

Fall 2019

微分方程 Differential Equations

Unit 03.3
Modeling with Systems of DEs

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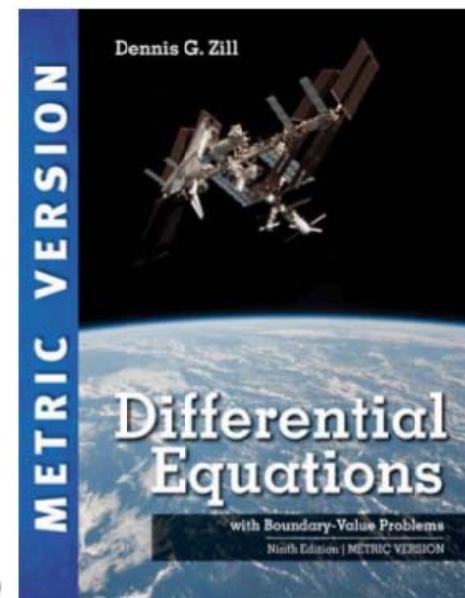
NTU-EE

Sep19 – Jan20

$$\frac{dx}{dt} = g_1(t, x, y)$$

$$\frac{dy}{dt} = g_2(t, x, y)$$

Figures and images used in these lecture notes are adopted from
[Differential Equations with Boundary-Value Problems](#), 9th Ed., D.G. Zill, 2018 (Metric Version)



- 3.1: Linear Models
- 3.2: Nonlinear Models
- **3.3: Modeling with Systems of DEs**

- 2 interacting/competing species: rabits & foxes

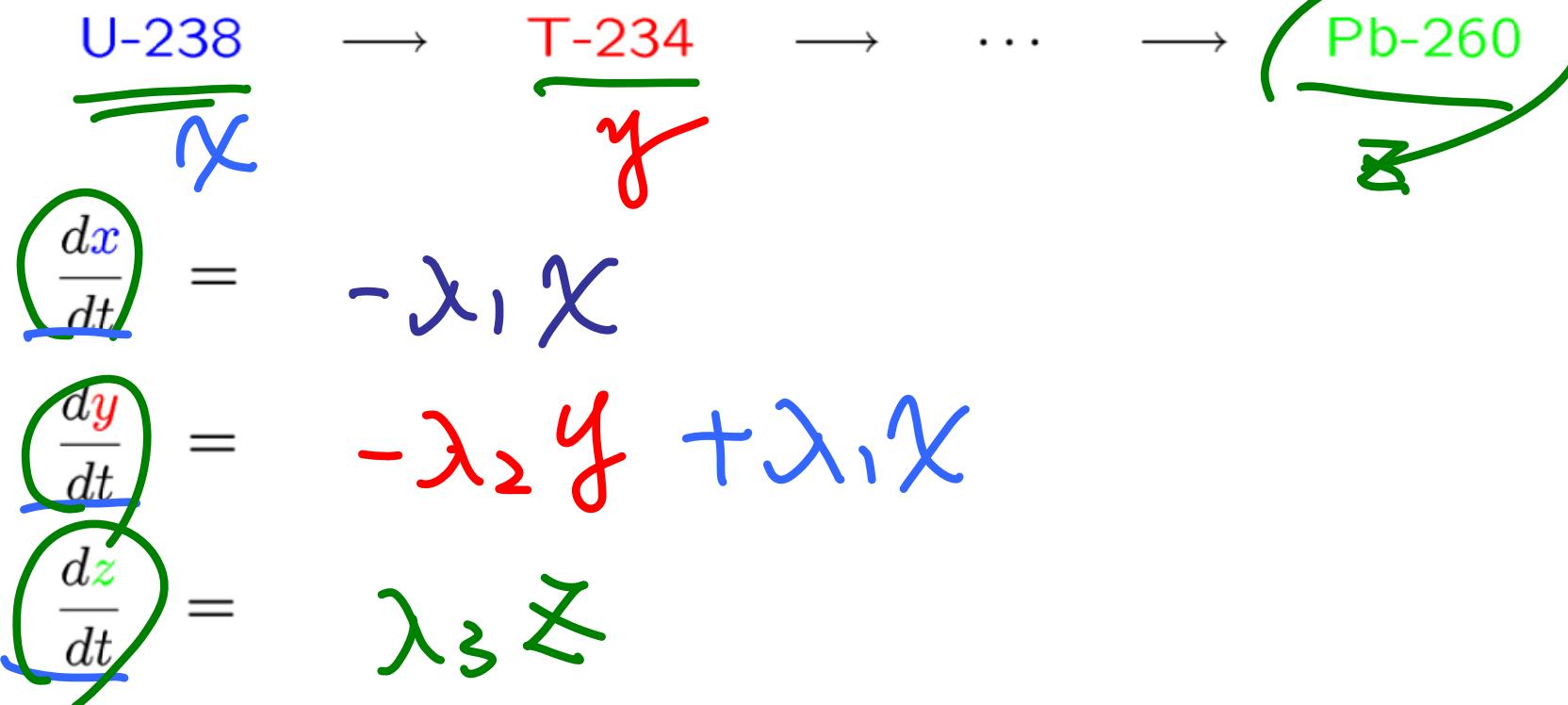
$$\frac{dx}{dt} = g_1(t, x, y)$$

~~\equiv~~ ~~\equiv~~ ~~\equiv~~

$$\frac{dy}{dt} = g_2(t, x, y)$$

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- Radioactive series:



- Predator and Prey

foxes rabbits

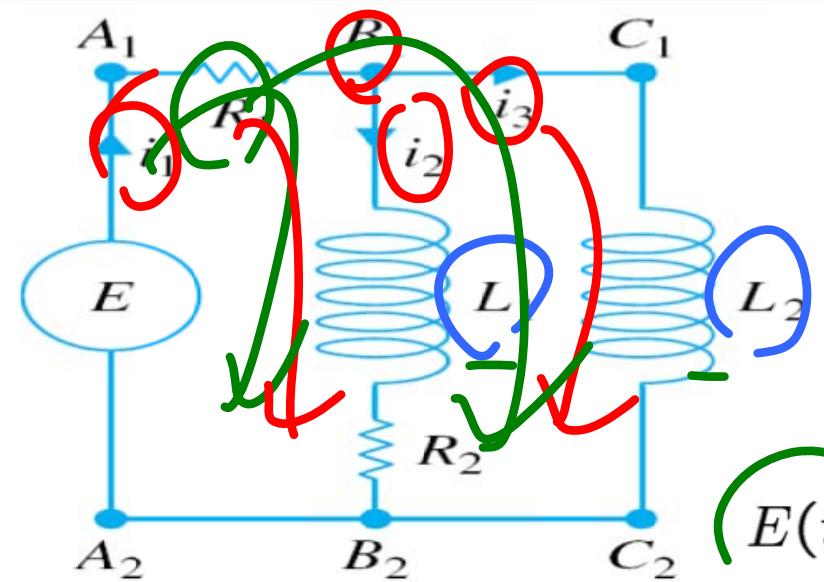
The image shows the Lotka-Volterra predator-prey model equations and their components. The equations are:

$$\frac{dx}{dt} = -\alpha x$$
$$\frac{dy}{dt} = \epsilon y$$

The term $\frac{dx}{dt}$ is enclosed in a green oval. The term $\frac{dy}{dt}$ is enclosed in a green oval. The term $-\alpha x$ is enclosed in a green oval. The term ϵy is enclosed in a green oval.

On the right side, there are two orange ovals. The top oval contains $+ b x y$, where b is green, x is blue, and y is red. The bottom oval contains $- c x y$, where c is green, x is blue, and y is red. Each x and y has a wavy line underneath it.

Lotka-Volterra predator-prey model



$$i_1(t) = i_2(t) + i_3(t)$$

$$E(t) = i_1 R_1 + L_1 \frac{di_2}{dt} + R_2 i_2$$

$$E(t) = i_1 R_1 + L_2 \frac{di_3}{dt}$$

$$L_1 \frac{di_2}{dt} + (R_1 + R_2) i_2 + R_1 i_3 = E(t)$$

$$L_2 \frac{di_3}{dt} + R_2 i_2 + R_1 i_3 = E(t)$$

- Linear Models:

$$\frac{dP(t)}{dt} = k P(t)$$

$$P(0) = P_0$$

- Nonlinear Models:

$$\frac{dP(t)}{dt} = f(t, P)$$

$$P(0) = P_0$$

e.g., $= P(a - bP)$

or, $= k(a - P)(b - P)$

and, $= k(P - a)(P - b)(P - c) \dots$

$\frac{1}{P-a}$, $\frac{1}{P-b}$, $\frac{1}{P-c}$

$$\frac{dP}{dt} = k(P-a)(P-b) = 0 \quad P=a \quad P=b$$

$$\Rightarrow \frac{dP}{(P-a)(P-b)} = k dt$$

$$\Rightarrow \left(\frac{A}{(P-a)} + \frac{B}{(P-b)} \right) dP = k dt$$

$$\Rightarrow A \ln |P-a| + B \ln |P-b| = k t + c_1$$

$$\Rightarrow P = F(t)$$

- 2 interacting/competing species: rabbits & foxes

$$\boxed{\frac{dx}{dt}} = \underline{g_1(t, x, y)}$$

$$\boxed{\frac{dy}{dt}} = \underline{g_2(t, x, y)}$$

- Predator and Prey

foxes



rabbits



$$\frac{dx}{dt} = -a x + b xy$$

$$\frac{dy}{dt} = +c y - e xy$$

Lotka-Volterra predator-prey model