

(4.1)

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$$x^2 y'' - xy' + y = 0, \quad y(1) = 3, \quad y'(1) = -1$$

$$y = C_1 x + C_2 x \ln x \quad I = (0, \infty)$$

$$\begin{aligned} y' &= C_1 + C_2 \ln x + C_2 x \frac{1}{x} \\ &= C_1 + C_2 (1 + \ln x) \end{aligned}$$

$$y(1) = C_1 = 3$$

$$y'(1) = C_1 + C_2 = -1 \Rightarrow C_2 = -4$$

$$y = 3x - 4x \ln x, \quad I = (0, \infty)$$

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(4.1) \uparrow

$$x'' + \omega^2 x = 0, \quad I = (-\infty, \infty)$$

$$\begin{aligned}x(0) &= x_0 \\x'(0) &= x_1\end{aligned}$$

$$x(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

$$x(0) = C_1 + 0 = x_0 \Rightarrow C_1 = x_0$$

$$x'(t) = -C_1 \omega \sin(\omega t) + C_2 \omega \cos(\omega t)$$

$$x'(0) = 0 + C_2 \omega = x_1 \Rightarrow C_2 = \frac{x_1}{\omega}$$

$$x(t) = x_0 \cos(\omega t) + \frac{x_1}{\omega} \sin(\omega t)$$

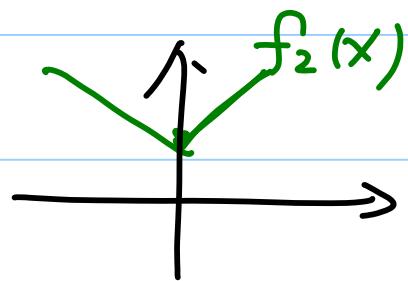
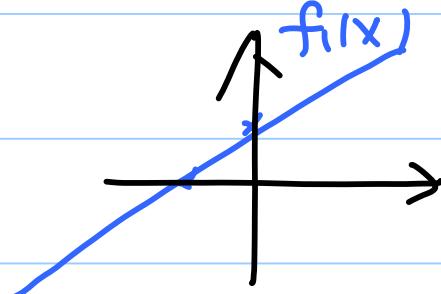
$$I = (-\infty, \infty)$$

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$$f_1(x) = 2+x \quad I = (-\infty, \infty)$$

$$f_2(x) = 2+|x|$$



$$f_1(x) \neq k f_2(x) \quad \forall x \in (-\infty, \infty)$$

\Rightarrow linearly independent

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$$y^{(4)} + y = 0$$

$1, X, \cos x, \sin x, I = (-\infty, \infty)$

$W(1, X, \cos x, \sin x)$

$$= \begin{vmatrix} 1 & X & \cos x & \sin x \\ 0 & 1 & -\sin x & \cos x \\ 0 & 0 & -\cos x & -\sin x \\ 0 & 0 & \sin x & -\cos x \end{vmatrix} = \omega s^2 x + \sin^2 x$$

on $I = (-\infty, \infty)$

∴ $1, X, \sin x, \omega s x$ forms a fundamental set of solutions of the DE.

The general solution is

$$y = C_1 + C_2 X + C_3 \cos x + C_4 \sin x$$

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$$y'' - 4y' + 4y = 2e^{2x} + 4x - 12$$

$$y(x) = \underbrace{c_1 e^{2x} + c_2 x e^{2x}}_{I = (-\infty, \infty)} + \underbrace{x^2 e^{2x} + x - 2}_{y_p}$$

$$y_c = y_{c_1} + y_{c_2}$$

 y_c

$$y_{c_1} = e^{2x} \quad y_{c_2} = x e^{2x}$$

AHE:

$$y_{c_1}' = 2e^{2x}, \quad y_{c_1}'' = 4e^{2x}$$

$$y_{c_1}'' - 4y_{c_1}' + 4y_{c_1} = 4e^{2x} - 4(2e^{2x}) + 4(e^{2x}) = 0$$

$$y_{c_2}' = e^{2x} + 2x e^{2x}, \quad y_{c_2}'' = 2e^{2x} + 2e^{2x} + 4x e^{2x}$$

$$y_{c_2}'' - 4y_{c_2}' + 4y_{c_2}$$

$$= (\cancel{4e^{2x}} + \cancel{4xe^{2x}}) - 4(\cancel{e^{2x}} + \cancel{2x e^{2x}}) + 4(\cancel{xe^{2x}}) = 0$$

$$\underline{y_c = c_1 y_{c_1} + c_2 y_{c_2}} = \boxed{c_1 e^{2x} + c_2 x e^{2x}}$$

 y_p

$$y_p = \cancel{x^2 e^{2x}} + x - 2$$

 $\times 4$

$$y_p' = \cancel{12x e^{2x}} + \cancel{2x^2 e^{2x}} + 1$$

 $\times (-4)$

$$y_p'' = 2e^{2x} + \cancel{4x e^{2x}} + \cancel{4x^2 e^{2x}} + \cancel{4x^3 e^{2x}} \times (1)$$

$$y_p'' - 4y_p' + 4y_p = \dots = \boxed{2e^{2x} + 4x - 12}$$

$$\boxed{y = y_c + y_p = c_1 e^{2x} + c_2 x e^{2x} + x^2 e^{2x} + x - 2}$$

$$(4,1) \quad 36$$

$$(a) \quad y'' + 2y = 10$$

$$y_p = C, \quad y'_p = 0, \quad y''_p = 0$$

$$0 + 2(C) = 10 \Rightarrow C = 5 \Rightarrow \underline{y_p = 5} \quad \cancel{\text{X}}$$

$$(b) \quad y'' + 2y = -4x$$

$$y_p = ax + b, \quad y'_p = a, \quad y''_p = 0$$

$$0 + 2(ax + b) = -4x \Rightarrow a = -2, \quad b = 0$$

$$\underline{y_p = -2x} \quad \cancel{\text{X}}$$

$$\text{or } y_p = ax^2 + bx + c, \quad y'_p = 2ax + b, \quad y''_p = 2a$$

$$(2a) + 2(ax^2 + bx + c) = -4x \Rightarrow a = 0, \quad b = -2, \quad c = 0$$

$$\underline{y_p = -2x} \quad \cancel{\text{X}}$$

$$(c) \quad y'' + 2y = -4x + 10$$

$$y_p = ax + b, \quad y'_p = a, \quad y''_p = 0$$

$$(0) + 2(ax + b) = -4x + 10 \Rightarrow a = -2, \quad b = 5$$

$$\underline{y_p = -2x + 5} \quad \cancel{\text{X}}$$

$$\text{OR } y_p = y_{p(a)} + y_{p(b)} = \underline{-2x + 5} \quad \cancel{\text{X}}$$

$$1d) \quad y'' + 2y = 8x + 5$$

$$y_p = ax + b, \quad y_p' = a, \quad y_p'' = 0$$

$$0 + 2(ax+b) = 8x + 5 \Rightarrow a=4, b=\frac{5}{2}$$

$$y_p = 4x + \frac{5}{2}$$

$$OR \quad y_p = (-2)y_p(b) + \frac{1}{2}y_p(a)$$

$$= (-2)(-2x) + \frac{1}{2}(5)$$

$$= 4x + \frac{5}{2}$$

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