z) = z^3 + p_1 z^2 + p_2 z + p_3$

$$\Rightarrow \begin{aligned} r_1 + \frac{h^2}{2} s_0 &= p_1 + 2 \\ -2r_1 + \frac{h^2}{2}(s_0 + s_1) &= p_2 - 1 \\ r_1 + s_1 \frac{h^2}{2} &= p_3 \end{aligned}$$

$$\Rightarrow \begin{aligned} r_1 &= \frac{3 + p_1 + p_2 - p_3}{4} \\ s_0 &= \frac{5 + 3p_1 + p_2 - p_3}{2h^2} \\ s_1 &= -\frac{3 + p_1 - p_2 - 3p_3}{2h^2} \end{aligned}$$

Simple Design Problem

- Example 5.1: (Control of a double integrator)

- $T(z) = t_0 A_o(z)$

  b/c: $A_{cl}(z) = A_c(z) A_o(z)$

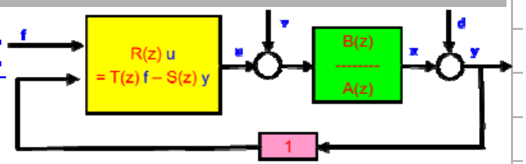
  $\Rightarrow$ $A_c(z)$ of 2nd order

  $A_o(z)$ of 1st order

  And, let $t_0 = \frac{A(1)}{B(1)}$

Realistic Design Problem

- Cancellation of Poles and Zeros:



$$A = A^+ \; A^- \qquad (z-0.1)(z+0.5)(z+2) \quad = \quad (z-0.1)(z+0.5)(z+2)$$
$$B = B^+ \; B^- \qquad (z-0.2)(z-3) \quad = \quad (z-0.2)(z-3)$$

- A^+, B^+ are nice polynomials

  i.e., their roots are inside the unit disc

  i.e., they can be canceled

$$\Rightarrow \quad \begin{aligned} R &= B^+ \; \bar{R} \\ S &= A^+ \; \bar{S} \\ T &= A^+ \; \bar{T} \end{aligned}$$

$$\Rightarrow \frac{BT}{AR + BS}$$

$$\Rightarrow \frac{(B^+B^-)(A^+\bar{T})}{(A^+A^-)(B^+\bar{R}) + (B^+B^-)(A^+\bar{S})}$$

$$\Rightarrow \frac{B^-\bar{T}}{A^-\bar{R} + B^-\bar{S}}$$

- Closed-Loop Characteristic Polynomial:

$A = A^+ \, A^-$
$B = B^+ \, B^-$
$R = B^+ \, \bar{R}$
$S = A^+ \, \bar{S}$
$T = A^+ \, \bar{T}$

- $A_{cl} = AR + BS$

$$= (A^+A^-)(B^+\bar{R}) + (B^+B^-)(A^+\bar{S})$$

$$= A^+B^+\left(A^-\bar{R} + B^-\bar{S}\right)$$

$$= A^+B^+(\bar{A}_{cl})$$

- $b/c: \; A_{cl} = A_c \, A_o \;\; = \left(B^+\bar{A}_c\right)\left(A^+\bar{A}_o\right)$

$$\Rightarrow \; A^-\bar{R} + B^-\bar{S} = \bar{A}_{cl} = \bar{A}_c\bar{A}_o$$

- Minimum-Degree Causal Controller:

$$\Rightarrow \; A^-\bar{R} + B^-\bar{S} = \bar{A}_{cl} = \bar{A}_c\bar{A}_o$$

  - It is unique, if $\deg(\bar{S}) < \deg(A^-)$

- Control Law:

$A = A^+ \, A^-$
$B = B^+ \, B^-$
$R = B^+ \, \bar{R}$
$S = A^+ \, \bar{S}$
$T = A^+ \, \bar{T}$

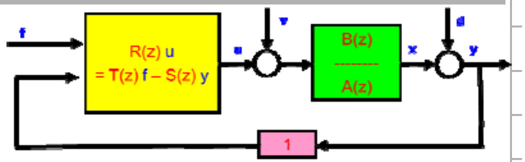
$$Ru = Tf - Sy$$

$$\Rightarrow \; B^+\bar{R}u = A^+\bar{T}f - A^+\bar{S}y$$

$$\Rightarrow u = \frac{A^+}{B^+}\left(\frac{\bar{T}}{\bar{R}}f - \frac{\bar{S}}{\bar{R}}y\right)$$

Realistic Design Problem

- Transfer Functions
  from command to output:



$$y(k) = \frac{BT}{AR + BS} f(k)$$

$$\Rightarrow \frac{BT}{A_{cl}} = \frac{(B^+ B^-)(t_0 A_o)}{A_c A_o}$$

$$= \frac{t_0 B^-}{\bar{A}_c}$$

$A = A^+ \; A^-$
$B = B^+ \; B^-$
$R = B^+ \; \bar{R}$
$S = A^+ \; \bar{S}$
$T = A^+ \; \bar{T}$
$T = t_0 \; A_o$
$A_c = B^+ \; \bar{A}_c$
$A_o = A^+ \; \bar{A}_o$

- If unstable modes are canceled,
  they are uncontrollable or unobservable

Realistic Design Problem

- Disturbance & Command Signal Response:

$$A = A^+ \; A^-$$
$$B = B^+ \; B^-$$

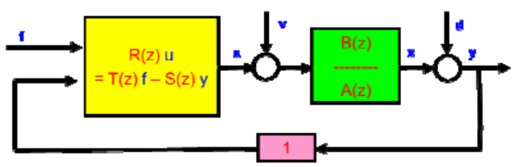
- A^+, B^+ are canceled nice polynomials

- Desired response to command signals:

$$y_m = G_m \; f = \frac{B_m}{A_m} \; f$$

- For perfect model following:

$$B_m = \bar{B}_m B^-$$



Realistic Design Problem

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$

- Let:

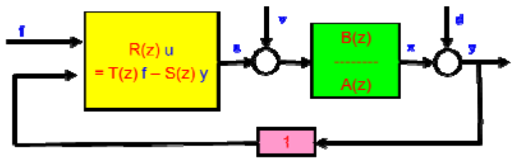
$$R = A_m \, B^+ \, \bar{R}$$
$$S = A_m \, A^+ \, \bar{S}$$
$$T = \bar{B}_m \, \bar{A}_o \, \bar{A}_c \, A^+$$

- Control Law:

$$R(z)u(k) = T(z)f(k) - S(z)y(k)$$

$$\Rightarrow \quad u(k) = \frac{T(z)}{R(z)}f(k) - \frac{S(z)}{R(z)}y(k)$$

$$\Rightarrow \quad u(k) = \frac{A^+}{B^+}\left(\frac{\bar{B}_m\bar{A}_o\bar{A}_c}{A_m\bar{R}}f - \frac{\bar{S}}{\bar{R}}y\right)$$



Realistic Design Problem

$$A = A^+ \, A^-$$
$$B = B^+ \, B^-$$
$$B_m = \bar{B}_m \, B^-$$
$$R = A_m \, B^+ \, \bar{R}$$
$$S = A_m \, A^+ \, \bar{S}$$
$$T = \bar{B}_m \, \bar{A}_o \, \bar{A}_c \, A^+$$

- From the Diophantine equation:

$$\bar{A}_o \, \bar{A}_c = A^- \, \bar{R} + B^- \, \bar{S}$$

- Hence:

$$\Rightarrow \quad \frac{\bar{B}_m}{A_m\bar{R}}(\bar{A}_o\bar{A}_c) = \frac{\bar{B}_m(A^- \, \bar{R} + B^- \, \bar{S})}{A_m \, \bar{R}}$$

$$= \frac{\bar{B}_m \, A^-}{A_m} + \frac{\bar{B}_m \, B^- \, \bar{S}}{A_m \, \bar{R}}$$

$$= \frac{B_m \, A^-}{A_m \, B^-} + \frac{B_m \, \bar{S}}{A_m \, \bar{R}}$$

$$y_m = \frac{B_m}{A_m} f$$

$$\Rightarrow \quad u(k) = \frac{A^+}{B^+}\left(\left(\frac{B_m \, A^-}{A_m \, B^-} + \frac{B_m \, \bar{S}}{A_m \, \bar{R}}\right) f - \frac{\bar{S}}{\bar{R}}y\right)$$

$A = A^+ A^-$
$B = B^+ B^-$
$B_m = \bar{B}_m B^-$
$R = A_m B^+ \bar{R}$
$S = A_m A^+ \bar{S}$
$T = \bar{B}_m \bar{A}_o \bar{A}_c A^+$

- Control Law:

$$\Rightarrow u(k) = \frac{A^+}{B^+}\left(\left(\frac{B_m A^-}{A_m B^-} + \frac{B_m \bar{S}}{A_m \bar{R}}\right) f - \frac{\bar{S}}{\bar{R}} y\right)$$

$$y_m = \frac{B_m}{A_m} f \qquad \to f = \frac{A_m}{B_m} y_m$$

$$\Rightarrow u = \frac{B_m A}{A_m B} f + \frac{A^+ \bar{S}}{B^+ \bar{R}}(y_m - y)$$

- Feedforward:

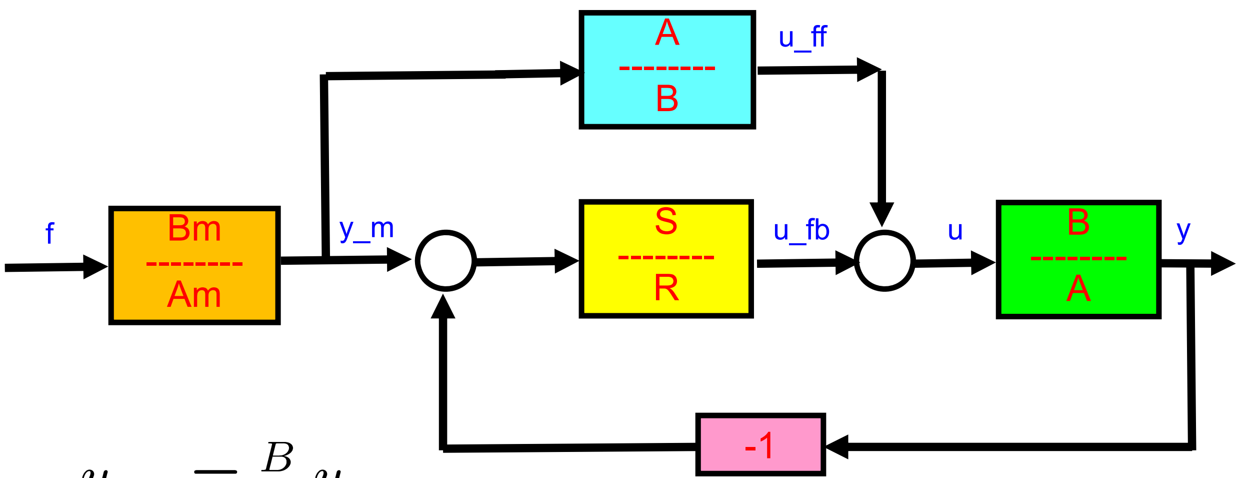
$$G_{\text{ff}} = \frac{B_m A}{A_m B} = \frac{\bar{B}_m A}{A_m B^+}$$

- Feedback from e = (y_m – y):

$$G_{\text{fb}} = \frac{A^+ \bar{S}}{B^+ \bar{R}}$$

- Control Law:

$$\Rightarrow u = \frac{B_m A}{A_m B} f + \frac{A^+ \bar{S}}{B^+ \bar{R}}(y_m - y)$$



$$\Rightarrow \begin{array}{l} y = \dfrac{B}{A} u \\ y_m = \dfrac{B_m}{A_m} f \end{array} \quad \Rightarrow y \to y_m \quad \Rightarrow y = \frac{B_m}{A_m} f$$

- Example 5.5 (motor with zero cancellation):

$$G(z) = \frac{K(z-b)}{(z-1)(z-a)}$$

$$G_m(z) = \frac{z(1+p_1+p_2)}{(z^2+p_1 z+p_2)}$$

$A = A^+ A^-$
$B = B^+ B^-$
$B_m = \bar{B}_m B^-$
$R = A_m B^+ \bar{R}$
$S = A_m A^+ \bar{S}$
$T = \bar{B}_m \bar{A}_o \bar{A}_c A^+$

- Cancel the zero z = b:

$$B^+ = z - b$$

$$B^- = K \qquad \bar{B}_m = B_m / K$$

$$A^+ = 1 \qquad A_o = 1 \qquad \bar{A}_c = A_m$$

- Example 5.5 (motor with zero cancellation):

- Control Law:

$A = A^+ A^-$
$B = B^+ B^-$
$B_m = \bar{B}_m B^-$
$R = A_m B^+ \bar{R}$
$S = A_m A^+ \bar{S}$
$T = \bar{B}_m \bar{A}_o \bar{A}_c A^+$

$$A_{cl} = AR + BS$$

$$\Rightarrow A_m = A\bar{R} + B^- \bar{S}$$

Try $\bar{R} = r_0, \bar{S} = s_0 z + s_1$

$$\Rightarrow \begin{array}{l} r_0 = 0 \\ s_0 = \frac{1+a+p_1}{K} \\ s_1 = -\frac{p_2-a}{K} \end{array}$$

And $T(z) = A_o(z)\bar{B}_m$

$$\Rightarrow \frac{z(1+p_1+p_2)}{K} = t_0 z$$

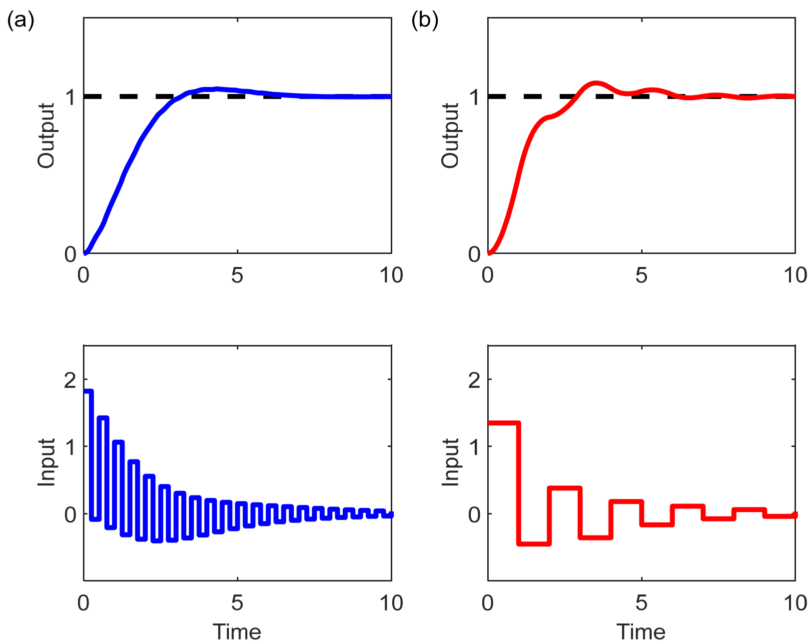
- Example 5.5 (motor with zero cancellation):
- Control Law:

$$A = A^+ A^-$$
$$B = B^+ B^-$$
$$B_m = \bar{B}_m B^-$$
$$R = A_m B^+ \bar{R}$$
$$S = A_m A^+ \bar{S}$$
$$T = \bar{B}_m \bar{A}_o \bar{A}_c A^+$$

$$\Rightarrow u(k) = \frac{A^+}{B^+}\left(\frac{\bar{B}_m \bar{A}_o \bar{A}_c}{A_m \bar{R}} f - \frac{\bar{S}}{\bar{R}} y\right)$$

$$\Rightarrow u(k) = t_0 f(k) - s_0 y(k) - s_1 y(k-1) + b u(k-1)$$

- Example 5.5 (motor with zero cancellation):
- Step response for a motor with pole-placement control
- Specification: ζ = 0.7, ω = 1
- (a) h = 0.25, (b) h = 1.0.



- Example 5.6 (motor w/o zero cancellation):

$$G(z) = \frac{K(z-b)}{(z-1)(z-a)}$$

$$G_m(z) = \frac{(1+p_1+p_2)}{(1-b)} \frac{(z-b)}{(z^2+p_1 z+p_2)}$$

$$\begin{aligned} B^+ &= 1 \\ B^- &= K(z-b) \\ A^+ &= 1 \\ A_c &= A_m \\ A_o &= z \\ \bar{B}_m &= \tfrac{1+p_1+p_2}{K(1-b)} \end{aligned}$$

- Example 5.6 (motor w/o zero cancellation):

- Control Law:

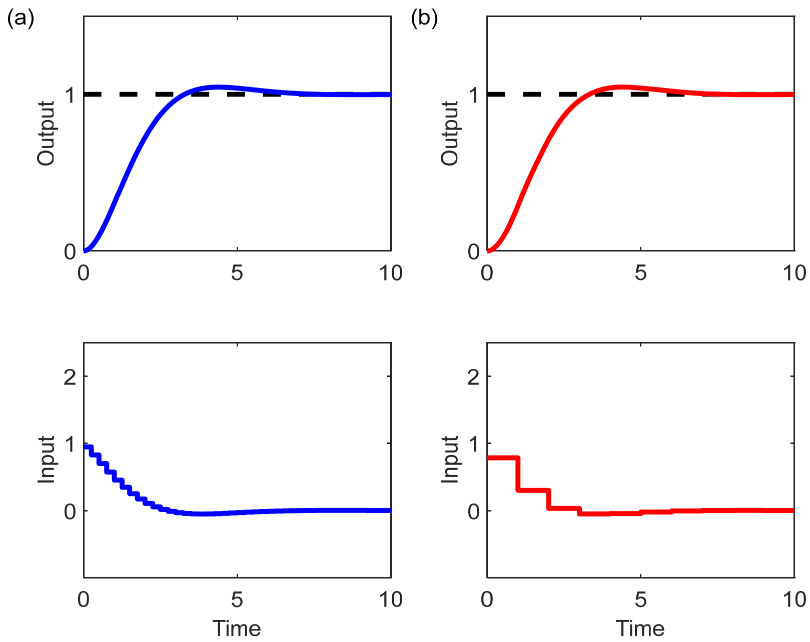
$$AR + B^- S = A_m A_o$$

$$\deg S = 1, \deg R = 1$$

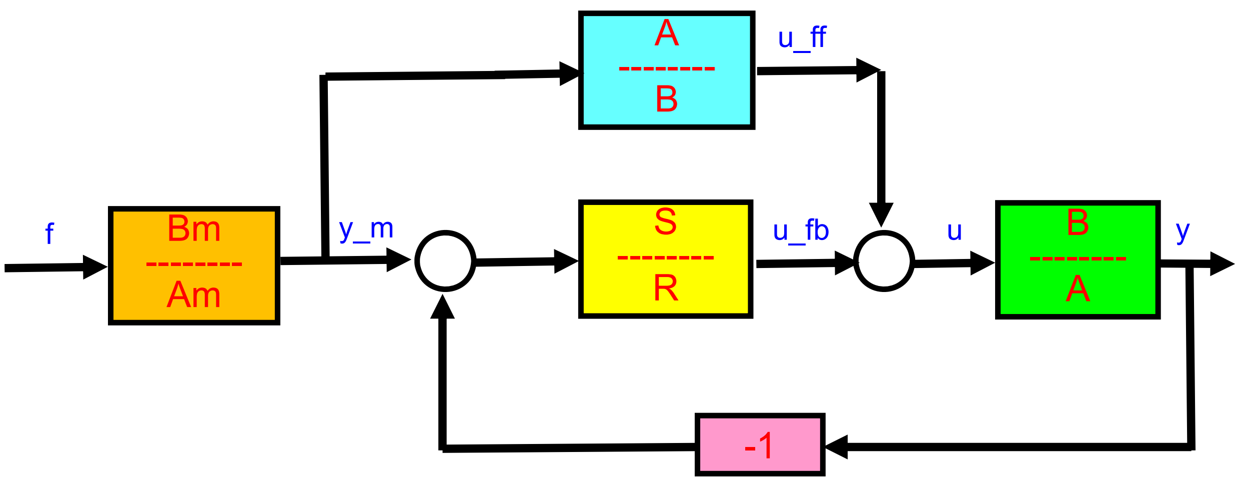
Try $R = z + r_1, S = s_0 z + s_1$

$$u(k) = t_0 u_c(k) - s_0 y(k) - s_1 y(k-1) - r_1 u(k-1)$$

- Example 5.6 (motor wi/0 zero cancellation):
- Step response for a motor with pole-placement control
- Specification: ζ = 0.7, ω = 1
- (a) h = 0.25, (b) h = 1.0.



- Algorithm 5.3 (General Pole-Placement Design):
- Block Diagram:



- Algorithm 5.3 (General Pole-Placement Design):
- Data:
  - Process Model: $\frac{B(z)}{A(z)}$ $\quad A(z), B(z)$: no common factors
  - Closed-Loop Chac. Poly.: $A_{cl}(z)$
  - Desired Response: $\frac{B_m(z)}{A_m(z)}$
  - $R_d(z), S_d(z)$ specify $R(z), S(z)$

- Algorithm 5.3 (General Pole-Placement Design):
- Pole Excess Condition:
  - $\deg A_m(z) - \deg B_m(z) \geq \deg A(z) - \deg B(z)$
  - $\deg A_m(z) + \deg B(z) \geq \deg A(z) + \deg B_m(z)$
- Model Following Condition: $\quad G_{\text{ff}} = \frac{B_m \, A}{A_m \, B}$
  - $B_m = B^- \bar{B}_m$
- Degree Condition:
  - $\deg A_{cl} = 2 \deg A + \deg A_m + \deg R_d + \deg S_d - 1$

- Algorithm 5.3 (General Pole-Placement Design):
- Step 1:
  - $A = A^{+} A^{-}$
  - $B = B^{+} B^{-}$

A^{+}, B^{+}: can be canceled by the controller

- Step 2:
  - Solve the Diophantine eqn:

$$A^{-} R_d \bar{R} + B^{-} S_d \bar{S} = \bar{A}_{cl}$$

- Algorithm 5.3 (General Pole-Placement Design):
- Step 3:
  - the controller:

$$Ru = Tu_c - Sy$$

  - the C.L. Chac. Poly.:

$$A_{cl} = A^{+} B^{+} A_m \bar{A}_{cl}$$

$$R = A_m B^{+} R_d \bar{R}$$
$$S = A_m A^{+} S_d \bar{S}$$
$$T = \bar{B}_m A^{+} \bar{A}_{cl}$$
$$A = A^{+} A^{-}$$
$$B = B^{+} B^{-}$$
$$B_m = \bar{B}_m B^{-}$$
$$A_{cl} = A^{+} B^{+} A_m \bar{A}_{cl}$$

- Algorithm 5.3 (General Pole-Placement Design):
- Calculating the Control Law:

$R = A_m\, B^+\, R_d\, \bar{R}$
$S = A_m\, A^+\, S_d\, \bar{S}$
$T = \bar{B}_m\, A^+\, \bar{A}_{cl}$
$A = A^+\, A^-$
$B = B^+\, B^-$
$B_m = \bar{B}_m\, B^-$
$A_{cl} = A^+\, B^+\, A_m\, \bar{A}_{cl}$

$$A^-\, R_d\, \bar{R} + B^-\, S_d\, \bar{S} = \bar{A}_{cl}$$

- $AR^0 + BS^0 = A_{cl}^0$
- $AU + BV = 0$ — U, V: min-degree sol
- Define $\begin{cases} R = XR^0 + YU \\ S = XS^0 + YV \end{cases}$ — X: stable monic poly.

$$\Rightarrow\ AR + BS = XA_{cl}^0 = A_{cl}$$

- Process Model:

- $G(s) = \frac{K}{s^2}$ $\qquad K = 1$
- $G(z) = \frac{h^2}{2}\,\frac{z+1}{(z-1)^2}$

- Specifications:

- Closed-Loop Polynomial A_c:
  - $A_c(s) \;\Rightarrow\; s^2 + 2\zeta w s + w^2$
  - $$A_c(z) = z^2 - 2ze^{-\zeta w h}\cos\left(wh\sqrt{1-\zeta^2}\right) + e^{-2\zeta w h} = z^2 + a_{c1}z + a_{c2}$$

- Closed-Loop Polynomial A_o:
  - $A_o(s) \;\Rightarrow\; (s+\alpha)^2$
  - $$A_o(z) = (z - e^{-\alpha h})^2 = z^2 + a_{o1}z + a_{o2}$$

- Controller Design:

- The Diophantine Eqn:

$$(z-1)^2R(z) + \frac{h^2}{2}(z+1)S(z) = A_o(z)A_c(z) = A_{cl}(z)$$

- For Integral Action:
  - $R(z)$ should have $(z-1)$

- For Minimum-Degree Solution:
  - $R(z) \;\Rightarrow\;$ 2nd order
  - $S(z) \;\Rightarrow\;$ 2nd order

- Controller Design:
  - $R(z) = (z + r)(z - 1) = z^2 + r_1 z + r_2$
  - $S(z) = s_0 z^2 + s_1 z + s_2$
- Straightforward calculations give:

$$
\begin{aligned}
s_0 &= \frac{A_{cl}(1) - 2A'_{cl}(1) + 2A''_{cl}(1)}{4h^2} \\
s_1 &= \frac{-A_{cl}(1) + 2A'_{cl}(1) - A''_{cl}(1)}{h^2} \\
s_2 &= \frac{7A_{cl}(1) - 6A'_{cl}(1) + 2A''_{cl}(1)}{4h^2} \\
r_1 &= \frac{A_{cl}(-1)}{8} \\
r_2 &= 1 - \frac{A_{cl}(-1)}{8}
\end{aligned}
$$

- And:
  - $T(z) = \frac{A_c(1)A_o(z)}{B(1)} = \frac{(1 + a_{c1} + a_{c2})A_o(z)}{h^2}$

- Nominal Design:
- Closed-Loop Parameters:

$$
\begin{aligned}
\zeta &= 0.707 \\
w &= 0.2 \\
\alpha &= 2 \\
h &= 1
\end{aligned}
$$

$$
e^{-\alpha h} = 0.135
$$

Simulation Study:

Command Signal: Unit Step

Load Disturbance: Step of -0.05 at time 50

Measurement Noise: $0.01 \sin 2t$ at time 100

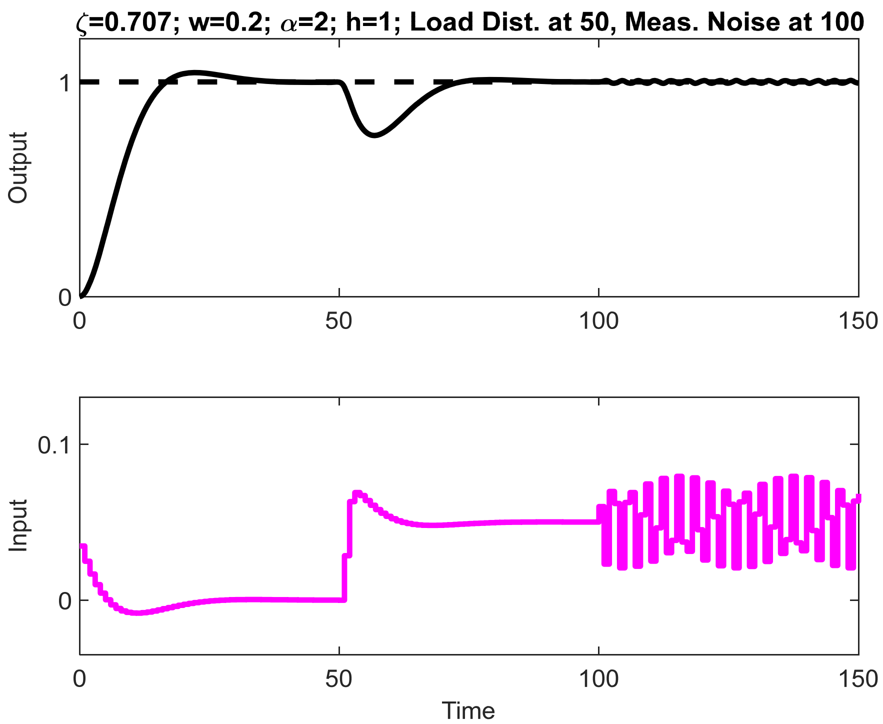
Frequency Folding:

Nyquist Frequency: 0.5 Hz $= \pi$ rad/s

Measurement Noise: 2 rad/s

Simulation Study:

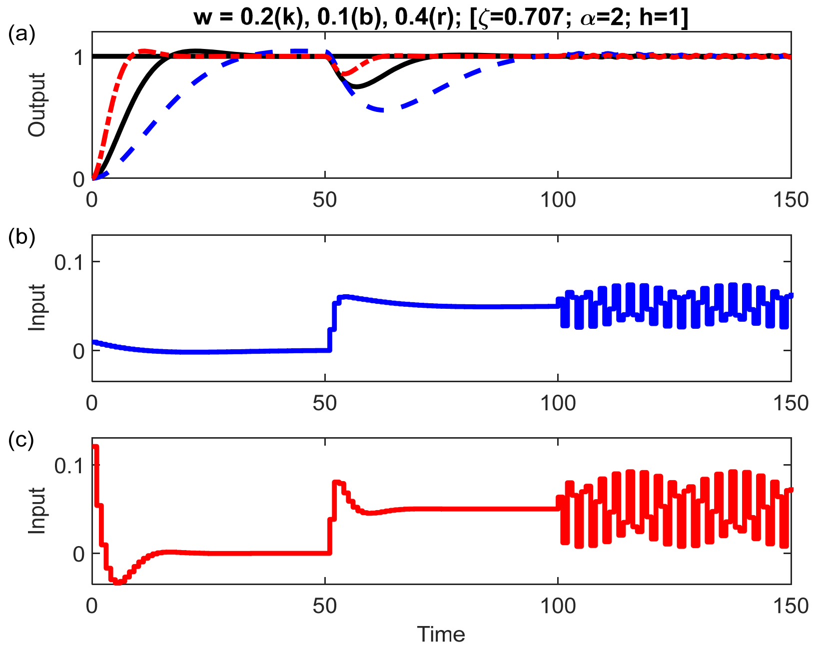
$$\begin{aligned} \zeta &= 0.707 \\ w &= 0.2 \\ \alpha &= 2 \\ h &= 1 \end{aligned}$$



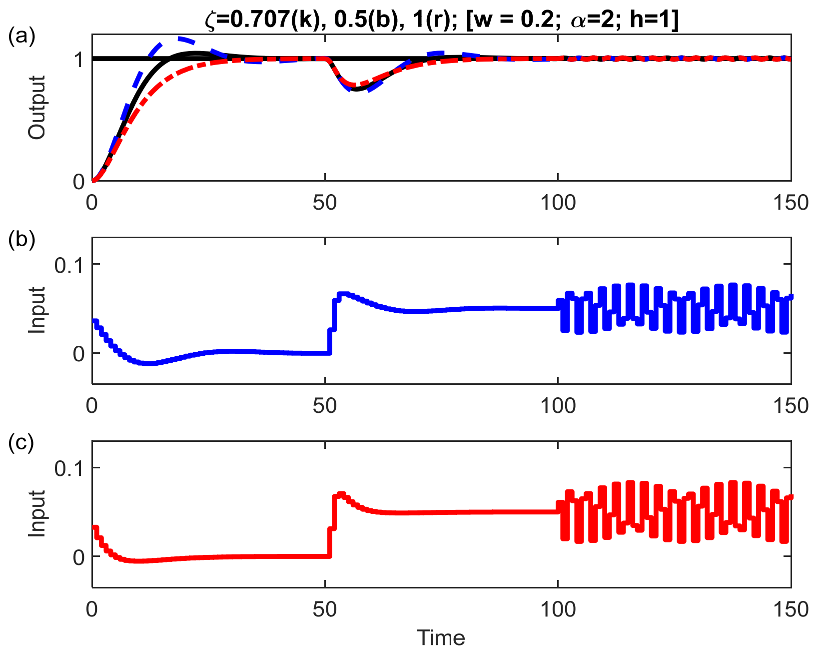
Disturb at 50

Noise at 100

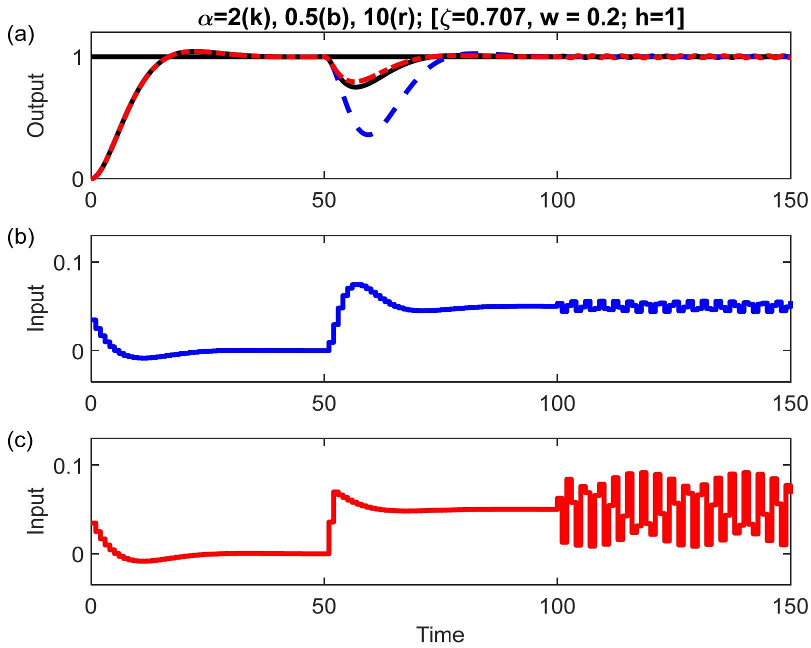
- Changing Natural Frequency ω:
- (a) ω = 0.2, 0.1, 0.4; [ζ = 0.707; α = 2; *h* = 1]
- (b) control signal when ω = 0.1
- (c) control signal when ω = 0.4



- Changing Damping Ratio ζ:
- (a) ζ = 0.707, 0.5, 1.0; [ω = 0.2; α = 2; *h* = 1]
- (b) control signal when ζ = 0.5
- (c) control signal when ζ = 1.0



- Changing Observer Poles α: $z = e^{-\alpha h}$
- (a) α = 2, 0.5, 10; [ζ = 0.707; ω = 0.2; h = 1]
- (b) control signal when α = 0.5
- (c) control signal when α = 10



- Changing Sampling Period h: $z = e^{-\alpha h}$
- (a) h = 1, 2, 0.1; [ζ = 0.707; ω = 0.2; α = 2]
- (b) control signal when h = 2
- (c) control signal when h = 0.1