(5.1) $m\frac{d^2x}{dt^2} + Kx = 0$

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$m = \frac{20\,N}{9.8\,N/kg} = \frac{100}{49}\,kg$

$K = \frac{F}{l} = \frac{20\,N}{49/320\,m} = \frac{6400}{49}\,\frac{N}{m}$

$\Rightarrow \frac{100}{49}\frac{d^2x}{dt^2} + \frac{6400}{49}x = 0$

$\frac{d^2x}{dt^2} + 64x = 0$

$\Rightarrow x(t) = c_1\cos(8t) + c_2\sin(8t)$

$x'(t) = -8c_1\sin(8t) + 8c_2\cos(8t)$

$x(0) = 0.5 = c_1\cdot 1 + c_2\cdot 0 = c_1$

$x'(0) = 0 = -8c_1\cdot 0 + 8c_2\cdot 1 = 8c_2$

$\Rightarrow c_1 = 0.5, \quad c_2 = 0$

$\Rightarrow x(t) = 0.5\cos(8t)$

(a) $x(\frac{\pi}{12}) = \frac{1}{2}\cos(8\cdot\frac{\pi}{12}) = \frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}$

$x(\frac{\pi}{8}) = \frac{1}{2}\cos(8\cdot\frac{\pi}{8}) = \frac{1}{2}(-1) = -\frac{1}{2}$

$x(\frac{9\pi}{32}) = \frac{1}{2}\cos(8\cdot\frac{9}{32}\pi) = \frac{1}{2}\cos(\frac{9}{4}\pi) = \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$

(b) $x'(\frac{3\pi}{16}) = -4\sin(8\cdot\frac{3}{16}\pi) = -4\sin(\frac{3}{2}\pi) = 4\,m/s$

(c) $x(t) = 0.5\cos(8t) = 0 \Rightarrow t = \frac{\pi}{16}(2n+1)$ "+" 向下

$n = 0, 1, 2, \cdots,$

(5.1) $\frac{1}{8}x'' + x' + 2x = 0$

2 $x(0) = -1$ (m), $x'(0) = 8$ (m/s)

$\Rightarrow x'' + 8x' + 16x = 0$

$x \triangleq e^{mt}$

$m^2 e^{mt} + 8m e^{mt} + 16 e^{mt} = 0$

$e^{mt}(m^2 + 8m + 16) = 0$

$e^{mt}(m+4)^2 = 0 \Rightarrow m = -4, -4$

$x(t) = c_1 e^{-4t} + c_2 t e^{-4t}$

$x'(t) = -4c_1 e^{-4t} + c_2 e^{-4t} - 4c_2 t e^{-4t}$

$x(0) = -1 \Rightarrow c_1 + c_2 \cdot 0 = -1 \Rightarrow c_1 = -1$

$x'(0) = 8 \Rightarrow -4c_1 + c_2 = 8 \Rightarrow -4(-1) + c_2 = 8$

$\Rightarrow x(t) = -e^{-4t} + 4te^{-4t}$ $\quad c_2 = 4$

(a) $x(t) = 0 \Rightarrow 0 = -e^{-4t} + 4 \cdot t \cdot e^{-4t} \Rightarrow \boxed{t = \frac{1}{4}}$ (s)

(b) $x'(t) = 0 \Rightarrow 4e^{-4t} + 4e^{-4t} - 16te^{-4t} = 0$

$\Rightarrow 8e^{-4t} - 16te^{-4t} = 0$

$\Rightarrow \boxed{t = \frac{1}{2}}$ (s)

(c) $x(\frac{1}{2}) = -e^{-2} + 4\frac{1}{2}e^{-2} = \boxed{e^{-2}}$ (m)

(5,1) $2x'' + 0 \cdot x' + 32x = 68e^{-2t}\cos 4t$

3 $x(0) = 0, \quad x'(0) = 0$

$x_c(t) \triangleq e^{mt}$

$\Rightarrow AHE = 2(m^2 e^{mt}) + 32(e^{mt}) = 0$

$e^{mt}(2m^2 + 32) = 0$

$e^{mt}(m^2 + 16) = 0$

$m = \pm 4i$

$\Rightarrow x_c(t) = c_1 \cos 4t + c_2 \sin 4t$

$x_p(t) = Ae^{-2t}\cos 4t + Be^{-2t}\sin 4t$

$x_p' = -2Ae^{-2t}\cos 4t - 4Ae^{-2t}\sin 4t$
$\quad -2Be^{-2t}\sin 4t + 4Be^{-2t}\cos 4t$

$= (-2A + 4B)e^{-2t}\cos(4t) + (-4A - 2B)e^{-2t}\sin 4t$

$x_p'' = (4A - 8B)e^{-2t}\cos(4t) - (-8A + 16B)e^{-2t}\sin(4t)$
$\quad + (8A + 4B)e^{-2t}\sin(4t) + (-16A - 8B)e^{-2t}\cos(4t)$

$= (-12A - 16B)e^{-2t}\cos(4t) + (16A - 12B)e^{-2t}\sin(4t)$

$$\Rightarrow \quad 2x_p'' + 32x_p =$$

$$e^{-2t}\cos(4t)\left[-24A - 32B\right] + e^{-2t}\sin(4t)\left[32A - 24B\right]$$

$$+ e^{-2t}\cos(4t)\left[32A\right] + e^{-2t}\sin(4t)\left[32B\right]$$

$$= e^{-2t}\cos(4t)\left[8A - 32B\right] + e^{-2t}\sin(4t)\left[32A + 8B\right]$$

$$= 68\, e^{-2t}\cos(4t)$$

$$\Rightarrow \begin{cases} 68 = 8A - 32B \\ 0 = 32A + 8B \end{cases} \Rightarrow \begin{array}{l} A = \frac{1}{2} \\ B = -2 \end{array}$$

$$x_p(t) = \frac{1}{2} e^{-2t}\cos(4t) - 2e^{-2t}\sin(4t)$$

$$x(t) = x_c + x_p$$

$$= c_1\cos 4t + c_2 \sin 4t$$

$$+ \frac{1}{2} e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$$

$$x(0) = 0 \Rightarrow c_1 + \frac{1}{2} = 0 \Rightarrow c_1 = -\frac{1}{2}$$

$$x'(0) = 0 \Rightarrow 4c_2 + \left(\frac{1}{2}\right)(-2) - 2(4) = 0$$

$$4c_2 - 9 = 0 \Rightarrow c_2 = \frac{9}{4}$$

$$\Rightarrow \boxed{\begin{aligned} x(t) = &-\frac{1}{2}\cos 4t + \frac{9}{4}\sin 4t \\ &+ \frac{1}{2} e^{-2t}\cos 4t - 2e^{-2t}\sin 4t \end{aligned}}$$

(5.1) $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E$

4 $0.05\,q'' + 2q' + \frac{1}{0.01}q = 0$

$\Rightarrow q'' + 40q' + 2000q = 0$

$q \triangleq e^{mt}$

$\Rightarrow m^2e^{mt} + 40me^{mt} + 2000e^{mt} = 0$

$e^{mt}(m^2 + 40m + 2000) = 0$

$\Rightarrow m = \frac{-40 \pm \sqrt{40^2 - 4\times 2000}}{2} = -20 \pm 40i$

$\Rightarrow q(t) = e^{-20t}(c_1\cos 40t + c_2\sin 40t)$

$q(0) = 5, \quad i(0) = 0 \Rightarrow q'(0) = 0$

$\Rightarrow 5 = c_1$

$q'(t) = (-20)e^{-20t}(c_1\cos 40t + c_2\sin 40t)$
$\quad + e^{-20t}(-40c_1\sin 40t + 40c_2\cos 40t)$

$\Rightarrow 0 = (-20)(c_1) + (40c_2)$

$c_2 = \frac{1}{2}c_1 = \frac{1}{2}5 = \frac{5}{2}$

$q(t) = e^{-20t}(5\cos 40t + \frac{5}{2}\sin(40t))$

$q(0.01) = \cdots = \boxed{4.5636\,(C)}$ ※

$q(t) = 0 = e^{-20t}(5\cos(40t) + \frac{5}{2}\sin(40t))$

$\Rightarrow 40t + 1.1071 = \pi \Rightarrow \boxed{t = 0.0509\,(s)}$ ※