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$$y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 0$$

$$u(x, y) = \underline{X}(x) \underline{Y}(y)$$

$$\frac{\partial u}{\partial x} = \underline{X}' \underline{Y}$$

$$\frac{\partial u}{\partial y} = \underline{X} \underline{Y}'$$

$$y \underline{X}' \underline{Y} + x \underline{X} \underline{Y}' = 0$$

$$\frac{\underline{X}'}{\underline{X}} = - \frac{\underline{Y}'}{y \underline{Y}} \stackrel{\Delta}{=} -\lambda$$

(a) if $\lambda \neq 0$

$$\begin{cases} \underline{X}' = -\lambda x \underline{X} \\ \underline{Y}' = \lambda y \underline{Y} \end{cases} \Rightarrow \begin{aligned} \underline{X}(x) &= C_1 e^{\lambda \frac{x^2}{2}} \\ \underline{Y}(y) &= C_2 e^{-\lambda \frac{y^2}{2}} \end{aligned}$$

$$\begin{aligned} \therefore u(x, y) &= \underline{X}(x) \underline{Y}(y) = C_1 e^{\lambda \frac{x^2}{2}} C_2 e^{-\lambda \frac{y^2}{2}} \\ &= C_1 C_2 e^{\lambda (x^2 - y^2)/2} \\ &= \boxed{C_3 e^{\lambda (x^2 - y^2)/2}} \quad C_3 \equiv C_1 \cdot C_2 \end{aligned}$$

(b) if $\lambda = 0$

$$\begin{cases} \underline{X}' = 0 \\ \underline{Y}' = 0 \end{cases} \Rightarrow \begin{aligned} \underline{X}(x) &= C_4 \\ \underline{Y}(y) &= C_5 \end{aligned}$$

$$u(x, y) = \underline{X}(x) \underline{Y}(y) = C_4 C_5 \boxed{\equiv C_6}$$

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$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

$$u(x,t) \equiv \underline{X}(x) T(t)$$

$$\frac{\partial u}{\partial x^2} = \underline{X}'' T$$

$$\frac{\partial^2 u}{\partial t^2} = \underline{X} T''$$

$$a^2 \underline{X}'' T = \underline{X} T''$$

$$\Rightarrow \frac{\underline{X}''}{\underline{X}} = \frac{T''}{a^2 T} \stackrel{\Delta}{=} -\lambda$$

$$\Rightarrow \begin{cases} \underline{X}'' = -\lambda \underline{X} \\ T'' = -\lambda a^2 T \end{cases}$$

(a) If $\lambda = 0$

$$\begin{cases} \underline{X}' = 0 \Rightarrow \underline{X}(x) = c_1 x + c_2 \end{cases}$$

$$\begin{cases} T' = 0 \Rightarrow T(t) = c_3 t + c_4 \end{cases}$$

$$u = \underline{X} T = \boxed{(c_1 x + c_2)(c_3 t + c_4)}$$

(b) If $\lambda = -\alpha^2 < 0$

$$\underline{X}'' - \alpha^2 \underline{X} = 0$$

$$\Rightarrow \underline{X} = c_5 \cosh(\alpha x) + c_6 \sinh(\alpha x)$$

$$T'' - \alpha^2 a^2 T = 0$$

$$\Rightarrow T = C_7 \cosh(\alpha at) + C_8 \sinh(\alpha at)$$

$$\Rightarrow \boxed{U = X T = (C_5 \cosh(\alpha x) + C_6 \sinh(\alpha x)) \cdot (C_7 \cosh(\alpha at) + C_8 \sinh(\alpha at))}$$

(c) If $\lambda = \alpha^2 > 0$

$$X'' + \alpha^2 X = 0$$

$$\Rightarrow X = C_9 \cos(\alpha x) + C_{10} \sin(\alpha x)$$

$$T'' + \alpha^2 a^2 T = 0$$

$$\Rightarrow T = C_{11} \cos(\alpha at) + C_{12} \sin(\alpha at)$$

$$\Rightarrow \boxed{U = X T = (C_9 \cos(\alpha x) + C_{10} \sin(\alpha x)) \cdot (C_{11} \cos(\alpha at) + C_{12} \sin(\alpha at))}$$

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$$3 \frac{\partial^2 u}{\partial x^2} + 5 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\Rightarrow A = 3, B = 5, C = 1$$

$$\Rightarrow B^2 - 4AC = 5^2 - 4 \cdot 3 \cdot 1 \\ = 13 > 0$$

\Rightarrow hyperbolic

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$$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

$$\Rightarrow A = \alpha^2, B = 0, C = -1$$

$$\Rightarrow B^2 - 4AC = 0 - \alpha^2(-1) = \alpha^2 > 0$$

\Rightarrow hyperbolic